Automorphic Functions And Fermat's Last Theorem（5）

Chun-Xuan Jiang

P. O. Box 3924, Beijing 100854, P. R. China
jiangchunxuan@sohu.com

Abstract: In 1637 Fermat wrote: "*It is impossible to separate a cube into two cubes, or a biquadrate into two biquadrates, or in general any power higher than the second into powers of like degree: I have discovered a truly marvelous proof, which this margin is too small to contain.*" This means: $x^n + y^n = z^n (n > 2)$ has no integer solutions, all different from 0(i.e., it has only the trivial solution, where one of the integers is equal to 0). It has been called Fermat's last theorem (FLT). It suffices to prove FLT for exponent 4 and every prime exponent P. Fermat proved FLT for exponent 4. Euler proved FLT for exponent 3[8]. In this paper using automorphic functions we prove FLT for exponents $6P$ and $2P$, where P is an odd prime. The proof of FLT must be direct. But indirect proof of FLT is disbelieving.

[Chun-Xuan Jiang. **Automorphic Functions And Fermat's Last Theorem（5）.** *Rep Opinion* 2012;4(8):21-24]. (ISSN: 1553-9873). http://www.sciencepub.net/report. 5

Keywords: automorphic function; cyclotomic field; Fermat; theorem

In 1974 Jiang found out Euler formula of the cyclotomic real numbers in the cyclotomic fields

$$\exp\left(\sum_{i=1}^{2n-1} t_i J^i\right) = \sum_{i=1}^{2n} S_i J^{i-1}, \tag{1}$$

where J denotes a $2n$ th root of negative unity, $J^{2n} = -1$, n is an odd number, t_i are the real numbers.

S_i is called the automorphic functions (complex trigonometric functions) of order $2n$ with $(2n-1)$ variables [5,7].

$$S_i = \frac{(-1)^{i-1}}{n}\left[e^H \cos\left(\beta + \frac{(i-1)\pi}{2}\right) + \sum_{j=0}^{\frac{n-3}{2}} e^{B_j} \cos\left(\theta_j + \frac{(i-1)(2j+1)\pi}{2n}\right)\right]$$

$$+\frac{1}{n}\sum_{j=0}^{\frac{n-3}{2}} e^{D_j} \cos\left(\phi_j - \frac{(i-1)(2j+1)\pi}{2n}\right), \tag{2}$$

where $i = 1, \ldots, 2n$;

$$H = \sum_{\alpha=1}^{n-1} t_{2\alpha}(-1)^\alpha, \quad \beta = \sum_{\alpha=1}^{n} t_{2\alpha-1}(-1)^{1+\alpha}$$

$$B_j = \sum_{\alpha=1}^{2n-1} t_\alpha (-1)^\alpha \cos\frac{(2j+1)\alpha\pi}{2n}, \quad \theta_j = \sum_{\alpha=1}^{2n-1} t_\alpha (-1)^{1+\alpha} \sin\frac{(2j+1)\alpha\pi}{2n},$$

$$D_j = \sum_{\alpha=1}^{2n-1} t_\alpha \cos\frac{(2j+1)\alpha\pi}{2n}, \quad \phi_j = \sum_{\alpha=1}^{2n-1} t_\alpha \sin\frac{(2j+1)\alpha\pi}{2n},$$

$$2H + 2\sum_{j=0}^{\frac{n-3}{2}} (B_j + D_j) = 0. \tag{3}$$

From (2) we have its inverse transformation[5,7]

$$e^{H}\cos\beta=\sum_{i=1}^{n}S_{2i-1}(-1)^{1+i},\ e^{H}\sin\beta=\sum_{i=1}^{n}S_{2i}(-1)^{1+i}$$

$$e^{B_j}\cos\theta_j=S_1+\sum_{i=1}^{2n-1}S_{1+i}(-1)^{i}\cos\frac{(2j+1)i\pi}{2n},$$

$$e^{B_j}\sin\theta_j=\sum_{i=1}^{2n-1}S_{1+i}(-1)^{1+i}\sin\frac{(2j+1)i\pi}{2n},$$

$$e^{D_j}\cos\phi_j=S_1+\sum_{i=1}^{2n-1}S_{1+i}\cos\frac{(2j+1)i\pi}{2n},$$

$$e^{D_j}\sin\phi_j=\sum_{i=1}^{2n-1}S_{1+i}\sin\frac{(2j+1)i\pi}{2n}. \qquad (4)$$

(3) and (4) have the same form.

