

Zen and the Art of Founding Mathematics

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Abstract: The process of attaching a ribbon to each tree in a field will be analysed in order to construct the notion 'number' which apparently involves two components; separating an inside from an outside and linear order. Instead of using a worldview that involves separation, a method based on synthesis will be introduced as a means to reveal a thought pattern which has a fractal-like structure. This method, which is associated with fractal logic, leads up to an alternative worldview and the term 'fractalism' is coined. Fractalism does not appear to contradict science and is contrasted with philosophies such as formalism, constructivism and platonism to make clear that fractalism belongs to a different school of thought. The fractilian 'school' will be described and its difficulties discussed. In a search for the 'foundations of mathematics', Gödel's incompleteness theorems are highlighted and it will be put forward that the Gödel sentence has the characteristic of a fractal, i.e., self-similarity on the level of not being provable. Consequently, fractalism offers a possible solution for the foundational crisis within mathematics. It will be explained how fractalism results in a theory of invariance, although to accept this theory one has to break free from the old-established view that self-evidence suits truth. Ultimately, the unreasonable effectiveness of mathematics in the natural sciences is discussed so as to demonstrate the illusiveness behind this effectiveness due to the creation of a reality-within-reality.

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1. INTRODUCTION

Even when the bamboos are growing thick,
they do not obstruct the running stream.

Anonymous

1.1 A plain introduction to the notion 'number'

Imagine a field of trees in such a way that it is clear what is meant by the concept 'tree' and it is also clear which of these trees are inside this field. A tree found inside this field can be labelled by attaching a ribbon to it. The process of attaching a ribbon to each tree in a field can be formulated with the following procedure:

- Go to a random tree in the field and check that it has a ribbon.
- If the tree does have a ribbon then restart the procedure.
- If the tree does not have a ribbon then take a ribbon and attach it to this tree.
- Check that all trees in the field have a ribbon, if this is not the case then restart the procedure.
- If all trees in the field do have a ribbon then stop the procedure.

Of course, one has to find a way of checking whether all trees in the field have a ribbon in order to know if this recursive procedure can be stopped.

To make things a little bit more interesting, suppose now that each ribbon has a different colour which makes each ribbon unambiguously unique. Moreover, the ribbons are taken in a certain sequence, for example: blue, pink, green, orange, purple, red, violet, yellow, white. The fore mentioned procedure remains the same only now each ribbon is taken from the top of an ordered pile. On top of the pile lies the blue ribbon, then comes the pink ribbon, and so on. If one runs out of colours then one could also use ribbons with stripes drawn on them, for example: I, II, III, IIII, IIIII, IIIIII, IIIIIII, etc. Since more marks appear on each succeeding ribbon it becomes more and more difficult to differentiate between the collection of marks on each ribbon which ultimately may result in confusion. A more advanced approach to (temporarily) overcome this problem would be found in the following coding: I, II, III, IV, V, VI, VII, VIII, IX, X, XI, XII, XIII, etc., yet this roman numbering system is again exhaustible. In order to make the series inexhaustible a more sophisticated set of symbols has to be arranged, for example: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, etc.—it is taken here that for instance '12' is only one symbol although it consists of two characters. All these colours and symbols remain, up to this stage, meaningless and valueless and their apparent occurrence in a sequence only arises from a certain chosen order and/or from a certain chosen set of notation rules.

If the process of attaching a ribbon to each tree in a field results in that the red ribbon is attached to the last tree for a certain field then the process of attaching ribbons will always result in that for this specific field; the red ribbon will be the last ribbon to be attached to a tree, independent of the order in which the trees are chosen to be ribboned (supposing that the colour sequence remains the same and in order, the field/tree situation remains unchanged over the course of time and no error or fraud occurs). This universally acknowledged truth suggests that a field of trees has a general property with respect to a certain procedure because this property does not depend upon the order in which the trees are ribboned and thus, theoretically, is invariant whoever executes the procedure. This leads up to the idea of a certain unique total amount of trees for a specific field. This 'universally' acknowledged truth, however, has been built up here from the following essentials:

1. the concepts 'field', 'tree' and 'ribbon'
2. a specific sequence of colours/symbols and tasks
3. the unambiguous uniqueness of each colour/symbol
4. the process of attaching a ribbon to each tree in a field

○ Comment on 1. A field of trees can be seen as an entity which has a boundary so it can be defined. A single tree can also be seen as an entity with its own boundary so that it may be defined. The concept 'tree' (and 'ribbon') and the concept 'field' are therefore all sustained by *the mental activity of separating an inside from an outside*. The concept 'field of trees', though, occurs on a different conceptual scale from the concept 'tree'. Simultaneous awareness of both different conceptual scales is essential if one wants to be able to fulfil the process of attaching a ribbon to each tree in a field. There also seems to be a certain order in which these concepts come into mind; first the tree and then the field of trees.¹

○ Comment on 2. For the procedure to be successful it is necessary to keep the sequence of colours/symbols, which is materialised in an ordered pile of ribbons, in its specific order. Yet it is also important to realise that the procedure itself can be seen as a sequence of various tasks through which the person who executes the procedure moves.

○ Comment on 3. Each ribbon, just like each tree, is already unique in itself. The colours (or symbols) are only there to make them unambiguously unique so the ribbons can be distinguished. The symbols on the ribbons can be seen as names that label particular

mental entities and their uniqueness results in labels with unvarying names for each of these mental entities.

○ Comment on 4. The process of attaching a ribbon to each tree in a field can only be accomplished if it is executed by a person who is capable of comprehending its procedure and also willing to be subdued to this procedure, that is, voluntarily willing to obey the tasks of the procedure.

Being capable of mentally comprehending the order of the ribbons (or symbols) and also being capable of mentally comprehending the order of the tasks of the procedure will hereafter be denoted by '*maintaining inner linear order*'—a certain state of mental order. And the possibility of comprehending the idea of obeying the tasks of the procedure will be denoted by '*maintaining outer linear order*'—a certain mental state in which one is voluntarily willing to obey orders. Without a person maintaining inner linear order the mentioned procedure cannot be completed, and without this person maintaining outer linear order as well (e.g., not being distracted) this procedure cannot be fulfilled either. The actual comprehension of the tasks of the procedure (i.e., understanding language) is already a mental state which is taken here for granted. Simultaneous awareness of both *different* linear orders is essential if one wants to be able to correctly fulfil the process of attaching a ribbon to each tree in a field.

The idea of a certain unique total amount of trees for a specific field can be represented by a colour or a symbol and one can use the colour or symbol that appears on the ribbon attached to the last tree in the field to indicate this amount. Hence, the process of attaching a ribbon to each tree in a field (i.e., counting²) results in that a specific symbol (i.e., numeral) can be attached to this field as to reveal the total amount of trees in this field which opens the doorway to the idea of natural numbers. What happens here, though, is that the valueless symbol that appeared on the ribbon attached to the last tree in the field metamorphoses into a symbol with an intrinsic value. This metamorphosis that leads to the consciousness of natural numbers, however, can only be conceived whilst perpetuating a certain mental activity and a certain mental state; *the activity of separating an inside from an outside and the state of maintaining linear order* (inner as well as outer).

1.2 The murmur of coherent insight

To attain an alternative view of this, take the following analogy. Most human beings are capable of crossing their eyes by looking at the tip of their nose. Whilst being in this state there is still a vision achieved, though, coherency is lost, depth perception disappears

and objects can be seen twice, each from a different point of view. Clearly, this option of non-overlapping double-vision is not preferred by human beings, yet it is not forbidden and can be practised. The process of attaching a ribbon to each tree in a field, as presented in the introduction, appears to demand a similar kind of non-overlapping double-vision with respect to insight. Double-vision was necessary because *simultaneous* awareness of *both* different conceptual scales is essential if one wants to be able to fulfil the process of attaching a ribbon to each tree in a field. And *simultaneous* awareness of *both* inner and outer linear order is also essential if one wants to be able to correctly fulfil the process of attaching a ribbon to each tree in a field. Yet, both conceptual scales will not merge into one because they are experienced in a linear order; first the tree and then the field of trees. And both linear orders will not merge into one because inner linear order is experienced differently from outer linear order; to master the procedure versus to obey the procedure. So, apparently no coherent vision with respect to insight is attained here and a perception of the natural numbers that involves a *non-overlapping* double-vision seems to be the convention—this convention, which involves a kind of “sustained incoherence”, does not (in principle) necessarily imply the existence of any inconsistency. As it happens, this analogy would suggest that alongside the conventional non-overlapping double-vision there could also exist the possibility of experiencing *overlapping* double-vision as a means to attain a coherent insight. Moreover, the need for an alternative view is underscored by three warning signs of a possible collapse of contemporary mathematics (Nelson, 2006). The following deliberation is a journey away from the conventional insight and its imperium of natural numbers towards the strange idea of this unknown coherent insight and its corresponding worldview. So, unfasten the safety belt, hold fast to nothing and enjoy the ride.

