A Mathematical Model for Assessing the Control of and Eradication strategies for Malaria in a Community

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ABSTRACT: This study discusses the Stability of equilibrium states of a model in an attempt to suggest control of and Eradication strategies for malaria in a community. The model exhibits two equilibria namely; the trivial equilibrium state and diseases free equilibrium state. The Model analysis shows that combination of both treatment and preventive method can eradicate malaria cases.[Abdullahi Mohammed Baba: A Mathematical Model for Assessing the Control of and Eradication strategies for Malaria in a Community.
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> "For centuries Malaria has out ranked warfare as a source of human suffering over the pass generation it has killed millions of human beings and sapped the strength of hundreds of millions more, it continues to be a heavy drag on man's efforts to advance his agriculture and industry" John F. Kennedy (1962) US former President.

INTRODUCTION

The human race has suffered from malaria since earliest times (Malaria 2010) and malaria Protozoa may have a human pathogen for the entire history of the species. (Malaria 2009), Malaria is one of the most common infectious diseases and enormous public health problem. The disease is caused by protozoan parasites of the genus plasmodium. Five species of the Plasmodium Parasite can infect humans; the most serious form of the disease is caused by Plasmodium parasite (Malaria 2009)(which is thought to have evolved in tropical Africa about 2.5million years ago (Malaria 2010)). Malaria caused by Plasmodium Vivax, Plasmodium Ovale and Plasmodium Malariae causes milder disease in humans that is not generally fatal (Malaria 2009) which is thought to have evolved between 2.5million and 3.0 million years ago. A fifth species, Plasmodium Knowlesi causes malaria in macaques but can also infect humans this group of human pathogenic plasmodium Species is usually referred to as malaria parasites. Malaria (2009).

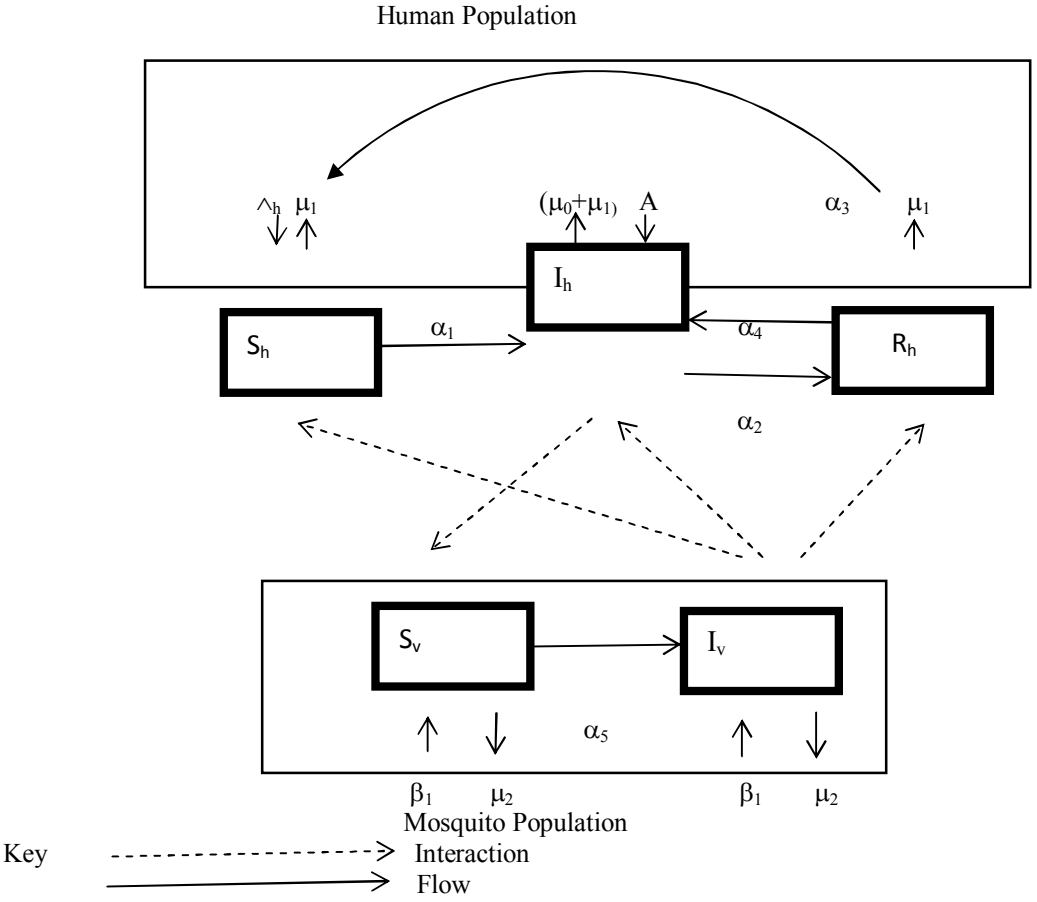
At present the disease affects more than 300 million human and kills 1.5 to 3 million people every years (Ngwa and Shu (2000), malaria (2010) and Malaria (2009), Chitnis (2004), Chittnis, Chushings and Hyman (2006), and Pongsum Pun and tang (2001). An African child dies of the Mosquito-borne disease every 20 seconds, according to some estimates, making it the number one killer of infants less than five years of age on the continent.