Let $n=1$. We have $H=0$ and $\beta=t_1$. From (2) we have

$$S_1=\cos t_1,\quad S_2=\sin t_1 \qquad (5)$$

From (5) we have

$$\cos^2 t_1+\sin^2 t_1=1 \qquad (6)$$

(6) is Pythagorean theorem. It has infinitely many rational solutions.

From (3) we have

$$\exp[2H+2\sum_{j=0}^{\frac{n-3}{2}}(B_j+D_j)]=1. \qquad (7)$$

From (4) we have

$$\exp\left[2H+2\sum_{j=0}^{\frac{n-3}{2}}(B_j+D_j)\right]=\begin{vmatrix}S_1 & -S_{2n} & \cdots & -S_2\\ S_2 & S_1 & \cdots & -S_3\\ \cdots & \cdots & \cdots & \cdots\\ S_{2n} & S_{2n-1} & \cdots & S_1\end{vmatrix}=\begin{vmatrix}S_1 & (S_1)_1 & \cdots & (S_1)_{2n-1}\\ S_2 & (S_2)_1 & \cdots & (S_2)_{2n-1}\\ \cdots & \cdots & \cdots & \cdots\\ S_{2n} & (S_{2n})_1 & \cdots & (S_{2n})_{2n-1}\end{vmatrix} \qquad (8)$$

where

$$(S_i)_j=\frac{\partial S_i}{\partial t_j}\ [7]$$

From (7) and (8) we have circulant determinant

$$\exp\left[2H+2\sum_{j=0}^{\frac{n-3}{2}}(B_j+D_j)\right]=\begin{vmatrix}S_1 & -S_{2n} & \cdots & -S_2\\ S_2 & S_1 & \cdots & -S_3\\ \cdots & \cdots & \cdots & \cdots\\ S_{2n} & S_{2n-1} & \cdots & S_1\end{vmatrix}=1 \qquad (9)$$

If $S_i\neq 0$, where $i=1,2,\ldots,2n$, then (9) has infinitely many rational solutions.

Assume $S_1\neq 0, S_2\neq 0, S_i=0$, where $i=3,\ldots,2n$. $S_i=0$ are $(2n-2)$ indeterminate equations with $(2n-1)$ variables. From (4) we have

$$e^{2H}=S_1^2+S_2^2,\quad e^{2B_j}=S_1^2+S_2^2-2S_1S_2\cos\frac{(2j+1)\pi}{2n},$$

$$e^{2D_j}=S_1^2+S_2^2+2S_1S_2\cos\frac{(2j+1)\pi}{2n}. \qquad (10)$$

Example. Let $n = 15$. From (9) and (10) we have Fermat's equation

$$\exp[2H + 2\sum_{j=0}^{6} (B_j + D_j)] = S_1^{30} + S_2^{30} = (S_1^{10})^3 + (S_2^{10})^3 = 1. \quad (11)$$

From (3) we have

$$\exp[2H + 2\sum_{j=0}^{1} (B_{3j+1} + D_{3j+1})] = [\exp(-t_{10} + t_{20})]^{10}. \quad (12)$$

From (10) we have

$$\exp[2H + 2\sum_{j=0}^{1} (B_{3j+1} + D_{3j+1})] = S_1^{10} + S_2^{10}. \quad (13)$$

From (12) and (13) we have Fermat's equation

$$\exp[2H + 2\sum_{j=0}^{1} (B_{3j+1} + D_{3j+1})] = S_1^{10} + S_2^{10} = [\exp(-t_{10} + t_{20})]^{10} \quad (14)$$

Euler prove that (11) has no rational solutions for exponent 3[8]. Therefore we prove that (14) has no rational solutions for exponent 10.

