

**Information Amount and Entropy of Black Holes(BH)  $M_b$  and its  
Hawking Quantum Radiation(HQR)  $m_{ss}$**

== The total information amount  $I_m$  of a BH of  $M_b$ ,  $I_m = 4GM_b^2/C$ . The minimum information unit  $I_0$  of any  $m_{ss}$  of any BH included  $M_{bm} = m_p$ ,  $I_0 = h/2 = 1 \text{ bit}$ . The entropy  $S_{Bbm}$  of  $M_{bm} = m_p$ ,  $S_{Bbm} = \pi$ . The total entropy  $S_{BM}$  of a BH of  $M_b$ ,  $S_{BM} = (I_0) I_m = (I_0) \times 4GM_b^2/C = 2\pi R_b^2 C^3/hG =$

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**【Abstract】** Hawking theories about BHs have been the epoch-making significances, they were build on the foundations of quantum mechanics and thermo-mechanics. Hawking proposed any BH being temperature on its Event Horizon (EH)  $R_b$ , and having Hawking quantum radiations(HQR)  $m_{ss}$  to be emitted out. As the result, BHs could lose its energy-matters  $M_b$ , reduce its  $R_b$  and disappear finally in Planck ERA. It is said, any BH must accord with the same general law of life and death as anything in the Universe. <sup>[1]</sup> Although Hawking derived out the famous temperature  $T_b$  formula on  $R_b$  of BHs, i.e,  $T_b M_b = (C^3/4G) \times (h/2)$ , it is the greatest contribution to the theories of BHs. The second famous formula about BHs is Schwarzschild solution to EGTR, i.e,  $GM_b/R_b = C^2/2$ , it is the existent condition of any BH. **However, those two formulas are not enough to solve many important problems about the properties and destiny of BHs, because the amount of  $m_{ss}$  could not be found out by Hawking.** He might be over-wholeheartedly busy to look for  $m_{ss}$  from virtual particles in Dirac's sea as to neglect to find out  $m_{ss}$  from classical theories. **In this article, author can find out the relationships of exact numerable values between HQR  $m_{ss}$  and  $M_b$ ,  $I_0$ ,  $I_m$ ,  $S_B$ ,  $S_{Bm}$  etc.**

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**【Key Words】** . Black Hole(BH); Hawking Quantum Radiation(HQR--  $m_{ss}$ ); Information amount  $I_m$  and  $I_0$  of BHs and  $m_{ss}$ ; Entropy of BHs--  $S_B$ ,  $S_{Bm}$

**【1】** Building the formula between mass  $M_b$  of a BH and its HQR--  $m_{ss}$ . New formulas(1d)  $m_{ss} M_b = hC/8 G = 1.187 \times 10^{-10} g^2$  and (1e)  $M_{bm} m_p = (hC/8 G)^{1/2} = 1.09 \times 10^{-5} g$  are derived as below:

$M_b$  -- mass of a BH,  $T_b$  --temperature on EH(Event Horizon) of a BH,  $m_{ss}$  --mass of a Hawking quantum radiation,  $R_b$  --radius of EH of a BH,  $m_p$  -- Planck particle,  $h$  --Planck constant =  $6.63 \times 10^{-27} g \cdot cm^2/s$ ,  $C$  --light speed =  $3 \times 10^{10} cm/s$ ,  $G$  --gravitational constant =  $6.67 \times 10^{-8} cm^3/s^2 \cdot g$ , Boltzmann constant =  $1.38 \times 10^{-16} g \cdot cm^2/s^2 \cdot k$ ,  $L_p$  ---Planck length,  $T_p$  ---Planck temperature,  $M_{bm}$  --minimum BH in the Universe, its corresponding parameters--  $R_{bm}$ ,  $T_{bm}$ ,

$$T_b M_b = (C^3/4G) \times (h/2) = 10^{27} gk, \quad [1][2] \quad (1a)$$

(1a) is famous Hawking temperature formula,

$m_{ss}$  is Hawking quantum radiation(HQR) on  $R_b$ ; according to formula of energy transformation,

