Similarity solution for natural convection from a moving vertical plate with internal heat generation and a convective boundary condition in the presence of thermal radiation and viscous dissipation

Olanrewaju, P.O.

Department of Mathematics, Covenant University, Ota, Nigeria.
oladapo_anu@yahoo.ie, philip.olanrewaju@covenantuniversity.edu.ng

Abstract: In this paper we analyze the influences of thermal radiation, viscous dissipation, buoyancy force and internal heat generation on steady laminar convection flow over a semi-moving vertical plate in the presence of a convective surface boundary condition. In the analysis, we assumed that the left surface of the plate is in contact with a hot fluid while the cold fluid on the right surface of the plate contains a heat source that decays exponentially with the classical similarity variable. We utilized similarity variable to transform the governing non-linear partial differential equations into a system of ordinary differential equations, which are solved numerically by applying shooting iteration technique alongside with sixth order Runge-Kutta integration scheme for better accuracy. The effects of the local Biot number, the Prandtl number, the internal heat generation parameter, the thermal radiation, the local Grashof number and the Eckert number on the velocity and temperature profiles are illustrated and interpreted in physical terms. A comparison with previously published results on similar special cases showed excellent agreement. Finally, numerical values of physical quantities, such as the local skin-friction coefficient and the local Nusselt number are presented in tabular form.
[Olanrewaju, P.O. **Similarity solution for natural convection from a moving vertical plate with internal heat generation and a convective boundary condition in the presence of thermal radiation and viscous dissipation.** *Rep Opinion* 2012;4(8):68-76]. (ISSN: 1553-9873). http://www.sciencepub.net/report. 12

Key words: **:** viscous dissipation; thermal radiation; moving vertical plate; local Grashof number; local Biot number; Eckert number; convective surface boundary condition.

1. Introduction

The problem of laminar hydrodynamic and thermal boundary layers over the flat plate in a uniform stream of fluid is a thoroughly researched problem in fluid mechanics. The velocity distribution in the hydrodynamic boundary layer is given by the well known Blasius similarity solution. The similarity solution for the thermal boundary layer for the case of constant surface temperature at the plate is also well established and widely quoted in heat transfer textbooks such as [1]. For the boundary condition of constant heat flux at the plate, Kays and Crawford [2] claimed that a similarity solution does not exist.

However, Bejan [3] refuted their claim by suggesting a different similarity temperature variable which reduced the energy equation to an ordinary differential equation. Although numerous studies such as [4,5] have considered different variations of temperature and heat flux at the plate.

The first and foremost work regarding boundary-layer behaviour in moving surfaces in a quiescent fluid was performed by Sakiadis [6] and many researchers [7-14] have worked on the problem of moving or stretching plates under different situations. In the boundary-layer theory, similarity solutions were found to be useful in the interpretation of certain fluid motions at large Reynolds numbers. Similarity solutions often exist for the flow over semi-infinite plates and stagnation point flow for two-dimensional, axisymmetrical and three-dimensional bodies. In special cases, when there is no similarity solution, one has to solve a system of non-linear partial differential equations. For similarity boundary-layer flows, velocity profiles are similar. But this kind of similarity is lost for non-similarity flows (see [15-18]. Obviously, the non-similarity boundary-layer flows are more general in nature and are more important, not only in theory but also in application.

The heat-transfer analysis of boundary-layer flows with radiation is also important in electrical power generation, astrophysical flows, solar power technology, space vehicle re-entry and other industrial areas. Extensive literature that deals with flows in the presence of radiation effects is now available. Raptis et al. [19] studied the effect of thermal radiation on the magnetohydrodynamic flow of a viscous fluid past a semi-infinite stationary plate. Hayat et al. [20] extended the analysis of reference [19] for a second-grade fluid.

Convective heat transfer studies are very important in processes involving high temperatures, such as gas turbines, nuclear plants and thermal energy storage. Recently, Ishak [21] examined the similarity solutions for flow and heat transfer over a permeable surface with convective boundary condition. Moreover, Aziz [22] studied a similarity solution for laminar thermal boundary layer over a flat plate with a convective surface boundary condition and also studied hydrodynamic and thermal slip flow boundary layers over a flat plate with a constant heat flux boundary condition. Makinde and Olanrewaju [23] investigated the buoyancy effects on a thermal boundary layer over a vertical plate with a convective surface boundary condition. More recently, Makinde [24] studied similarity solution for natural convection from a moving vertical plate with internal heat generation and a convective boundary condition. Olanrewaju et al. [25] examined the effects of internal heat generation, thermal radiation and buoyancy force on a boundary layer over a vertical plate with a convective surface boundary condition. Makinde and Olanrewaju [26] investigated the combined effects of internal heat generation and buoyancy force on boundary layer flow over a vertical plate with a convective surface boundary condition