Theorem [5,7]. Let $n = 3P$, where P is an odd prime. From (9) and (10) we have Fermat's equation.

$$\exp[2H + 2\sum_{j=0}^{\frac{3P-3}{2}} (B_j + D_j)] = S_1^{6P} + S_2^{6P} = (S_1^{2P})^3 + (S_2^{2P})^3 = 1. \quad (15)$$

From (3) we have

$$\exp[2H + 2\sum_{j=0}^{\frac{P-3}{2}} (B_{3j+1} + D_{3j+1})] = [\exp(-t_{2P} + t_{4P})]^{2P} \quad (16)$$

From (10) we have

$$\exp[2H + 2\sum_{j=0}^{\frac{P-3}{2}} (B_{3j+1} + D_{3j+1})] = S_1^{2P} + S_2^{2P}. \quad (17)$$

From (16) and (17) we have Fermat's equation

$$\exp[2H + 2\sum_{j=0}^{\frac{P-3}{2}} (B_{3j+1} + D_{3j+1})] = S_1^{2P} + S_2^{2P} = [\exp(-t_{2P} + t_{4P})]^{2P} \quad (18)$$

Euler prove that (15) has no rational solutions for exponent 3 [8]. Therefore we prove that (18) has no rational solutions for exponent $2P$ [5,7].

Remark. It suffices to prove FLT for exponent 4. Let $n = 4P$, where P is an odd prime. We have the Fermat's equation for exponent $4P$ and the Fermat's equation for exponent P [2,5,7]. This is the proof that Fermat thought to have had. In complex hyperbolic functions let exponent n be $n = \Pi P$, $n = 2\Pi P$ and $n = 4\Pi P$. Every factor of exponent n has Fermat's equation [1-7]. In complex trigonometric functions let exponent n be $n = \Pi P$, $n = 2\Pi P$ and $n = 4\Pi P$. Every factor of exponent n has Fermat's equation [1-7]. Using modular elliptic curves Wiles and Taylor prove FLT [9,10]. This is not the proof that Fermat thought to have had. The classical theory of automorphic functions, created by Klein and Poincarè, was concerned with the study of analytic functions in the unit circle that are invariant under a discrete group of transformation. Automorphic functions are the generalization of trigonometric, hyperbolic, elliptic, and certain other functions of elementary analysis. The automorphic functions (complex trigonometric functions and complex hyperbolic functions) have a wide application in mathematics and physics.

Acknowledgments

We thank Chenny and Moshe Klein for their help and suggestion.

References

[1] Jiang, C-X, Fermat last theorem had been proved, Potential Science (in Chinese), 2.17-20 (1992), Preprints (in English) December (1991). http://www.wbabin.net/math/xuan47.pdf.
[2] Jiang, C-X, Fermat last theorem had been proved by Fermat more than 300 years ago, Potential Science (in Chinese), 6.18-20(1992).
[3] Jiang, C-X, On the factorization theorem of circulant determinant, Algebras, Groups and Geometries, 11. 371-377(1994), MR. 96a: 11023, http://www.wbabin.net/math/xuan45.pdf
[4] Jiang, C-X, Fermat last theorem was proved in 1991, Preprints (1993). In: Fundamental open problems in science at the end of the millennium, T.Gill, K. Liu and E. Trell (eds). Hadronic Press, 1999, P555-558. http://www.wbabin.net/math/xuan46.pdf.
[5] Jiang, C-X, On the Fermat-Santilli theorem, Algebras, Groups and Geometries, 15. 319-349(1998)
[6] Jiang, C-X, Complex hyperbolic functions and Fermat's last theorem, Hadronic Journal Supplement, 15. 341-348(2000).
[7] Jiang, C-X, Foundations of Santilli Isonumber Theory with applications to new cryptograms, Fermat's theorem and Goldbach's Conjecture. Inter. Acad. Press. 2002. MR2004c:11001, http://www.wbabin.net/math/xuan13.pdf. http://www.i-b-r.org/docs/jiang.pdf
[8] Ribenboim, P, Femat's last theorem for amateur. Springer, New York, 1999.
[9] Wiles A, Modular elliptic curves and Fenmat's last theorem, Ann of Math, (2) 141 (1995), 443-551.
[10] Taylor, R. and Wiles, A., Ring-theoretic properties of certain Hecke algebras, Ann. of Math., (2) 141(1995), 553-572.

7/1/2012