2. SEPARATION

Not knowing most closely approaches the Truth.
Lo-han Kuei-ch'en

2.1 The worldview of René Descartes

Under consideration will be the statement of the French philosopher, mathematician and scientist René Descartes: "*cogito, ergo sum*". The analysis here will be applied to the english translation of Descartes' statement: "I think, therefore I am"—also translated as "I think, therefore I exist". As will become clear, this analysis cannot be applied to the expression in latin, but then again, Descartes' statement was originally made in french: "je pense, donc je suis". As it turns out, some languages are better suited for expressing the process of thinking than others, but the appreciation of this fact

belongs to a different scientific field (e.g., the syntax of language) and is beyond the scope of this paper.

● **A self-fulfilling ego.** Descartes' approach to the mind-bubble 'I think' involved *separation* and *linear order* because he separated the clear thought "I think" from his thinking process, dissected this thought into distinct concepts (i.e., 'I' and 'think'), kept both concepts in a linear order (i.e., first 'I' and then 'think') and then conducted his thoughts in order (i.e., I think, therefore I am). The being (or existence) of the concept 'I' was justified by Descartes because independent of whether the thinking that is being done is correct or incorrect at the very least there must *be* an I who does the thinking. Nonetheless, the conclusion "I am" is *not* based solely on the premises "I think" but also on the postulates of separation and linear order which had to be incorporated in the worldview of this I. The Cartesian ego (i.e., Descartes' I) therefore maintains a worldview which is governed by separation and linear order. By means of this worldview Descartes managed to implement a self-analysis capable of *fulfilling* the I with the essence being (or existence). This crystallisation process that gives essence to the Cartesian ego, however, incorporates a well-known problematic aspect of our thought processes; the tendency to separate the observer (i.e., the thinker) from the observed phenomenon (i.e., the thinking process).

2.2 An alternative view

The view that the thinker cannot be separated from the thinking process also suggests that the thoughts of the thinker cannot be separated from the thinking process and this would indicate that thoughts reveal only part of a total which involves the thinking process as well. Consequently, as soon as the thinker tries to encapsulate this total into a thought then this new thought will again be part of a bigger whole. This alternative view would require a different approach, specifically, the approach of synthesising thoughts and thinking instead of analysing them. The mind-bubble 'I think' would in this case not be separated from the thinking process and dissected but instead left as a whole in order to realise that this mind-bubble is already a thought which is part of a bigger whole. So, if one 'zooms out' in order to get the bigger picture (i.e., one synthesises) then there will appear a more profound realisation which can be expressed by the statement: I think: "I think". However, this deeper realisation also manifests itself as a thought which again is part of a bigger whole and therefore can be zoomed out even further: I think: "I think: "I think". Obviously, the process of zooming out may be continued endlessly and the ever deepening reflection that slowly becomes revealed appears to have the characteristic of a fractal, viz., a certain pattern in which a similar pattern is

nestled.

The word 'fractal' was coined by the Polish French American mathematician Benoît Mandelbrot (1982) and is derived from the latin word 'fractus' meaning broken or fractured. Mandelbrot defined a fractal as “a rough or fragmented geometric shape that can be split into parts, each of which is (at least approximately) a reduced-size copy of the whole”. An exact self-similar fractal is a fractal that appears *identical* at different scales. In this case the process of zooming results in that sooner or later exactly the same geometric shape will reappear. An intriguing aspect of these exact self-similar fractals is that the ever reappearing form within such a fractal may be seen as its elementary building block which therefore must be capable of yielding its essence. So, with respect to exact self-similar fractals the terms 'essence' and 'form' are no longer independent notions.

The idea that there exists a parallel between the discrete thoughts that form part of coherent insight and the rough geometric shapes that form part of an *exact self-similar* fractal will hereafter be indicated by the term 'fractalism'. According to fractalism it would be unnecessary to introduce the third notion 'am' (or 'exist') because, as a consequence of what is mentioned earlier, in fractalism there already exists a certain mysterious correlation between essence and form. Or to put it differently, essence in itself and form in itself have ceased to exist and only a kind of union of the two preserves an independent reality.³ Any statement compatible with fractalism with respect to the mind-bubble 'I think' can be considered as a possible alternative to the statement of Descartes. In the case of fractalism there are (at least) two possible statements available:

1. I think: “I think”, therefore I is the thinking of thinking.
2. I think: “I think”, therefore I is the thinking of no thinking.

How can this be understood? First a kind of abstract understanding of both possibilities will be provided which might give the sensation of being a bit austere. Subsequently, a more intuitive approach will be supplied to append a somewhat right-brained comprehension of the disparity of both world views which will take the edge off fractalism.

With respect to the first statement, suppose that the resulting expression “I is the thinking of thinking” is correct then one is capable of substituting 'I' in the statement 'I think: “I think”' to reveal the statement: 'the thinking of thinking think: “the thinking of thinking think”'. But what does the last half of this statement

(i.e., “the thinking of thinking think”) signify? Evidently, “thinking”. Therefore, the statement 'the thinking of thinking think: “the thinking of thinking think”' can be simplified into the statement: 'the thinking of thinking think: “thinking”', and this last statement is true because it is a tautology. This means that the expression “I is the thinking of thinking” provides a possible solution with respect to the principal mind-bubble 'I think'. Even so, it is highly doubtful whether this option is actually an example of fractalism because the total pattern that appears is too smooth and lacks the roughness which is such an essential characteristic of a fractal—a straight line, for example, is exactly self-similar but is also too smooth to be classified as a fractal because it can simply be described by Euclidian geometry. This paper will consider this first possibility as a failed attempt of constructing a fractal-like structure and therefore will not refer to it as an example of fractalism.

What's more, the first possibility provides a conclusion which does not seem to diverge much from Descartes' original statement because it reveals that there is thinking although this thinking can be either correct or incorrect. Hence, there *is* a thinker, in other words, I am (or I exist). But the conclusion 'I am' is clearly also based upon the law of excluded middle (i.e., “Everything must either be or not be.”) because thinking is considered to be “either correct or incorrect”. This consideration appears in this case to be justified because thinking seems to have essence, to be specific, thinking does not consist of no thinking. Consequently, this analysis of the synthesis of thoughts and thinking already reveals a pre-understanding of Descartes' “cogito, ergo sum” because it emphasises the necessity of the law of excluded middle with respect to Descartes' worldview and it forewarns of the exceptional possibility that involves an empty mind (e.g., the thinking of no thinking) because it would make the law of excluded middle fallacious. This actually means that the Cartesian worldview is not only governed by separation and linear order, but that *dualism* and linear order prevail in the Cartesian worldview because the apparent indispensable law of excluded middle prevents any third possibility with respect to the separation of correct and incorrect—dualism means here the division of something conceptually into two opposed or contrasted aspects as in correct and incorrect, or, inside and outside. Hence, the Cartesian ego capacitates the activity of separating an inside from an outside in a dualistic way and harbours the idea of linear order and is therefore in principle capable of conceiving the natural numbers—in principle because the Cartesian ego still has to learn to maintain linear order (inside as well as outside).

• **A self-emptying ego.** The second suggested possibility can be verified in a similar way as the first one, in spite of the fact that the term 'no' has been inserted. The conclusion, though, is quite different as will be demonstrated, and this second possibility may be seen as a departure from the earlier mentioned Cartesian worldview. Suppose that the resulting expression "I is the thinking of no thinking" is correct then one is capable of substituting 'I' in the statement 'I think: "I think"' to reveal the statement: 'the thinking of no thinking think: "the thinking of no thinking think"'. But what does the last half of this statement (i.e., "the thinking of no thinking think") signify? Evidently, "no thinking". Therefore, the statement 'the thinking of no thinking think: "the thinking of no thinking think"' can be simplified into the statement: 'the thinking of no thinking think: "no thinking"', and this last statement is true because it is a tautology. This means that the expression "I is the thinking of no thinking" may also provide a possible solution with respect to the initial mind-bubble 'I think'.

