A COMPUTATIONAL ALGORITHM FOR THE SOLUTION OF NON HOMOGENEOUS LINEAR ORDINARY DIFFERENTIAL EQUATION: given \( f(t) = e^t \).

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ABSTRACT: Differential equations are quite inevitable in describing diverse kinds of biological and physical processes ranging from various forms to the other. The analytic solution of an homogenous differential equation with \( f(t) = e^t \) has been delineated in this work with presentation of a computational algorithm. Invariably, a biological or physical process that can be faithfully described by this particular form of linear non homogenous differential equation can be well solved.

Key words: O.D.E, non homogenous o.d.e, algorithm, matrix equation, linearity.

INTRODUCTION

The matrix equation can be formulated to solve a differential equation subject to some initial conditions. Diverse problems in physical and biological applications are built around some forms of differential equations which are expediently required to be solved in one instance or the other.

A mathematical problem describing oscillations and coupled system can be formulated as an eigen- value equation viz an appropriate differential equation and solved explicitly for its eigen - values and eigen - vectors which could give an elaborate or explicit delineation of the physical system in consideration. The non-homogenous case of the linear differential equation for which \( f(x) \) is non zero is extremely important in diverse practical or physical applications, for instance a forced oscillation is under the influence of an external forcing frequency subject to the form of the external forcing frequency \( f(x) \).

DISCUSSION

A differential equation can be described homogenous or non homogenous. Considering a system of linear ordinary differential equation of the form:

\[
Ly(x) = f(x) \quad \text{..........................................................} (1)
\]

,where \( Ly(x) = a_0(x) \frac{d^n y(x)}{dx^n} + a_{n-1}(x) \frac{d^{n-1} y(x)}{dx^{n-1}} + \cdots + a_1(x) \frac{dy}{dx} + a_0(x) y. \quad \text{.......} (2)
\]

If \( f(x) \) is equivalent to zero i.e \( f(x) = 0 \), equation (2) is said to be homogenous equation.

If \( f(x) \) is not equal to zero i.e \( f(x) \neq 0 \), then the equation is said to be non-homogenous.
Considering an equation of the form;
\[ \frac{d^2x}{dt^2} + x = 0 \] .................................(3)

The above equation can be expressed as a matrix equation in matrix notation as follows:

**ANALYTIC SOLUTION & COMPUTATIONAL ALGORITHM**

A computational algorithm engendered towards solution of a non-homogenous differential equation of the form:
\[ \frac{d^2x}{dt^2} - b \frac{dx}{dt} = f(t) \] .................................(4)

which can be solved by applying the following formula;
\[ x = e^{A[t]}x_0 + e^{A[t]} \int_0^t e^{-A[t]} f(t') dt' \] .................................(5)

\[ x = A[x_0] + \{f(t)\} \] .................................(5b)

By setting \( A=\begin{bmatrix} 1 & 0 \\ 0 & b \end{bmatrix} \) in equation (..) above, \( b=2 \) subject to the initial condition; \( x(0)=1, \frac{dx(0)}{dt} = -1 \).

And for a given function \( f(t)=e^t \), the solution of the non-homogenous equation can be explicitly obtained. Thus;
\[ [x]=\begin{bmatrix} e^t & 0 \\ 0 & e^{2t} \end{bmatrix} \{x_0\} + \begin{bmatrix} e^t & 0 \\ 0 & e^{2t} \end{bmatrix} \int_0^t \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{bmatrix} \{f(t)\} dt \] .................................(6.1)

\[ = \begin{bmatrix} e^t & 0 \\ -e^{2t} & e^{2t} \end{bmatrix} \int_0^t e^{-t} e^t dt \]
\[ = \begin{bmatrix} e^t & 0 \\ -e^{2t} & e^{2t} \end{bmatrix} \int_0^t dt \]
\[ = \begin{bmatrix} 0 & 0 \\ e^t & e^{2t} \end{bmatrix} \]
\[ = \begin{bmatrix} 0 & 0 \\ e^t & e^{2t} \end{bmatrix} \]
\[ [x] = \begin{bmatrix} 1 + t \cdot e^t \\ -e^{2t} \end{bmatrix} \] .................................(6.2)

**COMPUTATIONAL ALGORITHM**

The analytic solution of the above non homogenous equation is presented and a computational algorithm for its numerical computation is presented subsequently:

EXTERNAL FUNC 2
DO 11 1 = 2, 4, 6, 8
SS=0
T=1.0*I
B=2
CALL QGAUSS (FUNC 2, 0, T, SS)
XX1=Exp(T) +SS*Exp(T)
XX2=Exp(B*T)
XX3=(1+T)*Exp(T)
XX4=Exp(B*T)
WRITE (*,*), T, XX1, XX2, XX3, XX4.
11 CONTINUE
STOP
END
FUNCTION FUNC 1 (X)
FUNC 1= Exp(X)
RETURN
END
FUNCTION FUNC 2 (X)
EXTERNAL FUNC 1
FUNC 2=Exp(-X)*FUNC 1 (X)
RETURN
END

CONCLUSION
A computational algorithm which can be implemented based on the analytic solution of a non homogenous equation subject to f(t)=e^t has been presented here. This solution undoubtedly will be extremely useful in obtaining the explicit numerical solution of a related biological or physical process that conform to this non homogenous linear differential equation.

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