A Deterministic Mathematical Modeling Approach Of Cholera Transmission Analysis In Nigeria

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Abtract: Cholera has long been, and continues to be, a world health issue especially in third world countries which Nigeria is included, and a key indicator of lack of social development. In this essay, a new mathematical model (S, I, R and B) for cholera transmission dynamics is developed and analyzed. Transmission means, global impact of the disease and control mechanisms of cholera disease are brief discussed. We established the existence of equilibrium states and analyze the disease free equilibrium state for stability using linearization theorem. The disease will die out if that rate at which people exposed to contaminated water and food, and the contribution of those infected with cholera to concentration of *v.cholerae* are checked..i.e $T < 0$ and $D > 0$ had given disease- free state to be asymptotically stable.

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1. Introduction

Cholera is an infection of the small intestine that is caused by the bacterium *Vibrio cholerae* 01 and 0139(Riyan 2004 AND WHO 2010). The main symptoms are profuse watery diarrhea and vomiting. Transmission is primarily through consuming contaminated drinking water or food. The severity of the diarrhea and vomiting can lead to rapid dehydration and electrolyte imbalance. Every year there is an estimated 3-5 million cholera cases and 100,000-120,000 deaths due cholera. The short incubation period of two to five days, enhance the potentially explosive pattern of out breaks (Faruque 2008 and WHO 2010). Cholera transmission is closely linked to inadequate environmental management. Typical at-risk areas include peri-urban slums, where basic infrastructure is not available, as well as camps for internally displaced people or refugees, where minimum requirements of clean water and sanitation are not met. The consequences of a disaster – such as disruption of water and sanitation systems, or the displacement of populations to inadequate and overcrowded camps – can increase the risk of cholera transmission should the bacteria be present or introduced. Epidemics have never arisen from dead bodies. Cholera remains a global threat to public health and a key indicator of lack of social development. Recently, the re-emergence of cholera has been noted in parallel with the ever-increasing size of vulnerable populations living in unsanitary conditions (Emch 2008 and WHO2010).

Two serogroups of *v. cholera* - 01 and 0139 - causes out breaks (Alexander 2008). *v. cholera* 01 causes the majority of outbreak, while 0139 -first indentified in Bangladash in 1992 –is confined to South-East Asia. Non-01 and non-0139 v. cholera can cause mild diarrhea but dot not generate epidemics. The bacteria are transmitted via contaminated drinking water or food. Pathogenic *v. cholera* can survive refrigeration and freezing in food supplies. (Reildl et al 2002) The dosage of bacteria required to cause an infection in healthily volunteers via oral administration of living vibrios is greater than ~ 1000 organisms (Hartely 2006). After consuming an antacid, however, cholera development in most volunteers after consumption of only ~100 cholera *vibrios* experiments also show that *vibrios* consumed with food are more likely to cause infection than those from water alone (Finkelstein 1996). Cases tend to be clustered by location as well as season, with most infections occurring in children ages 1-5 years (WHO 2010).

Cholera is severe water-born infectious disease caused by the bacterium *vibrio cholerae*. In 2005, 131,943 cases including 2,272 deaths have notified from 52 countries. The year was marked by a particular significant series of outbreaks in West Africa, which affected 14 countries and accounted for 58% of all cholera cases world-wide (WHO 2006). In the same year Nigeria had 4,477 cases and 174 deaths. There was reported case of cholera in 2008 in Nigeria in which 429 death out of 6,330 cases. More so, 2,304 cases in Niger State in which 114 were

reported death in 2008 (NBS 2009). Recent years have seen a strong trend of cholera outbreak in developing countries, including among others, those in India (2007), Iraq (2008), Congo (2008), Zimbabwe (2008-2009), Haiti (2010), Kenya (2010). In Nigeria, according to UN figure, 1,555 people have died since January and 38,173 cases have been reported. The figure is more than four times the death toll the government reported in August (Guardian. 2010)

2.0 Material and Method

2.1 Model Description: The population is partition into four compartment namely; S(t), I(t), R(t) and B(t) be the number susceptible, infected, recovery individuals and toxigenic *v. cholera* in water at time t respectively. Let Λ be the recruitment rate into the susceptible class, which could include immigrants and/ or new born that are uninfected. We assume that μ_0 and μ_1 are the capital natural human death and cholera death rate. 'N' is the total population and ' a' be the capital exposure rate to contaminated water. λ(B) the probability of any one exposed contaminated water and food to catch cholera, while γ the rate of recovering from cholera. 'n_b' is the net growth of

bacteria and ' e' is the capital contribution of the infected to the population of *v. cholera*.