Ninety percent of the 1.5 to 3.0 million people who die of malaria per year are in sub Saharan Africa and it is still a problem that it did not attract as much attention as the Human Immune Deficiency Virus/Acquire Immune Deficiency Syndrome (HIV/AIDS). People in poor remote Villages are usually unable to get treatment.

The yearly Mortality from malaria is about 9 times that of HIV/AIDS. Each year, if 1/3 of a million people die of HIV/AIDS, 3 million people die of malaria. Malaria ranked third among the major infectious diseases causing deaths, after pneumococca acute respiratory infections Collectively designated as pneumonia, and tuberculosis (Malaria 2009)

People get malaria by being bitten by an infective female Anopheles Mosquito. Only anopheles Mosquitoes can transmit malaria because they feed on blood meal. About 60 of the 390 Species of anopheles mosquito transmit the malaria parasite. Out of these only a dozen species are important in the transmission of malaria worldwide. Usually just one or two species of anopheles play a role in Malaria transmission in a particular region where the disease prevalent as pointed out by Williams (2006).

Schematic illustration below shows the interaction and flow from one class to another (Figure 1).



Susceptible human, S_h, can be infected when they are bitten by infected mosquitoes. They then progress through the infected, in to recovered, R_h compartments, via intake of anti-malaria drugs. The recovered compartment will have immediate return path to infected I_h when bitten by infected mosquito I_v, those not bitten re-enter the susceptible compartment after losing immunity. Susceptible mosquitoes, S_v can be infected when they bite infected or recovered humans. Birth, death and recruitment rate into and out of the population are shown in the schematic illustration.
wedefine the following parameters as follows.
$\wedge_h$: The recruitment rate
μ_1: Natural mortality of humans
μ_2: Natural mortality of Mosquitoes
μ_0: Infectious mortality of humans
α_1: The probability that a susceptible individual becomes infected by one infected Mosquito per contact unit time.
α_2:The recovery rate of the infected compartment
α_3: The rate of the recovery individual becomes susceptible.
α_4: The probability that a recovery individual become infected by one infected mosquito per contact per unit time
α_5: The probability that a susceptible mosquito becomes infected
β_1: Natural birth rate of mosquitoes
A: Measure of the effectiveness of anti-malaria drug
θ: Proportion of the offspring of the infected mosquito that are infected
(1- θ): Proportion of the offspring of the infected mosquito that are susceptible

Figure 1. Schematic illustration below shows the interaction and flow from one class to another

DEFFECTS OF EARLIER MODELS

A number of Mathematical Models had been developed and published. Significantly of interest are the Mathematical Models formulated by Ngwa and Shu (2000), Chitnis, Chushings and Hyman (2006) and Mohammed Model (2008). The former was a system of seven ordinary differential equations i. e Susceptible-Exposed-Infections-Recovered-Susceptible(SEIRS) pattern for humans and Susceptible-Infectious (SEI) for mosquito a susceptible-Exposed-Infections-Recovered-Susceptible (SEIRS) for Mosquito. Chitnis et al (2006) analysis a similar model with same number of differential equations, Mohammed (2008), analysed a similar model for malaria transmission with five ordinary differential equations i. e Susceptible-Exposed-Infections-Recovered-Susceptible (SEIRS) pattern for human and a Susceptible-Infectious (SI) for Mosquito.

In this paper we extend the Mohammed Model (2008). The new model (Figure 1) divide the human population to three classes; susceptible, S_h, Infected I_h, and Recovered, R_h. We divide the mosquito population into two classes; Susceptible S_v, and Infected I_v. we consider only Female Mosquitoes (because only female Mosquitoes feed on blood). The extension of the Ngwa and Shu model include human immigration, excludes direct human recovery from the infectious to the Susceptible class. The birth of mosquitoes is not all to susceptible but some portion belongs to the infectious class.

The extension of Chitniset al Model (2006) include recruitment rate (the immigration and human birth) should be presented with a parameter. The birth rate of Mosquitoes as in Ngwa and Shu (2000). There is no interaction in both human and mosquito exposed classes, which shows that there is no need for the class. The extension of Mohammed's Model (2008), include recruitment rate as in above a return part from Recovered class, R_h to infected class if there is interaction between the infected Mosquitoes with the Recovered individuals.