$$m_{ss} = T_b / C^2 \quad [5] \quad (1b)$$

$T_b$  is also valve temperature on  $R_b$ . according to Schwarzschild solution to EGTR,

$$GM_b / R_b = C^2/2 \quad [1][5] \quad (1c)$$

From (1a) and (1b), it is easily got ,

$$m_{ss} M_b = hC/8 G = 1.187 \times 10^{-10} g^2 \quad (1d)$$

(1d) is a new generally effective formula on  $R_b$

of any BHs. Now  $m_{ss} M_b = \text{constant}$ , according to thermo-mechanics, certainly  $T_b = 0$ , thus,  $M_b = 0$ ,  $R_b = 0$  and  $m_{ss} = 0$ . Consequently,  $m_{ss}$ ,  $M_b$  and  $R_b$  are all impossibly equal to zero or infinity. Then,  $m_{ss}$ ,  $R_b$  and  $M_b$  must have its respective limit. From (1d), **According to axiom of any part the whole**, at the limited condition,  $M_b = m_{ss} = M_{bm} = (hC/8 G)^{1/2}$ . **Owing to  $(hC/8 G)^{1/2} = m_p = \text{Planck particle}$ , <sup>[3]</sup> so, (1e) is another new important formula,**

$$M_{bm} = m_p = m_{ss} = (hC/8 G)^{1/2} = 1.09 \times 10^{-5} g \quad (1e)$$

$$\frac{m_{ss} R_b}{L_p} = \frac{h/(4 C)}{(Gh/2 C^3)^{1/2}} = 1.61 \times 10^{-33} cm \quad (1f)$$

$$R_{bm} = L_p \quad [3] \quad (1g)$$

$$T_{bm} = T_p \quad [3] \quad 0.71 \times 10^{32} k \quad (1h)$$

$$R_{bm} m_{ss} = h/(4 C) = 1.0557 \times 10^{-37} cmg \quad (1i)$$

Compton time  $t_c = \text{Schwarzschild } t_s$ ,

$$t_c = t_s = R_{bm}/C = 1.61 \times 10^{-33} / 3 \times 10^{10} = 0.537 \times 10^{-43} s \quad (1j)$$

$$\rho_{bm} \approx 10^{93} g/cm^3 \quad (1k)$$

From  $M_b = 4 R_b^3/3$  and (1c), for any BHs, (1n) is only and always valid,

$$\rho_b R_b^2 = 3C^2/(8 G) = \text{constant} \quad (1n)$$

It can be seen from above formulas,

A. once formulas (1d) and (1e) are build up, all the relationship between  $M_b$  and  $R_b$ ,  $T_b$ ,  $m_s$  and the final destiny of any BHs are well known. They can only finally become  $M_{bm} = m_p = m_{ss}$  and disappear in Planck Era.

B. The relationship between  $M_b$  and  $R_b$ ,  $T_b$ ,  $m_{ss}$  are all the simplest and liner relationship. Thus, BHs are the simplest objects in the Universe.

C. All BHs with the same mass-  $M_b$  can have the same numerable values of  $R_b$ ,  $T_b$ ,  $m_{ss}$ ,  $I_o$ ,  $I_m$ ,  $S_B$ ,  $S_{Bm}$ , etc, but the states and structures inside any BHs can be very great different.

【2】 According to analogy of thermo-dynamics in the theory of BHs, the entropy  $S_B$  of a BH in Einstein gravity theory is as follow:

$$S_B = A/4l^2 \quad [2] = 2\pi^2 R_b^2 C^3/hG \quad (2a)$$

In above (2a), A—surface of a BH-- $M_b$ ,  $A = 4\pi R_b^2$ .  $l$ —Planck length,  $S_B$ -- entropy of a BH,

$$l = (HG/C^3)^{1/2} \quad [3] \quad (2b)$$

(2a) is the famous Bekenstein-Hawking formula. From (1c)  $GM_b/R_b = C^2/2$ , then,

$$S_B = A/4l^2 = 4\pi R_b^2 / (4GH/C^3) = 4\pi R_b^2 \times C^3 / 4GH = \pi R_b R_b C^3 / GH = \pi \times C t_s \times 2GM_b C^3 / GHC^2 = \pi 2t_s \times M_b C^2 / H, \quad t_s \text{--Schwarzschild time, } C t_s = R_b. \text{ So,}$$

$$S_B \times (h/2\pi) = \pi(2t_s \times M_b C^2), \text{ or } S_B = \pi(2\pi/h) \times (2t_s \times M_b C^2) \quad (2c)$$