Still, the expression "I is the thinking of no thinking" gives the understanding that the 'I', although thinking, has an empty mind due to 'no thinking'. That being so, the analysis of the synthesis of thoughts and thinking done here does not add up to the idea that this 'I' (i.e., the thinking of no thinking) has the capacity to analyse at all, which raises the question: Who did the analysis?^{4, 5} The presence of this paradoxical situation, which Skinner also came up against (Barab et al., 1999, p. 356), finds its origin in that 'the thinking of no thinking' is incommensurable with a dualistic worldview because the thinking of no thinking can actually neither be correct nor incorrect—the law of excluded middle is no longer valid here. Thus an 'I' that has lost the Cartesian worldview cannot achieve a *self*-analysis in the Cartesian sense. So on that account this 'I' cannot fulfil its *self* in the way Descartes did.

The selfless appearance of a self-emptying ego does not automatically mean to say that fractalism is *self*-contradicting because the correctness of the independent concept 'I' (or 'self') is exactly what was being questioned here to begin with—the term 'self' is integral to the Cartesian paradigm and has a different connotation in the fractilian paradigm. One would therefore need to master the unknown fractilian worldview and unlearn the conventional Cartesian worldview in order to be capable of affirming whether this alternative worldview is *coherent*—the expression 'consistent from within itself' would not be appropriate here.

2.3 The disparity between a Cartesian worldview and a fractilian worldview

The possible correctness of the Cartesian worldview or the fractilian worldview pivots on the intensity of the concept 'separation' with respect to the world one is found in. For example, the thick end of a stick is not the thin end of this stick and both ends can be distinguished by names whilst still being connected via a middle. Hence, the separation between the thick end and the thin end is in this case only based on *naming* distinguished features of a whole. Now if the stick were cut open in the midpoint then the two remaining parts of the stick would no longer be connected via the middle. This would be a more intense kind of separation because it is based on a physical *disunion*. In this case the thick end of the thin end is exactly what used to be the midpoint of the whole and therefore may indicate the actual point of disunion—the point of disunion can also be indicated with 'the thick end of not the thick end' or 'the thin end of not the thin end'. In the case that separation turns out to be based on denomination only and hence not on physical disunion then this would mean that the apparent point of disunion would in actual fact be a point of synthesis.

Briefly, if thoughts can be separated from the thinking process by means of disunion then the Cartesian ego appears to stand on solid ground and in this case its dualistic and linear ordered worldview would appear to supply an adequate insight for analysing and reflecting itself. On the other hand, if thoughts cannot be separated from the thinking process by means of disunion then this would suggest that the appearance of separation is due to denomination only and therefore that the worldview of the Cartesian ego supplies an inappropriate insight for analysing itself due to a point of synthesis (e.g., the point in which the thinking of no thinking occurs). This would mean that the alternative insight of a self-emptying ego should at least also be considered—the possibility of fluid reasoning to revise crystallised intelligence.

2.4 On the threshold of fractal logic

The idea of fractalism, which theoretically should be compatible with the idea of fractal logic, is not something unforeseen and shall be an inevitable mathematical development. The term 'fractal logic' is not new (e.g., Bjorvand, 1995), yet it remains a muddled concept and is still not considered to be a logic in the strict mathematical sense of the word—to date it does not show up on Wikipedia. The analogy with non-Euclidean, non-classical geometry had already led to the belief of a possible non-Aristotelian, non-classical logic which eventually resulted in a variety of new 'curved' logics (Bazhanov, 1990); intuitionistic logic, fuzzy logic, multi-valued logic, paraconsistent logic, to name just a view. Likewise, fractal geometry suggests the idea of fractal logic; it is the next logical step.

Unquestionably, fractal logic should in some sense be different from all former 'plain' and 'curved' logics because fractal logic would embody the characteristic of self-similarity. One of the distinctive features of fractal logic is therefore that it either defies, alters, or reinterprets the law of identity (i.e., "Whatever is, is.") due to the trait of self-similarity—as will become clear in the following passage of this paper, the law of noncontradiction (i.e., "Nothing can both be and not be.") and the law of excluded middle (i.e., "Everything must either be or not be.") are both also touched by fractal logic. The tribulations of describing fractal logic are found in that if the law of identity becomes modified then it becomes quite problematic to define any concept whatsoever which makes any description in general indefinite and consequently turns fractal logic into an ambiguous subject. This probably is one of the main reasons why up to now fractal logic remains all but a mere vague concept and to give a perspicuous representation of fractal logic will be quite a pitted pathway, yet this trail is not impossible to hike. An easy, though informal, way to become more acquainted with fractal logic would be to follow the guided tour through the strange fractal landscape presented in this paper whilst, if the ride is not too bumpy, experiencing an inner appreciation of the unfolding fractilian worldview. An effort towards a somewhat basic systematic understanding of fractal logic is supplied in the endnotes of this paper—more structural details concerning fractalism are saved for the endnotes to make sure that the excursion proceeds as fluently as possible.

3. CONCEPTION

Above, not a piece of tile to cover the head;
Beneath, not an inch of earth to put one's foot on.
Zenrin Kushu

3.1 A hatch of fractalism

Aside from the earlier mentioned seemingly paradoxical situation that involves a self-emptying ego as a result of fractalism (i.e., "Who did the analysis?"), the apparent emptiness of form might also strike someone as being incongruous with reality. Modern science, though, does not detect any contradiction with respect to this idea of 'emptiness'. Energy, for example, comes in many *forms*; gravitational energy, kinetic energy, radiant energy, mass energy, electrical energy, chemical energy, etc. Experiments show that it is possible for a certain form of energy to transform into another form of energy, all according to the principle of energy conservation. Nevertheless, energy in *itself*, as a monadic building block of the universe has not yet been detected by a single experiment. The Feynman lecture on energy, in which energy is compared to the playing

blocks of Dennis the Menace, is very clear about this issue: "there are no blocks". The idea that the universe consists of forms without an independent essence, just like certain fractals, is not contradicted by scientific experiment but only by a dualistic and linear ordered worldview. An 'empty' universe, though, appears to be an unbearable situation for a human being to be found in, thus the searching continues.

3.2 Passing the three schools of thought

The search outside for this 'block' has a counterpart inside although mathematicians prefer to call this inner block 'the foundations of mathematics'. Three important schools of thought with respect to the philosophy of mathematics are formalism, constructivism and platonism. Formalists claim that mathematics is no more than a manipulation of meaningless symbols by certain rules of inference; mathematics is a game. Constructivism asserts that it is necessary to find (or 'construct') a mathematical object (block) to prove that it exists and rejects for this construction the law of excluded middle; mathematics is founded on constructive methods. Platonism advocates that there is some kind of platonic realm where universals (ideal forms) find their existence independent of the human mind but which can be conceived by the human mind; mathematics is discovered. Fractalism values sections of all three -isms yet it also repudiates parts of all three. For example, fractalism uses the meaningless symbols of formalism, yet without its rules of inference because these meaningless symbols express exactly their own 'rules of inference'. Additionally, fractalism values the rejection of the law of excluded middle found in constructivism, yet fractalism declines its constructive method because it has abandoned the idea of a foundation of solid blocks—fractalism prescribes 'the foundation of no foundation'. Furthermore, fractalism uses the realm of platonism, yet without its independent existence because the human mind and the realm are considered to be without disunion. Hence, a fractal ordered mind might visit all three schools of thought but could not reside in any one of them and its search for an appropriate school would continue.

3.3 A natural manifestation that involves fractal order

The appearance and behaviour of a large shoal of fish, each fish without a rank, may be suggested as a manifestation that involves fractal order. Of course, the form of the whole shoal of fish is not identical to the form of a single fish but the shoal of fish may be seen as an entity whose dynamics is correlated to the motion of each single fish. If a diver approaches this shoal then the shoal as an entity might 'decide' to swim away from the diver or the shoal might 'decide' not to swim away, but there is also a third option. As the diver comes closer,

the shoal of fish might just open up and create a void in the close surrounding of the diver. This way the diver may swim in the shoal and out the other side whilst continuously being closely surrounded by water empty of fish; the diver does not touch a single fish whilst passing through the shoal. This third option demonstrates that it is possible for the shoal of fish to swim away from the diver without swimming away because the shoal supplies the cover of no cover for each fish. Thus if the diver is found in the middle of the shoal of fish then the diver is inside the shoal without being inside the shoal—the diver is physically inside but lacks the possibility of expressing the dynamics of the shoal of fish in order to cohere with the shoal. Hence, statements like 'everybody must either be inside the shoal or not be inside the shoal', 'nobody can both be inside the shoal and not be inside the shoal' and 'whoever is inside the shoal, is inside the shoal' would all become ambiguous within the fractilian paradigm.