Design of the Mathematical Model

Variable and parameters: we use the following notations and/or definitions for the humans population and mosquito population

Susceptible (S_h): the number of individuals who can be infected but have not yet contacted the malaria but may contact it if exposed to any mode of its transmission.

Infected (I_h): The number of individuals who have contacted the malaria and are actively or capable of transmitting it to others.

Recovery (R_h): the number of individuals who are recovered after treatment and are immune to the disease.

Assuming in the given population at time say t. $S_h(t)$, $I_h(t)$ and $R_h(t)$ denote susceptible, infected and recovery, respectively and mosquito population $S_v(t)$, $I_v(t)$.

Susceptible Mosquito (S_v): the number of mosquitoes which can infected if exposed to the mode of transmission.

Infected Mosquito (I_v): the number of mosquito that can transmit malaria.

Assumptions of the Model: the following assumptions are taken into the consideration in the construction of the Model.

- All the people in the community are likely to be infected individually in case of contact.
- All immigrations and new born in the community are uninfected, hence they join the susceptible compartment
- Birth in the mosquito population are grouped into two susceptible and infected mosquitoes
- The community has fixed area size and only the population size will be varying
- The human population size is $N_h = S_h + I_h + R_h$.
- The mosquito population is $N_v = S_v + I_v$
- Compartmentalization of the model: the dynamics of malaria model can be described as in the compartment model fig

Equation of the model: In view of the above assumption and interrelations between the variables and parameters as described in the Compartmentalized model in fig 1, the followingsystem of differential equations were obtained.

$$\frac{dS_h}{dt} = \Lambda_h - \alpha_1 I_v S_h - \mu_1 S_h + \alpha_3 R_h \qquad 1$$

$$\frac{dI_h}{dt} = \alpha_1 I_v S_h \left(\mu_1 + \alpha_2 A + \mu_0\right) I_h + \alpha_4 I_v R_h \quad 2$$

$$\frac{dR_h}{dt} = \alpha_2 I_v S_h - \left(\mu_1 + \alpha_3 + \alpha_4 I_v\right) R_h \qquad 3$$

$$\frac{dS_v}{dt} = S_v\left\{\beta_1 + (1-\theta)\right\} I_v - \mu_2 S_v - \alpha_5 S_v I_h \qquad 4$$

$$\frac{dI_v}{dt} = \theta\beta_1 I_v - \mu_2 I_v + \alpha_5 S_v I_h \qquad 5$$

Analysis of the Model

Normalization

$$\left.\begin{aligned} &X = \frac{S_h}{N_h}, Y = \frac{I_h}{N_h}, Z = \frac{I_v}{N_v} \\ &S_h + I_h + R_h = 1, S_v + I_v = 1 \\ &Rh = (1 - X - Y), Sv = (1 - z) \end{aligned}\right\} \qquad 6$$

Subsequently substituting equation (6) into the system, (1), (2) and (5) respectively gives.

$$X^1 = \Lambda_h + \alpha_3 - \left(\alpha_1 z + \mu_1 + \alpha_3\right) X - \alpha_3 y \qquad 7$$

$$Y' = \left(\alpha_4 - \alpha_4 x - \alpha_4 y + \alpha_1 x\right) z - \left(\mu_1 + \alpha_2 A + \mu_0 + \alpha_4 z\right) y \quad 8$$

$$Z^1 = -\left(\mu_2 - \theta\beta_2\right) z + \alpha_5 y (1 - z) \qquad 9$$

Subsequently, we shall consider the equation (7), (8) and (9) for the purpose of our analysis

Equilibrium States of the Model

At the equilibrium State, let

$$X(t) = x, Y(t) = y, Z(t) = z$$

So that from equation (7), (8) and (9), we have then the system of equation become

$$\Lambda_h + \alpha_3 - \left(\alpha_1 z + \mu_1 + \alpha_3\right) x - \alpha_3 y = 0 \qquad 10$$

$$\left(\alpha_4 - \alpha_4 x - \alpha_4 y + \alpha_1 x\right) z - \left(\mu_1 + \alpha_2 A + \mu_0 + \alpha_4 z\right) y = 0 \quad 11$$

$$-\left(\mu_2 - \phi\beta_2\right) z + \alpha_5 y (1 - z) = 0 \qquad 12$$

We solve this system of equations simultaneously to obtain the following values for (x, y, z) at critical points

a. The existence of the trivial equilibrium point; for as long as the recruitment term Λ_h is not zero; the population will not be extinct. This implies that there is no trivial equilibrium state i.e (x, y, z) ≠ (0, 0, 0)

b. The existence of the disease free equilibrium

State, (x, y, z): At the disease free equilibrium, we have

$$(x, y, z) = \left(\frac{\Lambda_h + \alpha_3}{\mu_1 + \alpha_3}, 0, 0\right)$$

$X = \dfrac{\Lambda_h + \alpha_3}{\mu_1 + \alpha_3}$ is the asymptotic carrying capacity of the population