Model diagram:

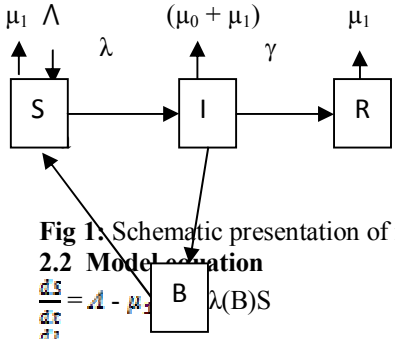

Fig 1: Schematic presentation of model

2.2 Model equation

$\frac{dS}{dt} = \Lambda - \mu_1 S - \lambda(B)S$ (1)

$\frac{dI}{dt} = a\lambda(B)S - (\mu_0 + \mu_1 + \gamma)I$ (2)

$\frac{dR}{dt} = \gamma I - \mu_1 R$ (3)

$\frac{dB}{dt} = Bn_b + e\,I$ (4)

2.3 Equilibrium Solution: We now solve the model equation to obtain the equilibrium states as by Sirajo (2009).At the equilibrium states:

$\frac{dS}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = \frac{dB}{dt} = 0$

Let: S(t) = w, I(t) = x, R(t) = y and B(t) = z

Then the system of equations become:

$\Lambda - \mu_1 w - a\lambda zw = 0$ (5)

$a\lambda zw - (\mu_0 + \mu_1 + \gamma)x = 0$ (6)

$\gamma x - \mu y = 0$ (7)

$zn_b + ex = 0$ (8)

From equation (8), we have

$z = -\frac{ex}{n_b}$

(9)

substitute equation (9) into equation (6)

$a\lambda zw - (\mu_0 + \mu_1 + \gamma)x = 0$

$-\frac{a\lambda exw}{n_b} - (\mu_0 + \mu_1 + \gamma)x = 0$

$[-\frac{a\lambda ew}{n_b} - (\mu_0 + \mu_1 + \gamma)]\, x = 0$

Either x = 0 or $[-\frac{a\lambda ew}{n_b} - (\mu_0 + \mu_1 + \gamma)] = 0$

(10)

$\Rightarrow x = 0$

(11)

equation (9) yields

$z = 0$

(12)

from equation (5)

$\Lambda - \mu_1 w - a\lambda zw = 0$

$W = \frac{\Lambda}{\mu_1 + a\lambda z}$ (13)

But z = 0

$\therefore w = \frac{\Lambda}{\mu_1}$ (14)

From equation (7)

$y = \frac{\gamma x}{\mu_1}$ (15)

But x = 0

$\therefore y = 0$ (16)

From equation (10)

$[-\frac{a\lambda ew}{n_b} - (\mu_0 + \mu_1 + \gamma)] = 0$

$a\lambda ew = -n_b(\mu_0 + \mu_1 + \gamma)$

$w = \frac{-n_b(\mu_0 + \mu_1 + \gamma)}{a\lambda e}$ (17)

from equation (5)

$\Lambda - \mu_1 w - a\lambda zw = 0$

$\Lambda + \mu_1 n_b(\mu_0 + \mu_1 + \gamma) + a\lambda z \frac{n_b(\mu_0 + \mu_1 + \gamma)}{a\lambda e}$

$\Lambda a\lambda e + \mu_1 n_b(\mu_0 + \mu_1 + \gamma) + a\lambda z n_b(\mu_0 + \mu_1 + \gamma) = 0$

$Z = -\frac{[\Lambda a\lambda e + \mu_1 n_b(\mu_0 + \mu_1 + \gamma)]}{a\lambda n_b(\mu_0 + \mu_1 + \gamma)}$

$Z = -\frac{\Lambda a\lambda e + \mu_1 n_b \mu_0 + \mu_1^2 n_b + \mu_1 n_b \gamma}{a\lambda n_b(\mu_0 + \mu_1 + \gamma)}$ (18)

From equation (8)

$zn_b + ex = 0$

$-\frac{\Lambda a\lambda e + \mu_1 n_b \mu_0 + \mu_1^2 n_b + \mu_1 n_b \gamma}{a\lambda n_b(\mu_0 + \mu_1 + \gamma)}]\, n_b + ex = 0$

$-[\Lambda a\lambda e + \mu_1 n_b \mu_0 + \mu_1^2 n_b + \mu_1 n_b \gamma]\, n_b + ex\, a\lambda n_b(\mu_0 + \mu_1 + \gamma) = 0$

$$x = \frac{[\Lambda a\lambda e + \mu_1 n_b \mu_0 + \mu_1^2 n_b + \mu_1 n_b \gamma]n_b}{ea\lambda n_b(\mu_0 + \mu_1 + \gamma)}$$

$$x = \frac{\Lambda a\lambda e + \mu_1 n_b \mu_0 + \mu_1^2 n_b + \mu_1 n_b \gamma}{a\lambda e(\mu_0 + \mu_1 + \gamma)} \quad (19)$$

from equation (15)

$$y = \frac{\gamma x}{\mu_1}$$

$$y = \frac{\gamma[\Lambda a\lambda e + \mu_1 n_b \mu_0 + \mu_1^2 n_b + \mu_1 n_b \gamma]}{a\lambda e(\mu_0 + \mu_1 + \gamma)\mu_1} \quad (20)$$