3.4 The order of the universe

Being found in the universe yet abiding a different order than the one present in the universe (i.e., the cosmos) means to not really cohere with the universe due to an alienated order—like the diver who cannot cohere with the shoal of fish whilst it lacks the capability of expressing the dynamics of the shoal. Hence, to gradually master the discipline of expressing the rhythm of the universe would eventually mean that one suddenly coheres with the universe in concordance (e.g., cosmic dancing).

Unfortunately, this explanation introduces the notion 'to cohere with the universe' which labels the point of synthesis with a name. Once light is thrown on the point of synthesis by giving it a name, it is destined to fall victim to the dualistic mind of a Cartesian ego and doomed to be torn apart. For example, either one coheres with the universe or one does not cohere with the universe—if for an instant the diver and the shoal of fish remained both motionless (i.e., the dynamics of no dynamics) then would the diver temporarily cohere with the shoal of fish or not? Such a dualistic dissection of a point of synthesis fosters an inconsistency with respect to the fractilian worldview and is therefore a silent conversion back to the Cartesian worldview. Hence, to name the point of synthesis makes it possible to present a complete narration of a fractilian worldview which might cause someone with a Cartesian worldview to see the light. Having said that, such a narration would always involve some kind of *inconsistency* (e.g., it would bear a point of disunion) and trying to imitate the description of this 'light' would therefore be bound to result in a caricature. Yet any explanation that does not name the point of synthesis leaves one with a Cartesian worldview unfulfilled and in the dark because it

provides an *incomplete* description of the facts of the matter. Such an abstruse description, however, may be consistent.

The fore mentioned dilemma of incompleteness or inconsistency also finds its origin in a point of disunion; to name or not to name. This point of disunion, however, may actually be a point of synthesis but in the eyes of a Cartesian ego gives the impression of disunion due to a non-overlapping double perception. A fractilian plan of action to solve this dilemma would be to find the pathway back to the origin of this appearing disunion and then try to discover whether this origin is actually a point of synthesis. In general, a self-emptying ego would consider any dilemma (i.e., mental fork) as the beginning of a pathway towards the origin of separation as a means to challenge the nature of this origin. Moving backwards in the direction of an apparent point of disunion and trying to dissolve this point is a road less travelled, yet it will also be ventured with respect to a comparable dilemma that is found in mathematics.

4. TRANSFIGURATION

Before the first step is taken the goal is reached.
 Before the tongue is moved the speech is finished.
 More than brilliant intuition is needed
 To find the origin of the right road.
 Mumon

4.1 A glimpse of the revolution within physics versus a confronting dilemma within mathematics

The theory of relativity was a scientific breakthrough in the sense that it demolished the foundations of classical mechanics and replaced them with new foundations. In these fundamentals is found the idea of 'space-time' and the synthesis of space and time was uncovered by the Russian German mathematician Hermann Minkowski (1908): "Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality." This profound realisation matched the simplicity of the relativity principle, it was the underlying reason behind the shift in scientific 'language' from plain geometry to curved geometry and it showed the gateway to a new paradigm of understanding. The need for a new theory was to a large extent provoked by the discovery of an oddity revealed by the Michelson-Morley experiment. Prussian American physicist Albert Michelson and American scientist Edward Morley had built an ingenious apparatus sensitive enough to detect the motion of the earth within a supposed existing luminiferous ether. The result of their experiment, though, was the detection of no detection (i.e., evidence of absence) and the anomaly caused by this null result challenged the old ways of understanding. The old ways of understanding appeared

to be incorrect and had to be adjusted which led to the embarkment for a revolutionary scientific theory.

With respect to mathematics, it was the incompleteness theorems of the Austrian American logician, mathematician and philosopher Kurt Gödel that really challenged the contemporary ideas which intended to embrace the foundations of mathematics. In order to prove his theorems Gödel had to construct an ingenious 'apparatus' capable of encoding statements of a formal system about natural numbers into natural numbers, also known as Gödel-numbers, while this 'apparatus' is also capable of decoding these Gödel-numbers back again into their original statements—the statement/Gödel-number transformation produces a kind of affinity between natural numbers and statements about natural numbers. Consequently, properties of a statement about natural numbers can be detected by examining the properties of its Gödel-number. The core of Gödel's incompleteness theorems is a statement known as the Gödel sentence of the formal system. This sentence is a statement in the language of the formal system with complete self-affinity, that is, although the statement is about natural numbers it is also a statement about itself as a result of the statement/Gödel-number transformation. To be more specific, the Gödel sentence is devised in such a way that according to the statement/Gödel-number transformation it says of itself that it has the property of not being provable without giving rise to any infinite regress. The following sentence gives a rough insight into the structure of the Gödel sentence.

Is not provable when succeeded by itself in quotes “Is not provable when succeeded by itself in quotes”

Just like the Gödel sentence, this sentence also asserts its own unprovability whilst not directly referring to itself. The Gödel sentence, however, has a more substantial meaning because it is in actual fact an arithmetical statement about natural numbers.

Kurt Gödel managed in this way to prove the existence of a statement (i.e., the Gödel sentence) within the language of the formal system with the characteristic of exact self-similarity when transformed via the statement/Gödel-number transformation on the level of not being provable in the case that the formal system is consistent; that is, if the Gödel sentence cannot be proven by the formal system then, according to the statement/Gödel-number transformation, the Gödel sentence must be true for this specific formal system and therefore the formal system incomplete. And, in the case that the Gödel sentence does not have the characteristic of exact self-similarity when transformed via the statement/Gödel-number

transformation on the level of not being provable, then this would imply that the formal system under consideration must be inconsistent; that is, if the Gödel sentence can be proven by the formal system then, according to the statement/Gödel-number transformation, the formal system must contain a contradiction and therefore be inconsistent. Hence, the characteristic of self-similarity with respect to the property of not being provable turns out to be too fine for a consistent formal system to imitate and this results in that the formal system under consideration must either be incomplete or inconsistent.⁶

Gödel's incompleteness theorems can also be interpreted as the discovery of an anomaly within contemporary mathematical paradigms because the prevailing schemes of encapsulating mathematics suffer from inherent limitations, to be specific, any *consistent* (fixed) formal system that includes enough of the theory of natural numbers fails to take into account a certain arithmetical fact which can be expressed by the Gödel sentence of this formal system.

• **The Michelson-Morley experiment contrasted with Gödel's incompleteness theorems.** There seems to be a parallel between Gödel's incompleteness theorems and the Michelson-Morley experiment because both used an ingeniously built device that enabled them to reveal an anomaly with respect to a certain contemporary paradigm. The interferometer of Michelson and Morley was made sensitive enough to demonstrate 'the detection of no detection' while Gödel's machinery was constructed with sufficient delicacy to reveal 'the proof of no proof' with respect to a sufficiently strong formal system—assuming this formal system is consistent. But from here on the analogy breaks down. As a result of the incompleteness theorems there has not occurred any synthesis within mathematical understanding. There has not been a profound change with respect to the scheme of encapsulating mathematical phenomena; no concordant paradigm shift has taken place. Gödel's incompleteness theorems gave a boost to the philosophy of mathematics in the sense that the theorems affected many old ways of thinking, but the old ways have not been replaced by an overall satisfying new way, that is to say, no revolutionary mathematical break-through has ever been presented. Many an Einstein has confronted Gödel's incompleteness theorems, many of them contributing a lot of important and valuable suggestions to the philosophy of mathematics, but there has not occurred any profound realisation with respect to mathematics equivalent to that of Minkowski with respect to physics. Any formal theory designed to encapsulate all of mathematics will *always* lack coherency because nobody has found a coherent way of escaping the dilemma of incompleteness or

inconsistency. Yet, to loose faith in the possibility of an all-embracing *coherent* theory for mathematics would for a mathematician in effect mean ceasing to be a scientist.⁷

4.2 A pathway towards a synthesis with respect to mathematical insight

There appears to be some kind of a resemblance between the structure of a Gödel sentence and the formulation of the earlier mentioned reflection: I think: "I think". There is also a very important difference between these two structures with respect to the schemes that are involved. In the case of fractalism one zooms *out* for a wide view of the mind as a means to make a synthesis of thoughts and thinking (i.e., I think: "I think") which can be analysed. In the case of Gödel's incompleteness theorems, one zooms *in* for a close-up of a formal system in order to make an analysis of this system which uncovers the Gödel sentence. Once this is done it becomes possible to make the victorious synthesis of the Gödel sentence and the statement/Gödel-number transformation as a means to demonstrate the dilemma of incompleteness or inconsistency.