In above (2c),  $H = (h/2\pi) = I_o$ , According to Heisenberg's Uncertainty Principle, two complementary physical dimensions; such as time and energy, location and momentum, angle and angular momentum, if both have no way to be measured precisely, their product is equal to a constant  $= H = h/2\pi = 1.058 \times 10^{-34} \text{ Js} = 1.058 \times 10^{-27} \text{ g}\cdot\text{cm}^2/\text{s}$ . Then,

$$2t_s \times M_b C^2 = h/2\pi = I_o \quad (2d)$$

$$\Delta E \times \Delta t \quad h/2 = I_o \quad (2e)$$

Doing analogy between (2d) and (2e), (2e) is mathematical formula of Uncertainty Principle.  $2t_s$  is corresponding to  $\Delta t$ , and  $M_b C^2$  is corresponding to  $\Delta E$ . It shows that, BHs emitting  $m_{ss}$  are all quantum.

【3】 The information unit  $I_o$  and entropy  $S_{Bbm}$  of  $M_{bm} = m_{ss} = m_p = (hC/8 G)^{1/2}$

In above 【1】, it was proved that,  $M_{bm} = m_{ss} = m_p = (hC/8 G)^{1/2}$ , and  $R_{bm} = L_p = (Gh/2 C^3)^{1/2} = 1.61 \times 10^{-33} \text{ cm}$ ,  $t_{sbm} = R_{bm}/C = 0.537 \times 10^{-43} \text{ s}$ . Let check up data of  $M_{bm} = m_{ss} = m_p$ .

According to (2c) and (2d):

$$2t_{sbm} \times M_{bm} C^2 = 2 \times 0.537 \times 10^{-43} \text{ s} \times 1.09 \times 10^{-5} \text{ g} \times 9 \times 10^{20} = 1.054 \times 10^{-27} \text{ g}\cdot\text{cm}^2/\text{s}. \quad (3a)$$

$$h/2\pi = 6.63 \times 10^{-27} / 2\pi = 1.06 \times 10^{-27} \text{ g}\cdot\text{cm}^2/\text{s}. \quad (3b)$$

It can be seen, (3a) = (3b), so,

$$2t_{sbm} \times M_{bm} C^2 = h/2\pi = H = I_o \quad (3c)$$

Thus,  $h/2\pi = H = I_o = 1 \text{ bit}$  is the minimum information unit in the Universe, and  $I_o = \text{minimum information unit of } M_{bm} = m_{ss} = m_p \text{ and } m_{ss}$ . Owing to the lifetime of  $M_{bm} = m_p$  only being  $0.537 \times 10^{-43} \text{ s}$  and  $I_o = 1 \text{ bit}$ , the sole way for  $M_{bm} = m_p$  could disintegrate themselves into many smaller energy-particles for prolonging the lifetime of themselves.

If applying natural Planck constant, let  $h/2\pi = 1 = H = I_o$ , so,  $t_{sbm} \times M_{bm} C^2 = 1$ .

From (3c), the entropy  $S_{Bbm}$  of  $M_{bm} = m_p$ , due to  $S_B (h/2\pi) = \pi 2t_s \times M_{bm} C^2$ , so,

$$S_{Bbm} = \pi \text{ and } I_o = 2t_{sbm} \times M_{bm} C^2 = h/2\pi, \quad (3d)$$

It shows, the information unit  $< (I_o = h/2) \text{ of any BH could be impossible to exist in the Universe, and } I_o = h/2\pi = 1 \text{ bit} = \text{the minimum information unit of our Universe.}$

An amateur physicist, Ms. Feng (方舟之女) explained: [On philosophy, existence is just perceived by sensory organs, and perceptibility is just the information to be got and transformed. Anything bringing no information could have no way to be perceived. Thus, information is just existence. Thus,

Information = Existence = energy  $\times$  time .

