Basis For Symmetric Matrix And Pseudo-Potential Tensor In Data Gravity Interpretation

[1*]ADEWOLE O.O, [2]ALLI S.G .

[1] Department of Physics, University of Ibadan, Ibadan, Nigeria.
[2]Department of Computer, Mathematics & Statistics, The Polytechnic, Ibadan, Nigeria.

ABSTRACT: Gravity method has tremendously pervaded the field of geophysical exploration and thus discussed in this work. The basis for symmetric matrix data gravity interpretation is proposed. A symmetric matrix representation and pseudo-potential tensor were discussed and optimistically form a veritable tool for gravity data interpretation. The fundamental law of gravitational attraction is essential in delineating gravity data interpretation and the gravity effect or anomaly representation is evidently expressed by a finite difference matrix in the discussion. A computational extension can be done based on the symmetric matrix representation with available gravity data.

[ADEWOLE O.O, ALLI S.G. **Basis For Symmetric Matrix And Pseudo-Potential Tensor In Data Gravity Interpretation.** Report and Opinion 2011;3(10):1-3]. (ISSN: 1553-9873). http://www.sciencepub.net/report

Keywords: Symmetric matrix, pseudo-potential, gravity data, gravity effect, gravitational attraction.

Intrduction

Gravity method has been a veritable tool in geophysical exploration for several years. The gravity method is a non-destructible geophysical technique that measures differences in the earth's gravitational field at specific locations. In engineering and environmental sciences, it has found numerous applications among locating voids and karst features , buried stream valleys, water table levels and the determination of soil layer thickness. The gravity method depends on variations produced in the measured gravitational field produced by different earth materials of different bulk densities i.e mass. These variations can be interpreted by a variety of analytical and computer methods to determine the depth, geometry and density that causes the field variations.

The interpretation of Bourguer gravity anomalies ranges from just manually inspecting the grid or profiles for variations in the gravity field to more complex methods that involve separating the gravity anomaly due to an object of interest from some sort of regional gravity field. Several manual and computer techniques including graphical smoothing and polynomial surface fitting are applied in performing these techniques.

The gravity method can be a relatively easy technique to perform and interprete. It requires only simple but precise data for processing. The technique has good depth penetration when compared to ground penetrating radar, high frequencies electromagnetic and D.C resistivity techniques and is not affected by the high conductivity values of near surface clay rich soils. Additionally, lateral boundaries of sub surface features can be easily obtained especially through the measurement of the derivatives of the gravitational field.

Theory

The gravity method involves variation in acceleration due to gravity and thus thye Newton's law of universal gravitation becomes extremely important. The acceleration due to gravity, g can be obtained from Newton's law of motion; where m is a test mass in equation (1)b, hence the Newton's law of universal gravitation is obtained where M_e is the mass of the earth.

$$g = {GMm_o r'}/{R_e^2} \quad \text{.................... (1)}$$

$$F = ma \quad \text{.................................. (1)b}$$

The units for g are cm/s^2 in c.g.s system and are commonly known as Gals, where the average acceleration due to gravity at the earth's surface is 980Gals. Variations in gravity from 10^{-1} to 10^{-3} are considered in most gravity studies and milliGal (mGal) is used by most workers. Even some detailed engineering and environmental work involve microGal (μGal) variations.

Since the gravity method is concerned with determining sub-surface variations in mass distributions, most interpretation techniques involve the solution of the previous equation (1) due to some mass distribution. This can be accomplished by solving for the gravity field due to a generalized mass distribution using an integral equation. In most gravity applications, recent work on the application of the gravity gradient tensor to exploration problem (Mickus et al, 2002, Mikhailov et al, 2007) could involve all three components of the gravity field.

Delineating the contour for the integral:

$$\Delta g = \int_s K(x, x_0, z_0)\, \rho(x, x_0)\, dz_0\, dx_0 \;\ldots\ldots\ldots\ldots\ldots(2)$$

where $\Delta g(x)$ is the gravity effect of a two dimensional mass distribution bounded by the surface, s,x_0 and z_0 are the source coordinates, $K(x,x_0,z_0)$ is the kernel giving the gravity effect of an elementary block of unit density, and $\rho(x_0,z_0)$ is the actual density of the block.

In equation (2), it is tacitly assumed that the density is a continous function of space co-ordinates which renders the evaluation of the integral (2) very difficult. Thus, to perform numerical computations, a discrete variation of density with position is assumed. The integral (2) is then reduced to finite difference matrix:

$$\Delta g = \sum_{i=1}^{m} \sum_{i=1}^{n} kii\, \rho ii \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

which in matrix notation could be expressed viz;

$|\Delta| = |\mathbf{k}||\rho|$ **..(4)**

where the body consists of m rows and n columns.

The value of m is fixed through successive approximation to give a close fit between the calculated and observed gravity values. The limit on n is set by the memory space of the computer.

Further Discussion

Density is a very important parameter in gravity interpretation of data. The sub-surface variations of mass is very important and requires that the density of the material of interest or the density contrast between the material of interest and the surrounding material be known. The density can be determined by various techniques. The best technique is to acquire rock samples within the study area and determine their average density. The density logs obtained from drill logs can also be used but this is not always available. Density can also be estimated from experimental relationships relating compressional seismic velocities obtained from seismic refraction surveys and density (Nafe and Drake, 1957, Birch, 1961). Also, average density values from tables obtained from numerous measurements of rocks, soil and mineral samples (Johnson and Olhoeft, 1984, Telford et al, 1990).