An overview of the schemes that involve fractalism and Gödel's incompleteness theorems gives the impression that in both cases reasoning encounters a kind of similar pathway although it is followed in opposite directions; in fractalism one takes a step backwards—a conclusion is deduced from the analysis of a *synthesis of* thoughts *and* thinking—and every Gödel sentence takes a step forward into the rich profusion of mathematics—a conclusion is deduced from the *synthesis of* an analysis of the formal system which reveals the Gödel sentence *and* the statement/Gödel-number transformation. By revealing the pathway of self-affinity and moving towards profusion Kurt Gödel was capable of demonstrating that various old ways of understanding mathematics had been incorrect and that, just like fractals, mathematics incorporates an infinite complexity, too irregular to be easily described in traditional language. Fractals, though, can easily be described by a simple recursive definition; that is, a non-Euclidian geometric language. Hence, a non-traditional language for mathematics might also reveal a pathway to the 'foundations of mathematics'. This means, though, that one would have to follow the trail of self-affinity in the exact opposite direction of the Gödel sentence, away from profusion towards the sophistication of simplicity in order to encounter the source of the dilemma of incompleteness or inconsistency. And if necessary, pass the threshold of old-established views that barricade the straight and narrow pathway so that a possible synthesis with respect to mathematical insight could be achieved.

4.3 A hindering force

Strikingly, the struggle for a synthesis is in general not found in the strain of comprehending a synthesis, but in the psychological difficulty of accepting the consequences of such a comprehension and in avoiding the temptation of grasping at previous common-sense understandings which assert the independence of the observer from the observed phenomenon. In other words, the difficulty of maintaining balance whilst detaching from imaginary psychological models. This detaching from imaginary psychological models is necessary if one wants to move freely within the new worldview. For example, even though the theory of relativity had been fruitful from its early beginnings it still created some resistance among eminent scientists because of its seemingly absurd consequences, e.g., time dilation, length contraction, clock paradoxes. Consequently, the idea of the existence of a luminiferous ether, which requires the notion of an absolute frame of reference that is preferred above all other frames of reference, was only abandoned gradually after the discovery of the theory of relativity.

The principle of scale invariance would certainly eliminate another illustrious notion from the human mind, namely, the notion of a certain absolute scale of reference which is preferred above all other scales of reference. This highly esteemed notion brings forth the impression of a solid basis (i.e., a starting point) which is essential for the mind of a human being in order to make a stand for 'I' (e.g., "I exist"). The psychological difficulty of extinguishing this ardent notion may conceivably form a hindering force that shields against any possible synthesis with respect to mathematical insight. Therefore, in all likelihood, mathematicians will simply prefer to carry on with the imperial horse-and-buggy models which assert that the observer is absolutely separated from the observed phenomenon called mathematics. The life space of a mathematician will, with respect to the dilemma of the incompleteness or inconsistency of formal systems, simply remain without any further development whilst the mathematician lacks a boundary zone experience with the true nature of this 'solid basis' and the acceptance of such a possible experience.

4.4 A possible solution to the foundational crisis within mathematics

Crisis takes place when the old has not died and the new has still not been born—Bertolt Brecht. This also appears to count for the foundational crisis which is still present within mathematics. The terms 'zero', 'immediate successor of' and 'natural number' are still found in the axioms of the natural numbers, terms that date back to the era of classical mechanics. The term

'immediate successor of', however, is an emanation of the notion 'to immediately succeed'—compare with the term 'thinking of' which is an emanation of the notion 'to think'. Hence, in the case of fractalism the term 'zero' (or the term 'one' if this is a preferred starting point) becomes redundant because it can be substituted by the term 'the immediate successor of no immediate successor'—compare with the term 'I' which in the fractilian paradigm was substituted by the term 'the thinking of no thinking'. Accordingly, the term 'natural number' is the consequence of a dualistic mind trying to maintain linear order (inner as well as outer) and is brought to light in the same way the term 'am' (or 'exist') came to mind in the case of Descartes—compare “I am”, for example, with Peano's original axiom “one is a natural number”.

Consequently, mathematicians are confronted with a similar trinity which involves the appearance of a starting point. The root of this trinity, however, is found in a crystallisation process of thoughts rather than in a realisation of a fluent whole. It is a clear-cut case and a turnabout is straightforward; avoid introducing the term 'natural number' because there already exists a mysterious relationship that expresses the term 'zero' (or 'one') in terms that can be derived from the mere notion 'to immediately succeed', for example, 'one is the immediate successor of no immediate successor'. With this orientation one will avoid any unnecessary multiplication of entities—this unnecessary multiplication of entities appears to be the genesis of the trail of self-affinity which the Gödel sentence manages to track down. Crucially, *if* one is willing to adopt the principle of scale invariance *then* it must become clear that the axioms of the natural numbers will have to be modified, just as the formulas of classical mechanics had to be modified, in order to achieve a theory that abides fractal order. In this way a possible synthesis with respect to mathematical insight could be attained. Once more, the difficulty here is not found in comprehending the 'new' way but is mainly found in *not* grasping at previous common-sense understandings whilst being bamboozled by paradoxical appearances—compare with the 'difficulty' of breaking free from a Monkey Trap (Barab et al., 1999, p. 350).

5. REHABILITATION

In myriad forms, there is a single body revealed.
Hogen

5.1 The special theory of invariance

A better name for 'the theory of relativity', as was suggested by the German American theoretical physicist Albert Einstein, is 'the theory of invariance' because according to the theory, the general laws of physics are to be expressed by equations which hold good for all

systems of reference—this would actually be the case if the principle of relativity could also be extended to scales and efforts in this direction have been made notably by the French astrophysicist Laurent Nottale (1992; 2009). Fractalism also results in a theory of invariance, though on a mathematical level, because it entails that the general 'foundations of mathematics' are to be expressed by axioms which hold good for all systems of reference. The principle of scale invariance can be demonstrated by comparing two different scales of a fractal with one another and showing that both observed shapes are identical. The zooming in or out is done in a fixed step and this can therefore only lead to a special theory of invariance—stepwise scale invariance can be seen as a special case. The origin of this stepwise scale invariance is found in that there are certain fixed stages (i.e., discrete scales) of conceptual awareness available for a human being but 'nothing' in between. For example, the awareness of the concept 'tree' is immediately succeeded by the awareness of the concept 'field of trees' (i.e., collection of trees) while there is no concept in between the two, or, in between the two is found the concept of no concept.

5.2 The general theory of invariance

The general theory of invariance is found in that a smooth zooming in on the fractal, or a fluent zooming out for a wide view, will continuously unfold an identical 'shape' during the whole process of zooming—continuous scale invariance.⁸ This general theory leads to belief that during the process of zooming, the concepts that are perceived on the fixed stages of conceptual awareness are in fact the result of a continuous unfolding of one and the same. Human beings, though, can only conceive this continuous unfolding as manifestations displayed on the fixed stages of their conceptual awareness while there is no conception of this unfolding in between the stages. That is, in between the stages of conceptual awareness there appears to be 'nothing'. The conception of these manifestations is experienced as thoughts and these thoughts can be achieved by zooming in or by zooming out, both of which are called thinking; analysing (left-hemispherical brain activity) and synthesising (right-hemispherical brain activity) respectively.

With respect to the general theory of invariance, the identical 'shape' of the fractal that is continuously being unfolded by the smooth zooming in or out can no longer be considered a shape because in fractalism the notion 'shape' relates to the notion 'thought' which forms part of coherent insight; that is, 'nothing' only collapses into a thought in the eyes of a conceptually aware being on one of its conceptual stages—without this collapsing 'nothing' has the form of no form. Also, the splitting up of the fractal into parts turns out to be no more than a

conceptual stage performance while this general theory manages to take a look behind the scenery—a peek, for example, reveals that the bare formulation of the principle of scale invariance is 'the principle of no principle'. And finally, the general theory of invariance cannot be expressed completely and consistently in terms of fixed symbols and rigid relationships because fixed symbols cannot be smoothly broken up into, for example, half symbols (the difficulty of expressing a conceptual continuity) and equations have two sides instead of one (the difficulty of expressing an ultimate unification).⁹ A coherent way of expressing the general theory of invariance simply requires 'something' more profound because the essence of this theory is that there are no mental blocks—it requires the expression of no expression.