Correspondingly, Planck constant  $H = \text{energy Uncertainty} \times \text{time Uncertainty}$ .

Why does existence = energy  $\times$  time? It reflects existence has only two essential factors. Any existent thing must have its energy and its living time. A thing of no energy or no living time can be really no existence.] [4]

I think, her concept to information is rather correct and accepted.

【4】 The information amount of any different  $m_{ss}$  radiated by any BH of  $M_b$  is completely the same  $= I_o = h/2\pi$ , and has nothing to do with the amount of  $M_b$  and  $m_{ss}$ .

For getting the general formula of information amount of  $m_{ss}$ , from (1d),  $m_{ss} M_b = hC/8 G = 1.187 \times 10^{-10} \text{ g}^2$ . So,

$$I_o = m_{ss} C^2 \times 2t_s = C^2 hC / (8 GM_b) \times 2R_b / C = C^2 hC / (8 GM_b) \times 2 \times 2GM_b / C^3 = h/2 \quad (4a)$$

Above (4a) shows, the information unit of any  $m_{ss}$  of a BH is always equal to  $I_o = h/2\pi$ , no matter whether  $M_b$  and  $m_{ss}$  is big or small. Thus,  $I_o$  is the minimum, the most basic information unit = 1 bit.

To getting the total information amount  $I_m$  and total entropy  $S_{BM}$  of a BH of  $M_b$ , let  $n_i = M_b/m_{ss}$ .

$$I_m = n_i I_o \quad S_{BM} = n_i = (I_o) I_m, \quad (4b)$$

$$\text{Owing to } M_b = n_i m_{ss}, \quad I_m = I_o M_b / m_{ss}, \quad (4c)$$

From (1d) and (4c) ,  

$$I_m = I_0 M_b / m_{ss} = 4GM_b^2 / C \quad (4d)$$

From (4b) ,  $S_{BM} = ( I_0 / I_m ) I_m = ( I_0 / I_m ) \times 4GM_b^2 / C = 2\pi^2 R_b^2 C^3 / hG = S_B$  ,  
 (4e)

(4e) is completely the same with previous (2a) , it can be proved that, all formulas derived above are perfectly correct and very harmonious..

From energy transformation,  $m_{ss} C^2 = (h/2 ) \times C / \lambda_{ss}$  , so, any wave length  $\lambda_{ss}$  of  $m_{ss}$  is equal to the diameter of BH of  $M_{bu}$ .

$$\lambda_{ss} = 2 t_c C = 2R_b = D_b \quad (4f)$$

**【5】** It has been testified that our Universe is a really cosmic-BH(CBH).<sup>[6]</sup> As a real calculated example, the numerable values of various parameters of our Universe(CBH) as a real BH can be calculated out with all above formulas. The only known number of CBH is its current mass  $M_{bu} = 10^{56}$  g.

As the results: from(1c),  $R_{bu} = 1.5 \times 10^{28}$  cm; from(1a),  $T_{bu}$  on  $R_{bu} = 10^{-30}$  k; from(1d), Hawking quantum radiation on  $R_{bu}$ ,  $m_{ss} = 10^{-66}$  g; total numbers of  $m_{ss}$ ,  $n_{iu} = M_{bu} / m_{ss} = 10^{122}$ ; from(4f), the wave length of  $m_{ss}$ ,  $\lambda_{ss} = 2R_{bu} = 3 \times 10^{28}$  cm; from(4a), information amount of any different  $m_{ss}$ ,  $I = h/2 = 1.06 \times 10^{-27}$  g·cm<sup>2</sup>/s; from(3d), entropy of any different  $m_{ss}$ ,  $S_{Bbm} = \pi$ ; from(4d), the total information amount of CBH,  $I_{mu} = 10^{95}$  g·cm<sup>2</sup>/s; from(4e), the total entropy of CBH,  $S_{BMU} = 10^{122} \pi$ .