Viscous dissipation changes the temperature distributions by playing a role like an energy source, which leads to affected heat transfer rates. The merit of the effect of viscous dissipation depends on whether the plate is being cooled or heated. Heat transfer analysis over porous surface is of much practical interest due to its abundant applications. To be more specific, heat-treated materials traveling between a feed roll and wind-up roll or materials manufactured by extrusion, glass-fiber and paper production, cooling of metallic sheets or electronic chips, crystal growing just to name a few. In these cases, the final product of desired characteristics depends on the rate of cooling in the process and the process of stretching. The work of Sonth et al. [27] deals with the effect of the viscous dissipation term along with temperature dependent heat source/sink on momentum, heat and mass transfer in a visco-elastic fluid flow over an accelerating surface. Chen [28] examined the effect of combined heat and mass transfer on MHD free convection from a vertical surface with ohmic heating and viscous dissipation. The effect of viscous dissipation and Joule heating on MHD free convection flow past a semi-infinite vertical flat plate in the presence of the combined effect of Hall and non-slip currents for the case of power-law variation of the wall temperature is analyzed by Abo-Eldahab and El Aziz [29].

In many new engineering areas processes such as fossil fuel combustion energy processes, solar power technology, astrophysical flows, gas turbines and the various propulsion devices for aircraft, missiles, satellites, and space vehicle re-entry occur at high temperatures so knowledge of radiation heat transfer beside the convective heat transfer plays a very important role and hence its effect cannot be neglected. Also thermal radiation is of major importance in many processes in engineering areas which occur at a very high temperature for the design of many advanced energy conversion systems and pertinent equipment. The Rosseland approximation is used to describe the radiative heat flux in the energy equation. Pal and Mondal [30] investigate the unsteady two-dimensional MHD non-Darcian mixed convection heat and mass transfer past a semi-infinite vertical permeable plate embedded in a porous medium by taking into account of Soret and Dufour effects in the presence of suction or injection, thermal radiation and first-order chemical reaction. Hence the present study investigates the similarity solution for natural convection from a moving vertical plate with internal heat generation and a convective boundary condition in the presence of thermal radiation and viscous dissipation which is an extension of Makinde [24] with the addition of thermal radiation and viscous dissipation for more physical implications. Using a similarity approach, the governing equations are transformed into nonlinear ordinary equations and solved numerically using shooting iteration technique together with sixth order Runge-Kutta integration scheme. The pertinent results are displayed graphically and discussed quantitatively.

1. 2. Mathematical formulation

We consider the steady laminar incompressible natural convection boundary layer flows over the right surface of a vertical flat plate moving with uniform velocity U_0 in contact with a quiescence cold fluid at temperature T_∞. The cold fluid on the right surface of the plate generates heat internally at the volumentric rate $\dot{q}$. The left surface of the plate is heated by convection from a hot fluid at temperature T_f which provides a heat transfer coefficient h_f as shown in fig. 1. Under the Boussinesq for fluid density variation, the continuity, momentum, and energy equations describing the flow can be written as:

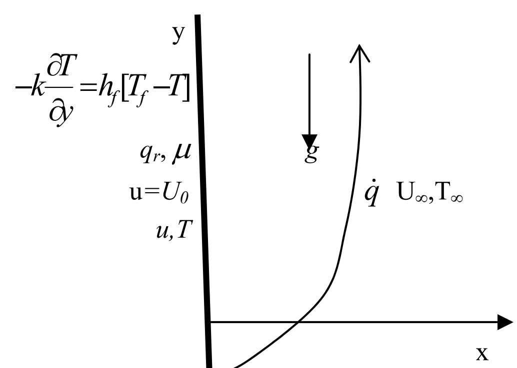

Fig. 1. Flow configuration and coordinate system

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty), \qquad (2)$$

$$\rho c_p\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+\dot{q}-\frac{\partial q_r}{\partial y}+\mu\left(\frac{\partial u}{\partial y}\right)^2, \quad (3)$$

where u and v are the x (along the plate) and the y (normal to the plate) components of the velocities, respectively, T is the temperature, μ is the fluid viscosity, υ is the kinematics viscosity of the fluid, and k is the thermal conductivity of the fluid and β is the thermal expansion coefficient, $\dot{q}$ is the internally generated heat at volumentric rate, g is the gravitational acceleration and q_r is the radiative heat flux, respectively. The velocity boundary conditions can be expressed as

$$u(x,0)=U_0, v(x,0)=0, \quad (4)$$

$$u(x,\infty)=0. \quad (5)$$

The boundary conditions at the plate surface and far into the cold fluid may be written as