6. LIBERATION

The clouds which are formed at the summit of the mountain do not exist,
But the reflection of the moon in the water does exist.
Tu Minh

6.1 But what about the unreasonable effectiveness of mathematics in the natural sciences?

Imagine a giant squid that lives in deep-sea waters that on a certain 'day' happens to stumble upon an object moving downwards in the water. The squid manages to grasp this object, a beaker, with one of its long tentacles and starts to examine this strange artefact by tapping the inside with several of its shorter tentacles. After a little investigation it comes to the conclusion that the form inside this object is flat and round on the bottom, then the bottom curves up perpendicular to form the wall which continues upwards until it makes a smooth end. Having become curious about this object the squid decides to move to higher 'grounds' where light is available so that it can visually observe this artefact. As the squid reaches water where visibility prevails it observes that this object is transparent, yet it also observes with great astonishment that the sea 'inside' the object has the exact same form as the form of the object on the inside which had been examined earlier on. Being an inquisitive squid, it starts to inspect the form of its surroundings through the glass of the beaker by means of holding the beaker in many other places as well, yet without the beaker touching the borders of the sea, viz., the bottom or the surface of the sea. After a series of tests the giant squid is confronted with the amazing impression that the form of the world in which it lives can be built up from the exact form that is found on the 'inside' of the miraculous object. The structure of the sea appears to involve the shape of a cylinder although the top of this cylinder remains for some strange reason undefined. Is this amazing impression legitimate? This impression may be called legitimate

although it can only provide a superficial understanding of what really is going on. A more profound realisation would be that in this case the notion 'form' is a reality-within-reality which is imposed upon the sea by the beaker because the sea is a liquid and therefore without a form of itself.

• **Observation.** Although the sea inside the beaker may be called 'inside' and the sea outside the beaker may be called 'outside' these two names do not really divide the sea into two parts because the inside and the outside are still connected via the mouth of the beaker—compare with the two ends of an unbroken stick. The idea of giving names to the 'parts' of a whole is an action of the intellect that causes an outlook in which the whole *appears* to dissipate resulting in a diffused worldview. The three classic laws of thought are in the whereabouts of the mouth of the beaker just folly dogmas because here the inside and the outside remain connected and therefore undefined—the actual point of synthesis. In order for all three laws to be completely effective one has to define a boundary in such a way that it is possible to divide the sea into two parts where one part corresponds with the inside of the beaker and the other part corresponds with the outside of the beaker—the point of synthesis becomes a point of disunion. If the three classic laws of thought were all universally valid then the diffused world would in fact be a shattered world. In such a shattered world, that is to say, a non-coherent universe or at best a universe upon which coherency is imposed by a separated external agency, separating the inside from the outside would be in order. And if the order of the universe were also linearity, then and only then would the Cartesian ego be justified.

• **Interpretation.** The encounter of the squid with the beaker in the story represents the mind of a human being which has been endowed with the concept 'self'. The inspection of the form inside this beaker reflects the examination of the concept 'self' which results in the idea of separation. The idea of linear order is introduced because the inside of the beaker becomes a starting point which is followed by the outside of the beaker, yet linear order is also inaugurated by the form of the beaker; the bottom is followed by the wall. Furthermore, in this analogy what 'form' means for the squid means 'essence' for the scientist and in this respect the analogy reflects reality from the point of view that the universe is without any essence of itself, just like the sea which is without any form of itself with regard to the giant squid who doesn't consider the borders of the sea. This would suggest that the unreasonable effectiveness of mathematics in the natural sciences can be explained with the idea that *any numerical measurement displays only a reality-within-reality* which is imposed upon the universe by the human mind

due to a Cartesian *outer* worldview, the exact same mind which is also capable of perceiving the natural numbers due to a Cartesian *inner* worldview.

- **Modification.** So in the case of fractal order, the law of identity (i.e., “Whatever is, is.”) is incorrect and this law should be modified as to reveal the principle of non-identity which can be formulated as: Whatever is, is in principle not—or to keep things homogeneous: Anything that is, is in principle not. Additionally, the law of noncontradiction needs to be reinterpreted because a point of synthesis will appear in the eyes of a Cartesian ego as *nothing* and therefore the statement “Nothing can both be and not be” remains correct once it is realised that the term 'Nothing' may actually correspond to a point of synthesis. And the law of excluded middle (i.e., “Everything must either be or not be.”) is only true as long as the term 'everything' does not include any point of synthesis; there is more than meets the Cartesian eye because the point of synthesis remains hidden for the Cartesian ego and therefore occult.

- **Example.** In order to measure distance, one first has to accept the (self-evident) idea that there exist two different points which are separated from one another as a means to find a certain procedure for measuring their distance. But this conception is not immaculate because it is already tainted with linear order and disunion; there exists a *first* point from which is measured up to a *separated second* point. Moreover, space is classically considered to consist of three dimensions and each of these three dimensions finds its origin in linear order. Time, the fourth dimension, also finds its origin in linear order (e.g., a present separated from a past), although it has to be treated slightly differently—as an imaginary dimension. Therefore, if linear order is a reality-within-reality forged by the human mind and imposed upon the universe then the unification of space and time should be less of a surprise because all these four dimensions share the same origin, namely, the human mind.

- **Dilemma.** The scientist tries to observe their diffused surroundings (caused by the self) by means of the concept 'self' (e.g., a quasi-enclosed system), comparable to the giant squid that tried to observe its diffused surrounding (caused by the beaker) through the crystal of the beaker. The giant squid is, due to the beaker, faced with an curious dilemma: hold on to the beaker (law of identity) and impose the law of noncontradiction as well as the law of excluded middle upon reality—dogmas which are necessary for a divide and conquer strategy—as well as the order of linearity as a means to master an understanding of its surrounding with the prospect of attaining a certain

supremacy to overcome the disquietude of disunion, or, to let go of the beaker so it gets completely lost in the vast darkness of the unfathomable sea which results in that the giant squid is found once again in the world of the formless whole.

- **Security by default.** This all could somehow give the understanding that maintaining a Cartesian ego implies that: “Everything we think might have an error in it because some demon somewhere is messing with our brains” (Brown). If this is really true then at least now it may become clear where exactly this demon is messing with our brains. Nevertheless, this does not mean to say that the reality described by science is illegitimate because demons are still half-gods. It only means that this reality-within-reality is based on a priori synthetic judgements which are experienced as true because they are *self-evident*. Such a priori synthetic judgements may find their origin in a kind of default configuration setting for the human mind for security reasons comparable with the umbilical cord which is also a kind of default setting for a foetus to secure its physical development in the womb which is made redundant after birth—the womb is also made redundant yet only relatively to the newborn. Security by default implies that the default configuration settings are the most secure settings possible although not necessarily the most user friendly settings. This would suggest that the earlier mentioned demon does not find its origin in evilness but rather in parental care, e.g., it provides a cause-and-effect worldview with the intention of securing the initial development of a human mind. If this were the case then once the human mind is sufficiently enhanced with earthly experience for survival (i.e., crystallised intelligence), reconciled with the illusiveness that maternal care brings (i.e., the warmth radiated from attachment) and found peace with the delusion brought in by paternal care (i.e., the safety casted by the threshold), the lofty Cartesian ego may just dissolve in order to give way to the maturation of the human mind.

6.2 In conclusion

Although science can be very accurate in its predictions, it can only provide a superficial understanding of what really is going on. On the other hand, the superficial understanding that results from science ultimately creates a certain esoteric undercurrent which may be experienced as a helping force. For example, quantum mechanics reached 'the mouth of the beaker' when it ran into spooky-action-at-a-distance; a certain phenomenon which defies the convention of separation. It is at this point (i.e., the point of synthesis) where the culmination of the theory was reached because a shadow of the true mechanics of the universe finally became highlighted and this

revelation resulted in 'the understanding of no understanding', or, as the American physicist Richard Feynman put it: "I think I can safely say that nobody understands quantum mechanics".