Stability of the Equilibra

Now to determine the stability of stability of trivial equilibrium state and disease free equilibrium, the following variational matrix is computed corresponding to equilibrium states as used by Lenka (2007) and Akinwande (1995)

$$M=\begin{bmatrix} -(\alpha_1 z+\mu_1+\alpha_3) & -\alpha_3 & -\alpha_1 x \\ (\alpha_1-\alpha_4)z & -(\mu_1+\alpha_2 A+\mu_0+\alpha_4 z) & \alpha_1 x-\alpha_4 x+\alpha_4-\alpha_4 y \\ 0 & \alpha_5(1-z) & -(\mu_2-\phi\beta_2+\alpha_5 y) \end{bmatrix}$$

We can easily see that the trace of the matrix is

$$T=-\left[(\alpha_1 z+\mu_1+\alpha_3)+(\mu_1+\alpha_2 A+\mu_0+\alpha_4 z)+(\mu_2-\phi\beta_2+\alpha_5 y)\right]<0$$

And the determinant of the matrix is

$$\Delta=-(\alpha_1 z+\mu_1+\alpha_3)[(\mu_1+\alpha_2 A+\mu_0+\alpha_4 z)(\mu_2-\phi\beta_1+\alpha_5 y)+(\alpha_1 x-\alpha_4 x+\alpha_4-\alpha_4 y)$$
$$(\alpha_5(1-z))]-\alpha_3[(\alpha_1-\alpha_4)(\mu_1+\alpha_2 A+\mu_0+\alpha_4 z)(\alpha_5(1-z))-$$
$$\alpha_1 x(\alpha_1-\alpha_4)(\alpha_5-\alpha_5 z)]<0$$

We have $T<0$ and $\Delta<0$ is the condition for linearized stability. According to the principle of linearised stability the trivial equilibrium states is locally asymptotically stable i.eR_0<1

$$M=\begin{bmatrix} -(\mu_1+\alpha_3) & -\alpha_3 & -\alpha_1\left(\frac{\Lambda_h+\alpha_3}{\mu_1+\alpha_3}\right) \\ 0 & -(\mu_1+\alpha_2 A+\mu_0) & (\alpha_1-\alpha_4)\left(\frac{\Lambda_h+\alpha_4}{\mu_1+\alpha_3}\right)+\alpha_4 \\ 0 & \alpha_5 & -(\mu_2-\phi\beta_1) \end{bmatrix}$$

We can easily see that the trace of the matrix is

$$T=-\left[(\mu_1+\alpha_3)+(\mu_1+\alpha_2 A+\mu_0)+(\mu_2-\phi\beta_1)\right]<0$$

And that its determinant

$$\Delta=(\mu_1+\alpha_3)\left[(\mu_1+\alpha_2 A+\mu_0)(-\mu_2+\phi\beta_1)-\alpha_5(\alpha_1-\alpha_4)\left(\frac{\Lambda_h+\alpha_4}{\mu_1+\alpha_3}\right)+\alpha_4\right]<0$$

According to the principle of linearized stability the disease free equilibrium is locally and asymptotically unstable i.eR_0>1

Conclusion

In the study we used the analytical method to determine the population equilibrium of a community having malaria, with reference to trivial equilibrium state and disease free equilibrium state. Furthermore we investigated the conditions of existence and stability of the equilibrium. By analysing the model, we found a threshold parameter R_0; It is noted that when $R_0<1$ then the epidemic will die out and when $R_0>1$ the disease will persist in the population and become endemic.

The Model has two non-negative equilibria namely:

$$(x,y,z)=\left(\frac{\Lambda_h+\alpha_3}{\mu_1+\alpha_3},0,0\right)$$

And(x, y, z) $\neq$ (0, 0, 0)

Conclusively, it is observed that the non-trivial equilibrium is unstable this indicates that it is very possible to control, or minimize or eradicate the disease in a community. This can be achieved if the recommendations are strictly adhered to.

Recommendations

- The two main control strategies against the spread of malaria in humans and mosquito reductionstrategies and personal protection against exposure to Mosquitoes. Mosquito reduction strategies include the elimination of mosquito breeding sites (through improved drainage and prevention of standing water). Larvaciding (killing Mosquito Larva before they become adult) and adulticiding (killing adult Mosquitoes Via fogging). Using appropriate biological agents.
- Personal protection, on the other hand entails the use of clothing protection. Insect repellents [containing diethyl-meta-toluamide (DEET)] and avoiding places where mosquitoes bite.
- Increasing the community awareness on the mode of transmission of malaria
- Ministry of Health should sensitize the public through the malaria control programmes and other NGO's who provide health care services about malaria

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