Hawking formula of BH's lifetime  $t_b$ ,

$$t_b \approx 10^{-27} M_b^3 \quad (5a)$$

So, if no energy-matters outside to be engulfed, the lifetime  $t_{bu}$  of our Universe will be,  $t_{bu} \approx 10^{134}$  years. It is said, Our Universe due to emitting Hawking Quantum Radiations(HQR) will finally become  $M_{bmu} = m_p$  , and disappear in Planck Era after  $10^{134}$  years. From(5a),

$$-d t_b \approx 3 \times 10^{-27} M_b^2 dM_b \quad (5b)$$

Let  $dM_b = 1 m_{ss}$ , then,  $-d t_b$  is the time needed by emitting 1  $m_{ss}$  of any BH. So, for our Universe(CBH),  $-d t_{bu} \approx 10^{12}$  years. It is said, our current Universe emitting 1  $m_{ss}$ (HQR) needs  $10^{12}$  years  $\approx 100$  times of current age of our Universe.

**【6】** Some very significant and effective conclusions from above calculations:

A; If our CBH having energy-matters outside , which will be thoroughly engulfed. After that, the bigger CBH in future will nonstop emit HQRs( $m_{ss}$ ) to contract its size finally to become  $M_{bmu} = m_p$  , and disappear in Planck Era after  $10^{134}$  years. However, the lifetime of the bigger CBH  $\gg 10^{134}$  years. Then, the destiny of our Universe from the viewpoints of BH's theory can be very great different with the

General Theory of Relativity. Thus,  $\Omega = 1$  got out from GTR can be a false proposition.

B; After new formulas(1d), (1e), (4a), (4b), (4d), (4f), (5b) derived by author, the theory of BHs will go to a rather complete system. The relationships between various parameters are very harmonious. Owing to the states and structures inside BHs having nothing to do with the mass of any BH, they can be very great different.

C; . The wave length of  $m_{ss}$ ,  $\lambda_{ss} = 2R_{bu} = 3 \times 10^{28}$  cm, so,  $m_{ss}$  emitted by our Universe should be the gravitational waves.

D. The information amount  $I_m$  of combinations of two BHs( $M_{b1} + M_{b2}$ ) can not be conservative.

From(4b), owing to  $I_m \propto M_b^2$  ( $I_m$  is directly proportional to  $M_b^2$ ), after combinations of two BHs of  $M_{b1} + M_{b2}$ , their total information amount  $I_{m1+m2}$

$(M_{b1} + M_{b2})^2$ ; but  $I_{m1} \propto M_{b1}^2$ ,  $I_{m2} \propto M_{b2}^2$ ; then,  $I_{m1+m2} > I_{m1} + I_{m2}$ . Similarly. If a BH of  $M_b$ , its original  $I_m$  of  $M_b$ ,  $I_m \propto M_b^2$ , after  $M_b$  emitting  $m_{ss}$  of  $0.5 M_b$ , the rest  $0.5 M_b$  will only have  $0.25 I_m$ , but the lost  $0.5 M_b$  bring away  $0.75 I_m$ . However, the original total information amount  $I_m$  of  $M_b$  does not be increases or decreases any more, and is equal to a constant in the process of emitting  $m_{ss}$ . Obviously, from(4f), owing to  $I_0$  of any  $m_{ss} = h/2$  , the bigger  $M_b$  emit the longer  $\lambda_{ss}$  of  $m_{ss}$ , and bring away the less mass of a  $m_{ss}$ . Therefore, the lost  $0.5 M_b$  emit out more information amount of  $0.75 I_m$ , but the rest  $0.5 M_b$  only keep  $0.25 I_m$ . Entropy is the same conditions with the information above.

E. Now that the information amount of any  $m_{ss}$  emitted by any BHs is always equal to  $h/2 = 1$  bit = minimum information unit, and BH can only emit 1  $m_{ss}$  each time, is the same condition available with all other objects emitting information?

====The End====

(5b)

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