7. CONCLUSION

Know the strength of a man,
But keep a woman's care!
Lao Tsu

7.1 Assessment

In order to be aware of the intrinsic value of the natural numbers it is necessary to maintain an insight with non-overlapping double-vision; simultaneous awareness of two different conceptual scales combined with simultaneous awareness of inner and outer linear order. The actual fact that numerous human beings are capable of having a corresponding arithmetical insight therefore suggests that they all espouse the same insight with non-overlapping double-vision. This would not mean to say that this engagement is illegitimate, for instance, chameleons are known for using non-overlapping double-vision which gives them a full 360-degree arc of vision around their body, as well as using overlapping double-vision which gives them depth perception—these creatures are true masters of both world views. It would mean to say, however, that the engagement with non-overlapping double-vision insight could only bear a superficial understanding of what really is going on. Nevertheless, a fractal ordered mind would experience a synthesis with respect to insight which results in a different worldview.¹⁰

Linear order is an order that differs from fractal order primarily in that, with respect to linear order the concept 'self' has an independent essence whereas with respect to fractal order the concept 'self' lacks independent essence; more specifically, the essence of the concept 'self' can in the fractilian paradigm be expressed in mere terms of form due to a mysterious correlation between form and essence. If a human mind could be found in the free-flowing condition of maintaining fractal order instead of being found in the mental state of maintaining linear order then a different vision with respect to insight would be attained because fractal order does not discriminate between scales. Applying fractal order to a diffused worldview could theoretically completely free one from diffusion and supply an *overlapping* double-vision with respect to insight; in other words, a coherent insight. Insight that involves overlapping double-vision is based on the principle of scale invariance and can be suggested as a possible alternative to the ordinarily used insight that involves *non-overlapping* double-vision which is based on the idea that *self-evidence* suits truth—the emperors new clothes.

7.2 Precience

The development of fractal logic as a logic in the strict mathematical sense of the word is both necessary and unavoidable.¹¹ Once fractal logic is established and mathematically accepted, the extension of the principle of relativity to scales will become more sensible. The current absence of fractal logic in the scientific enterprise might therefore explain the lack of coverage and discussion of Nottale's Scale Relativity as expressed in the *New Scientist*: "I do not understand why Nottale's theory has not been given wider coverage or discussion. If he is wrong it needs debunking, if he is right he deserves a nobel prize."¹²

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References and related works

1. Barab, Sasha A., Cherkes-Julkowski, Miriam, Swenson, Rod, Garrett, Steve, Shaw, Robert E. and Young, Michael [Principles of Self-organization: Learning as Participation in Autocatalytic Systems](#), The journal of the learning sciences, 8(3&4), pp. 349-390, 1999.
2. Bazhanov, Valentin A. [The fate of one forgotten idea: N.A. Vasiliev and his imaginary logic](#), Studies in Soviet Thought, 1990, vol. 39, N3-4, pp. 333-342.
3. Bjorvand, Anders T. *Fractal Logic A New Approach to Intelligent Systems Theory*, Knowledge Systems Group, Faculty of Electrical Engineering and Computer Science, The Norwegian Institute of Technology, The University of Trondheim, Trondheim, 1995.
4. Brown, Kevin *Is Arithmetic Consistent?* www.mathpages.com/home/kmath347/kmath347.htm
5. Campbell, Joseph *The Hero with a Thousand Faces*, 1st edition, Bollingen Foundation, 1949.
6. Cattell, Raymond B. *Abilities: Their structure, growth, and action*. New York, Houghton Mifflin, 1971.
7. Feynman, Leighton & Sands *The Feynman Lectures on Physics*, Volume I, chapter 4, (2) and Volume III, pp. 18-19.
8. Kuhn, Thomas S. *The Structure of Scientific Revolutions*, 3rd edition, Chicago, IL: University of Chicago Press, 1996.
9. Lewin, Kurt *Defining the "Field at a given time."* Psychological Review, 50: pp. 292-310, 1943.

Republished in *Resolving Social Conflicts & Field Theory in Social Science*, Washington, D.C.: American Psychological Association, 1997.

10. Mandelbrot, Benoît B. *The Fractal Geometry of Nature*, W.H. Freeman and Company, 1982.

11. Minkowski, Hermann *Space and Time*, a translation of an address delivered at the 80th assembly of German natural scientists and physicians, at Cologne, 21 September, 1908 in Lorentz, Hendrik A., Einstein, Albert, Minkowski, Hermann and Weyl, Hermann *The Principle of Relativity: A Collection of Original Memoirs on the Special and General Theory of Relativity*, New York, Dover Publications, 1952, p. 75.

12. Nagel, Ernest and Newman, James R. *Gödel's Proof*, New York University Press, 1958.

13. Nelson, Edward [Warning Signs of a Possible](#)

[Collapse of Contemporary Mathematics](#), Department of Mathematics, Princeton University, 2006.

14. New Scientist; Cutcliffe, Anthony *Scale relativity*, Magazine issue 2601, 28 april 2007.

15. Nottale, Laurent [The theory of Scale Relativity](#), International Journal of Modern Physics A, Vol. 7, No. 20, 1992.

16. Nottale, Laurent [Scale relativity and fractal space-time: theory and applications](#), Paris Observatory and Paris-Diderot University, Meudon, France, 2009.

17. Russell, Bertrand *The Problems of Philosophy*, Oxford University Press, New York, 1997 edition, p. 72 (first published 1912).

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Endnotes

¹ It is debatable whether a field of trees can only be perceived after the perception of a single tree. For instance, a forest can be seen from a distance before even a single tree can be seen. A country, though, may contain several forests which gives rise to the concept 'country of forests'. Any argument with respect to the idea of what comes first would therefore revolve around the idea of what should be taken as a starting point; is it possible to *choose* a starting point? The whole idea of the existence of a starting point, though, corresponds to linear order and not to fractal order. A fractal ordered mind considers a starting point as no more than a skyhook because in the fractilian worldview *any* focus point is a point of synthesis.

² The process of counting is strictly speaking different from the process of attaching a ribbon to each tree in a field in that for the latter, one still has to find a way of checking whether all trees in the field have a ribbon in order to know if the recursive procedure can be stopped. Thus if one counts the fingers on one hand, for example from the left to the right, then with each new finger that is counted the *total amount* of fingers increases with one finger because the 'field' that contains the counted fingers is each time extended in such a way that it also includes the next finger to be counted but no more than that. This next finger, which is the last finger in this specific 'field' is then labelled with the succeeding numeral and being the last finger in the 'field' this numeral therefore may be metamorphosed into a natural number which indicates the total amount of fingers in this 'field' upon which the 'field' gets further extended to include the following finger, but no more than that. In this case there is a systematic step-by-step extension of the 'field' which results in that after each count the process is temporarily stopped because the last finger in the 'field' has a numeral (ribbon) and, subsequently, this numeral transforms into a number. This extension can easily be done without confusion because the fingers on a hand are found on a 'line' and in a certain specified order—in this case from left to right. So theoretically speaking, every count involves a shift in focus from the concept 'finger' to the concept 'field' which results in a shift from numerals to numbers. With respect to the process of attaching a ribbon to each tree in a field, it is only the last ribbon that transforms into a total amount of trees with respect to the (fixed) field whilst the other ribbons are meaningless, that is to say, they do not refer to a total amount of trees with respect to this field while no other field has yet been defined.

³ For example, the dimension of a [Koch snowflake](#), which is an exact self-similar fractal, is neither one nor two (i.e., neither form nor essence) but somewhere in between. Consequently, the terms 'form' and 'essence' may in this case be united into a single term as a means to reflect this in-between (fractured) dimension.

⁴ The analysis of the reflection 'I think: "I think"' is only part (hence analysis) of the following possible infinite deep reflection: and so back ... I think: "I think: "I think: "I think".....". Nonetheless, the part 'I think: "I think"' in combination with the knowledge that one is handling the structure of an exact self-similar fractal contains all necessary information; in this case, a statical component 'I think' and a dynamical component which is found in how 'I think' and 'I think' relate to each other which is expressed by the double dot and the two quotes. The substitution ultimately led to the statement: the thinking of no thinking think: "no thinking", from which followed the realisation that this statement is true because it is a tautology. But if the original reflection was one step more profound (i.e., I

think: “I think: “I think””) then what would have been left over after the substitutions and the realisation of being a tautology would have been something like the following statement: the thinking of no thinking think: “true!”. Accordingly, this would suggest that 'no thinking' and 'true!' are the same because then this statement would also be a tautology and therefore 'true!'. So, if the reflection was much longer (deeper), or even infinite, then the realisation 'true!' would simply reoccur over and over again and therefore be finally true, or, infinitely be true—true? The oversight is that it is taken here that the possible infinite deep reflection under consideration, although without a beginning, does have an end to start from, yet fractalism refers to a fractal that is infinite with respect to zooming in as well as zooming out, otherwise there would still be a scale treated differently from all other scales and therefore preferred. Mandelbrot's earlier given definition of a fractal, though, would not include such a geometric shape that is infinite both ways (i.e., zooming in and out) because a part of this shape could no longer be scale invariant with respect to zooming out and therefore the property of being self-similar would fall through. Nevertheless, a lapse is found in the beginning of the line of reasoning in what appears to be evident: “But what does the last half of this statement (i.e., “the thinking of no thinking think”) signify? Evidently, “no thinking”.” This would mean that 'no thinking' is actually only an approximation of “true!”. Due to the fact that no thinking only approaches “true!” it becomes possible for the “Who did the analysis?” paradox to not really be paradoxical.

