A COMPUTATIONAL ALGORTIHM FOR THE SOLUTION OF NON HOMOGENOUS LINEAR ORDINARY DIFFERENTIAL EQUATION: given f(t)=e^t.

ALLI S.G.

Department of Mathematics, Statistics & Computer, The Polytechnic Ibadan, Nigeria.

ABSTRACT: Differential equations are quite inevitable in describing diverse kinds of biological and physical processes ranging from various forms to the other. The analytic solution of an homogenous differential equation with f(t)=e^t has been delineated in this work with presentation of a computational algorithm. Invariably, a biological or physical process that can be faithfully described by this particular form of linear non homogenous differential equation can be well solved.

[ALLI S.G. **A COMPUTATIONAL ALGORTIHM FOR THE SOLUTION OF NON HOMOGENOUS LINEAR ORDINARY DIFFERENTIAL EQUATION: given f(t)=e^t.** Report and Opinion 2012;4(3):49-51]. (ISSN: 1553-9873). http://www.sciencepub.net/report. 8

Key words: O.D.E, non homogenous o.d.e, algorithm, matrix equation, linearity.

INTRODUCTION

The matrix equation can be formulated to solve a differential equation subject to some initial conditions. Diverse problems in physical and biological applications are built around some forms of differential equations which are expediently required to be solved in one instance or the other.

A mathematical problem describing oscillations and coupled system can be formulated as an eigen- value equation viz an appropriate differential equation and solved explicitly for its eigen - values and eigen - vectors which could give an elaborate or explicit delineation of the physical system in consideration. The non-homogenous case of the linear differential equation for which f(x) is non zero is extremely important in diverse practical or physical applications, for instance a forced oscillation is under the influence of an external forcing frequency subject to the form of the external forcing frequency f(x).

Obviously, speaking vast number of natural and physical processes are describable by an explicit formulation of a rhobust system of differential equations. Appropriately solving the illustrative ordinary differential equation no doubt conspicuously gives a thorough and revealing delineation of the physical or natural system in consideration. A radio frequency and tuning to station of appropriate frequency entailed an external forcing frequency signal source being in resonance with the radio circuit and this system can be well delineated by a formulation of a linear ordinary differential equation for an R-L-C series circuit and appropriate solution is undoubtedly revealing.

Practically, the use of linear ordinary differential equations have enormously grown and even pervading biological applications and frequently encountered in chaotic systems as non linear forms, non linearity, solitons, etc.

DISCUSSION

A differential equation can be described homogenous or non homogenous. Considering a system of linear ordinary differential equation of the form:

$Ly(x) = f(x)$ ………………………………………………………….. (1)

,where $Ly(x) = a_0(x)\frac{d^n y(x)}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y(x)}{dx^{n-1}} + \cdots \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + a_1(x)\frac{dy}{dx} + a_0(x)y.$ ……. (2)

If f(x) is equivalent to zero i.e f(x)=0, equation (2) is said to be homogenous equation.

If f(x) is not equal to zero i.e f(x) $\neq$ 0, then the equation is said to be non-homogenous.

Considering an equation of the form;

$$\frac{d^2x}{dt^2} + x = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

The above equation can be expressed as a matrix equation in matrix notation as follows :

ANALYTIC SOLUTION & COMPUTATIONAL ALGORITHM

A computational algorithm engendered towards solution of a non-homogenous differential equation of the form;

$$\frac{d^2x}{dt^2} - b\frac{dx}{dt} = f(t) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

,which can be solved by applying the following formula;

$$x = e^{[A]t}x_0 + e^{[A]t}\int_0^t e^{-[A]t}\{f(t')\}dt' \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(5)$$

$$x = A\{x_0\} + \{f(t)\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(5)b$$

By setting A= $\begin{bmatrix} 1 & 0 \\ 0 & b \end{bmatrix}$ in equation (..) above, b=2 subject to the initial condition; x(0)=1, $\frac{dx(0)}{dt} = -1$.

,and for a given function f(t)=e^t, the solution of the non-homogenous equation can be explicitly obtained. Thus;

$$[x] = \begin{bmatrix} e^t & 0 \\ 0 & e^{2t} \end{bmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{bmatrix} e^t & 0 \\ 0 & e^{2t} \end{bmatrix}\int_0^t \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{bmatrix}\begin{pmatrix} e^t \\ 0 \end{pmatrix} dt \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(6.1)$$

$$= \begin{bmatrix} e^t \\ -e^{2t} \end{bmatrix} + \begin{bmatrix} e^t & 0 \\ 0 & e^{2t} \end{bmatrix}\int_0^t e^{-t}e^t dt$$

$$= \begin{bmatrix} e^t \\ -e^{2t} \end{bmatrix} + \begin{bmatrix} e^t & 0 \\ 0 & e^{2t} \end{bmatrix}\int_0^t dt$$

$$= \begin{bmatrix} e^t \\ -e^{2t} \end{bmatrix} + \begin{bmatrix} e^t & 0 \\ 0 & e^{2t} \end{bmatrix}\begin{bmatrix} t \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} e^t \\ -e^{2t} \end{bmatrix} + \begin{bmatrix} te^t \\ 0 \end{bmatrix}$$

$$[x] = \begin{bmatrix} (1+t)e^t \\ -e^{2t} \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(6.2)$$

COMPUTATIONAL ALGORITHM

The analytic solution of the above non homogenous equation is presented and a computational algorithm for its numerical computation is presented subsequently:

```
EXTERNAL FUNC 2
DO 11  I = 2, 4, 6, 8
SS=0
T=1.0*I
B=2
CALL QGAUSS (FUNC 2, 0, T, SS)
XX1=Exp(T) +SS*Exp(T)
XX2=-Exp(B*T)
XX3=(1+T)*Exp(T)
XX4=-Exp(B*T)
WRITE (*,*). T, XX1, XX2, XX3, XX4.
11 CONTINUE
STOP
END
FUNCTION FUNC 1 (X)
FUNC 1= Exp(X)
RETURN
END
FUNCTION FUNC 2 (X)
```

```
EXTERNAL FUNC 1
FUNC 2=Exp(-X)*FUNC 1 (X)
RETURN
END
```

CONCLUSION

A computational algorithm which can be implemented based on the analytic solution of a non homogenous equation subject to $f(t)=e^t$ has been presented here. This solution undoubtedly will be extremely useful in obtaining the explicit numerical solution of a related biological or physical process that conform to this non homogenous linear differential equation.

REFERENCES

- Numerical Recipes: William H. Press.
- Advanced Level Mathematics: Tranter C.J, Lambe C.G.
- Mathematics for Engineers and physicists.
- Intermediate Pure Mathematics: Blakey J.

2/2/2012