The regional-residual anomaly can be accomplished by many techniques. The simplest technique is the manual method such as the graphical smoothing where a simple smooth regional anomaly subtracted from the observed gravity anomaly to obtain a residual anomaly. The manual technique has the advantage of giving information on the lateral location of the source bodies which can be used select a correct regional anomaly.

Most other regional-residual anomaly separation techniques involve mathematical operations using a computer. The major limitation of the mathematical method is that they do not accurately represent the true residual gravity anomaly due to a specific body. Thus, they should not be used for quantitative interpretation of the sub-surface but only for qualitative interpretation (Ulyrch, 1968). The most common mathematical techniques are surface fitting and weighted averaging. Surface fitting involves a least-square fitting of 2-D polynomial (Beltrac et al,19991, Kim et al, 2008) or 2-D Fourier series (James, 1968) of different orders to the original gridded Bouguer gravity data to represent a regional gravity anomaly map.

The higher the surface order, the greater the fit to the original data, however, high-order surfaces are usually not desired, as they contain part of the anomaly that is desired.

The kernel in equation (2), is the effect of a point due to an elementary two-dimensional rectangular block of unit density and is given by the formula (Heiland, 1946).

$K_{ij} = 2y\ \{\ x'_{ij} \ln(z_{ij}^2 + x_{jj}^2 / z_{ij}^2 + x_{i,i}^2)^{1/2} - (x_{ij} - WS)\ln(z_{ij}^2 + (x_{ij} - WS)^2 / z_{ij-1}^2 + (x_{ii} - WS)^2)^{1/2} + z_{ij}[\tan^{-1} x_{ii}/z_{ij} - \tan^{-1}(x_{ii} - WS)/z_{ij}] - z_{ij-1}[\tan^{-1}/z_{ij-1} - \tan^{-1}(x_{ij} - WS)/z_{ij-1}]\ \}$ .... (5)

where y is the universal gravitation constant, z_{ij} is the z co-ordinate of the far horizontal face, x_{ij} is the x-co-ordinate of the far vertical face of j-th block reckoned from the observation point as origin, and WS is the width of the block.

Gravity satisfies the expression; $\vec{g} = -\nabla u$ and the gravity vector is $\vec{g} = (g_x, g_y, g_z)$ . ...(6)

$$\Gamma = \begin{bmatrix} \frac{\partial^2 u}{\partial x^2} & \frac{\partial^2 u}{\partial x \partial y} & \frac{\partial^2 u}{\partial z \partial x} \\ \frac{\partial^2 u}{\partial y \partial x} & \frac{\partial^2 u}{\partial y^2} & \frac{\partial^2 u}{\partial y \partial z} \\ \frac{\partial^2 u}{\partial z \partial x} & \frac{\partial^2 u}{\partial z \partial y} & \frac{\partial^2 u}{\partial z^2} \end{bmatrix} = \begin{bmatrix} g_{xx} & g_{xy} & g_{xz} \\ g_{xy} & g_{yy} & g_{yz} \\ g_{xz} & g_{yz} & g_{zz} \end{bmatrix}$$

.... (7)

The expression in (7) above represents the matrix tensor whose trace is given by;

Trace$(\Gamma) = \Gamma_{11} + \Gamma_{22} + \Gamma_{33}$ .... (7)b

Equating the trace to zero implies that the gravitational potential U satisfies the Laplace's equation: $\nabla^2 U(r) = 0$.

Since Γ is symmetric, it can be diagonalized as; $V^T \Gamma V = \Lambda$.

However, $V = \lfloor V_1, V_2, V_3 \rfloor$ and $\Lambda = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$ contain the eigen-vectors and eigen -values respectively.

Conclusion

Interpretation of gravity data is extremely important in geophysical exploration and related applications, thus it is expedient to have a rhobust tool for gravity data interpretation. The fundamental basis of symmetric matrix and the pseudo-potential matrix tensor have been elaborated and hopefully would be a veritable analytical tool in gravity data interpretation including those from aeromagnetic survey.

A numerical extension from pertinent theoretical or practical situations can be made and appropriate data can be thoroughly treated.

References

1. Birch FS. The velocity of compressional waves in rocks to 10 kilobars, Journal Geophysical Research. 1961:61:2199-2294.
2. Beltrac JF, Silva JBC, Costa JC. Robust polynomial fitting method for regional gravity estimation: Geophysics. 1991:56:80-89.
3. James WR. Effect of wavelength filtering on the shape of the residual anomaly; Geophysica. 1968:33; 1015-1018.
4. Johnson GR, Olhoek GR. Density of Rocks and minerals: in Carmicheal, R.S.,Ed., Handbook in physical properties of rocks, CRC Press. 1984: 3:1-38.
5. Kim H, Park H. Non negative matrix factorization based on alternating non negativity constrained least squares and active set method, ''SIAM J. Matrix Anal & Appl. 2008: 302(2), 713-30.
6. Mickus, Hinojos, KH. The complete gradient tensor derived from vertical gravity data: A Fourier transform technique: Journal of Applied Geophysics. 2002: 46:159-174.
7. Mikhailov V, Pjot G, Diament M, Price A. Tensor deconvolution: A method to locate equivalent sources from full tensor gravity data: Geophysics. 2007: 27:161-169.
8. Nafe. J, Drake.C. Variation with depth in shallow and deep water marine sediments of porosity, density and the velocities of compressional and shear waves: Geophysics. 1957:22: 523-552.
9. Telfoed, WM, Geldart LP, Sheriff RE. Applied Geophysics, Cambridge University Press. 1980.
10. Ulyrch, TJ. Effect of wavelength filtering on the shape of the residual anomaly, Geophysics, 33, 1968:1015-1018.

10/12/2011