3.0 The equilibrium states are:

(i) The disease free equilibrium state which is given by:-

$$(w, x, y, z) = \{ \frac{\Lambda}{\mu_1}, 0, 0, 0 \}$$

(ii) The endemic equilibrium state which is given by:-

$$w = \frac{-n_b(\mu_0 + \mu_1 + \gamma)}{a\lambda e}$$

$$x = \frac{\Lambda a\lambda e + \mu_1 n_b \mu_0 + \mu_1^2 n_b + \mu_1 n_b \gamma}{a\lambda e(\mu_0 + \mu_1 + \gamma)}$$

$$y = \frac{\gamma[\Lambda a\lambda e + \mu_1 n_b \mu_0 + \mu_1^2 n_b + \mu_1 n_b \gamma]}{a\lambda e(\mu_0 + \mu_1 + \gamma)\mu_1}$$

$$Z = -\frac{\Lambda a\lambda e + \mu_1 n_b \mu_0 + \mu_1^2 n_b + \mu_1 n_b \gamma}{a\lambda n_b(\mu_0 + \mu_1 + \gamma)}$$

4.0 Results and Discussions

4.1 Stability analysis of disease free equilibrium state.

4.2 The characteristic equation : Recall that the system of equations in this model at eqilibrium state is :

$$\Lambda - \mu_1 w - a\lambda zw = 0$$

$$a\lambda zw - (\mu_0 + \mu_1 + \gamma)x = 0$$

$$\gamma x - \mu y = 0$$

$$zn_b + ex = 0$$

The Jacobian matrix of this system of equation is given:

$$J = \begin{pmatrix} -\mu_1 - a\lambda z & 0 & 0 & -a\lambda w \\ a\lambda z & -(\mu_0 + \mu_1 + \gamma) & 0 & a\lambda w \\ 0 & \gamma & -\mu_1 & 0 \\ 0 & e & o & n_b \end{pmatrix}$$

$$(w, x, y, z) = \frac{\Lambda}{\mu_1}, 0, 0, 0$$

$$J = \begin{pmatrix} -\mu_1 & 0 & 0 & \frac{-a\lambda\Lambda}{\mu_1} \\ 0 & -(\mu_0 + \mu_1 + \gamma) & 0 & \frac{a\lambda\Lambda}{\mu_1} \\ 0 & \gamma & -\mu_1 & 0 \\ 0 & e & o & nb \end{pmatrix}$$

We consider the characteristic equation $\text{Det}(J - \lambda I) = 0$

$$J = \begin{pmatrix} -(\mu_1 + \lambda) & 0 & 0 & \frac{-a\lambda\Lambda}{\mu_1} \\ 0 & -(\mu_0 + \mu_1 + \gamma + \lambda) & 0 & \frac{a\lambda\Lambda}{\mu_1} \\ 0 & \gamma & -(\mu_1 + \lambda) & 0 \\ 0 & e & o & nb - \lambda \end{pmatrix}$$

To obtain the stability of the disease free equilibrium state, we use the principle of linearised stability (or so called decaying exponentials) (Lenka 2007).

Let us look at our Jacobian matrix which was given above we can easily see that the trace of the matrix is given by:-

$$T = -(\mu_1 + \mu_0 + \mu_1 + \gamma + \lambda + \mu_1 + \lambda - n_b + \lambda) < 0$$

and determinant of matrix is also given by

$$D = -2\lambda\mu_1\mu_0 n_b - 2ea\lambda^2\Lambda + 3\mu_1^2\lambda^2 + \mu_1^3\lambda - \mu_1^3 n_b + 3\mu_1\lambda^3 - \mu_1 ea\lambda\Lambda$$
$$-\mu_1{}^2\mu_0 n_b + \mu_1^2\mu_0\lambda - 3\mu_1^2\lambda n_b + 2\mu_1\mu_0\lambda^2 - 3\mu_1\lambda^2 n_b - 2\mu_1\lambda\gamma n_b - \lambda^2\mu_0 n_b - \lambda^2\gamma n_b$$
$$+\frac{a^2\lambda^3\Lambda^2 e}{\mu_1^2} + \frac{a^2\lambda^2\Lambda^2 e}{\mu_1} - \frac{ea\lambda^3\Lambda}{\mu_1} - \lambda^3 n_b + \gamma\lambda^3 + \mu_0\lambda^3 - \mu_1^2\gamma n_b + \mu_1^2\gamma\lambda + 2\mu_1\gamma\lambda^2 + \lambda^4 > 0$$

Therefore the roots of the characteristic equation have negative real parts. According to the principle of linearised stability; the disease free equalibrium state is locally asymptotically stable; Infact, the disease steady state is globally asymptotically stable. Meaning that if the rate at which people exposed to contaminated water and food, and the contribution of those infected with cholera to concentration of *v.cholerae* are checked, disease dies out .i.e $T < 0$ and $D > 0$ had given disease- free state to be asymptotically stable.

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