⁵ The mentioned “Who did the analysis?” paradox runs parallel with the clock paradox in the special theory of relativity, which is: How can a clock be capable of ticking slower than an exact similar clock and as a logical result of this simultaneously not tick slower (i.e., quicker) than this exact similar clock? Compare this with: How can I be capable of analysing an exact similar I and as a logical result of this simultaneously not be capable of analysing (i.e., synthesising) this exact similar I? Both paradoxes can only be interpreted as contradictions if one maintains believing in the simultaneous individual existence of both different instances, that is, to believe that the non-overlapping double-vision that occurs as a result of a certain way of thinking reveals true insight. With respect to the clock paradox, it has generally been accepted that the non-overlapping double-vision necessary to perceive the contradiction presents the scientist with an incorrect insight into the subject under consideration and this clock paradox is not regarded to involve any contradiction whatsoever. An insight with non-overlapping double-vision is in this case simply considered as incorrect insight. Logic would be the subject of discrimination if the “Who did the analysis?” paradox were to be given a different treatment.

⁶ Since the publication of Gödel's incompleteness theorems there have been developed new methods of proving the incompleteness of consistent formal systems that are complex enough to encompass arithmetic, although some of these methods do not depend on the idea of statements with complete self-affinity like the Gödel sentence. These new methods, though, would not necessarily contradict the idea of fractalism. They would only confirm that each new method requires its own specific analysis.

⁷ The idea of 'true arithmetic' whose axioms consists of the infinite list of all true statements about the natural numbers (and no false statements), for example, is consistent and complete by definition but is not seen in this paper as a coherent theory because it does not 'stick' together.

⁸ Scale invariance is more strict than self-similarity and it would be more precise to say that exact self-similar fractals are scale invariant for only a discrete set of scales. Hence, the special theory of invariance involves exact self-similarity and the general theory of invariance involves scale invariance.

⁹ At best one could try to symbolise the general theory of invariance with one single symbol which could be taken apart into smaller forms while these individual forms would express a certain relationship with one another. As it happens, such a symbol already exists.

¹⁰ A perception of the inner world with overlapping double-vision is in theory capable of giving an insight with depth. This depth would correspond with the profoundness of the universe in the case that universal order matches fractal order. The general theory of invariance is grounded in the idea that universal order matches fractal order because then there would not exist any stumbling block to obstruct insight from being completely synthesised with the outer worldview which would mean that 'non-overlapping' could become 'overlapping' in a natural way. This could provide a consciousness with a certain realisation of universal profoundness, the analogue of depth perception with respect to common human eyesight. The possibility of such a perception would depend on whether the human brain is hard-wired for such a possible synthesis (i.e., the left-hemispherical brain is capable of working in harmony with the right-hemispherical brain) as well as the validity of the thesis that universal order matches fractal order.

This thesis, though, would not require for the universe to actually *be* scale invariant. It would imply, however, that *being* would present a strong tendency to mimic a fractal-like form.

¹¹ An attempt will be made here to give an example of how it is possible to use the 'meaningless' symbols of formalism whilst expressing exactly their own 'rules of inference' and this example intends to provide a more systematic understanding of fractal logic. The interpretation that guides this example, however, uses a certain fixed meaning with respect to the term 'principle' and therefore can only be seen as a facile explanation because this interpretation would validate the law of identity whereas it has already been made clear that this law needs to be altered in the case of fractal logic. Any linguistic explanation of fractal logic will always be subdued to a certain inner conflict because without any validity of the law of identity the potency of language would simply cave in. This would mean that the mental state due to the understanding of language mentioned in the introduction of this paper (or the understanding of a symbolic representation) could no longer be granted for.

Take the following principle called P.

P: The principle of a principle is no principle.

What is the principle of this suggested principle? If the principle of principle P was also a principle then principle P would not really be a principle but the immediate consequence of a principle, that is, it would be a rule. Accordingly, the principle of principle P cannot also be a principle. This reveals that the former suggested conception with respect to the concept 'principle' corresponds with what has been encoded within principle P—special scale invariance. Now take the following rule called \sim P.

\sim P: The principle of no principle is a principle.

What is the principle of this suggested rule? The suggested rule \sim P, which is no principle, must be based upon some kind of principle in order for it to be a rule. Hence, the principle of this rule must be a principle. This shows that the former suggested conception with respect to the concept 'rule' has been encoded within rule \sim P. Thus, in the case of fractal logic one can read the meaning of the concept 'principle' with regard to this system from this system because the meaning is expressed by both statements.

The system that is made up from the mentioned two statements P and \sim P can further be extended by adding the following two statements:

P^{-1} : The un-principle of a principle is no principle.

$\sim P^{-1}$: The un-principle of no principle is a principle.

Here the term 'un-principle' designates the inverse of the meaning of the concept 'principle' (e.g., 'consequence'). Due to this extension, the actual meaning of the term 'principle' becomes completely formulated ascribable to a certain symmetry of the total, yet the concept 'principle' remains undefined and therefore nonessential—compare this idea with the logic puzzle [Knights and Knaves](#). An inner conflict, however, is found in the defined interpretation of the terms 'un-', 'of', 'no' and 'is'.

Although the four statements carry a certain symmetry, this symmetry has lost coherency. The origin of this loss in coherency is found in the difference in interpretation between the word 'no' and the prefix 'un-'. For instance, 'undoing' differs from 'no doing'. The use of the term 'no' gives the system built up from P, \sim P, P^{-1} and $\sim P^{-1}$ a more complete appearance, yet it breaks coherency with respect to symmetry, i.e., the top-down 'principle of/'un-principle of' symmetry differs from the left-right 'principle/'no principle' symmetry—for an interesting analogy consider [the mirror question](#). The breaking of coherency with respect to symmetry eventually results in the appearance of a hierarchy which in turn creates the illusion that the scaffolding of knowledge which is being raised is leaning safely against a certain existing edifice. The fact that this supposed existing edifice is no more than an imaginary psychological model and therefore non-existent predetermines an eventual collapse of fixed knowledge. *Any* logical analysis is like building up a framework with neither proper foundation nor genuine support and is therefore predestined to subside. The illusory consequences of moving away from a synthesis (i.e., moving away from the bigger picture) by means of raising an analysis is nicely demonstrated by the [two envelopes paradox](#).

The four statements P , $\sim P$, P^{-1} and $\sim P^{-1}$ also consist of four elementary segments in the case that symmetry remained coherent; namely, 'un-' which would replace 'no', 'of', 'is' and a meaningless concept (e.g., 'principle'). The occurrence of exactly four elementary segments is also typified by, for example, the four known fundamental interactive forces of nature (electromagnetism, strong interaction, weak interaction and gravitation) but also by the four types of possible nucleobases for each rung of the spiral DNA ladder. Even so, it is more often that four elementary segments appear only in a threefold while the fourth component remains hidden or disguised. As already pointed out, the three dimensions of space with time being the disguised fourth dimension—time can be seen as the space of no space. Also, in language the three categories used in the classification of pronouns are the first, second and third person. But there is also a hidden fourth category, namely, the person of no person which is nobody. Additionally, take the separation of powers with respect to trias politica into an executive, a legislature, and a judiciary. Yet there is also a hidden fourth power, knowledge (scientia potentia est), which can be seen as the power of no power. The four ways of answering a question according to Buddha; a categorical answer (i.e., straight forwardly yes, no, this, that), an analytical qualified answer (i.e., defining or redefining terms), asking a counter-question, or by putting the question aside—the answer of no answer. If anybody would ever stumble upon the 'foundations of mathematics' then surely this would cause a new wave of scientific revolutions. The revelation of the 'foundations of mathematics' would provide the 'beginning' of the third scientific revolution which means that science would 'start' entering its fourth phase. The prediction here would therefore be that this fourth phase would encompass 'the science of no science' and this fourth phase would also be the last phase of some kind of recursive procedure—this fourth phase would be like encountering the fourth side of the square staircase found in M.C. Escher's [Ascending and Descending](#).

¹² This paper is no more than a collage of gathered ideas displayed in a linear order—pardon error.