$$-k\frac{\partial T}{\partial y}(x,0)=h_f[T_f-T(x,0)], \quad (6)$$

$$T(x,\infty)=T_\infty. \quad (7)$$

The radiative heat flux q_r is described by Roseland approximation such that

$$q_r=-\frac{4\sigma^*}{3K}\frac{\partial T^4}{\partial y}, \quad (8)$$

where σ^* *and* K are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. Following Chamkha [31], we assume that the temperature differences within the flow are sufficiently small so that the T^4 can be expressed as a linear function after using Taylor series to expand T^4 about the free stream temperature T_∞ and neglecting higher-order terms. This result is the following approximation:

$$T^4\approx 4T_\infty^3 T-3T_\infty^4. \quad (9)$$

Using (8) and (9) in (3), we obtain

$$\frac{\partial q_r}{\partial y}=-\frac{16\sigma^*}{3K}\frac{\partial^2 T^4}{\partial y^2}. \quad (10)$$

Introducing a similarity variable η and a dimensionless stream function $f(\eta)$ and temperature $\theta(\eta)$ as:

$$\eta=y\sqrt{\frac{U_0}{\upsilon x}}=\frac{y}{x}\sqrt{\mathrm{Re}_x},$$

$$\frac{u}{U_0}=f', v=\frac{1}{2x}\sqrt{\mathrm{Re}_x}(\eta f'-f),$$

$$\theta=\frac{T-T_\infty}{T_f-T_\infty}, \quad (11)$$

where prime symbol denotes differentiation with respect to η and $Re_x=U_0x/\upsilon$ is the local Reynolds number. Eqs. (1) – (7) reduce to

$$f'''+\frac{1}{2}ff''+Gr_x\theta=0, \quad (12)$$

$$\theta''[1+\frac{4}{3}Ra]+\frac{1}{2}\mathrm{Pr}f\theta'+\lambda_x e^{-\eta}+Ec\mathrm{Pr}(f'')^2=0, \quad (13)$$

$$f(0)=0, f'(0)=1,$$

$$\theta'(0)=-Bi_x[1-\theta(0)], \quad (14)$$

$$f'(\infty)=0, \quad \theta(\infty)=0, \quad (15)$$

where

$$Bi_x=\frac{h_f}{k}\sqrt{\frac{\upsilon x}{U_0}}, \mathrm{Pr}=\frac{\upsilon}{\alpha}, Gr_x=\frac{xg\beta(T_f-T_\infty)}{U_0^2},$$

$$Ra=\frac{4\alpha\sigma^* T_\infty^3}{kK}, \quad \lambda_x=\frac{x^2\dot{q}e^{\eta}}{k\mathrm{Re}_x(T_f-T_\infty)}, Ec=\frac{U_0^2}{k(T_f-T_\infty)} \quad (16)$$

For the momentum and energy equations to have a similarity solution, the parameters Gr_x , λ_x, and Bi_x must be constants and not functions of x as in Eq. (16). This condition can be met if the heat transfer coefficient h_f is proportional to $x^{-1/2}$, the thermal expansion coefficient β is proportional to x^{-1} and the l internal generation $\dot{q}$ is proportional to x^{-1} . We therefore assume

$$h_f=cx^{-\frac{1}{2}}, \beta=mx^{-1}, \quad \dot{q}=lx^{-1}, e^{-\eta} \quad (17)$$

where *c, m, and l* are constants. Substituting Eq. (17) into Eq. (16), we have

$$Bi=\frac{c}{k}\sqrt{\frac{\upsilon}{U_0}}, \quad Gr=\frac{mg(T_f-T_\infty)}{U_0^2},$$

$$\lambda=\frac{l\upsilon}{kU_0(T_f-T_\infty)}, \quad (18)$$

With *Bi, λ, and* Gr defined by Eq. (18), the solutions of Eqs. (12)- (15) yield the similarity solutions, however, the solutions generated are the local similarity solutions whenever Bi_x , λ_x *and* Gr_x are defined as in Eq. (13).

3. Numerical solutions

The coupled non-linear equations (12) – (13) with the boundary conditions in Eqs. (14) – (15) are solved numerically using the sixth-order Runge-Kutta method with a shooting technique and implemented on Maple [32]. The step size 0.001 is used to obtain the numerical solution with seven-decimal place accuracy as the criterion of convergence.

4. Results and discussion

Numerical calculations have carried out for different values of the thermophysical parameters controlling the fluid dynamics in the flow regime. The Prandtl number used are 0.72, 1, 3, 7.1, convective parameter Bi_x used are 0.05, 0.10, 0.20, 0.40, 0.60, 0.80, 1, 5, 10, 20, the Grashof number (Grashof number Gr_x) used are $Gr > 0$ (which corresponds to the cooling problem), the thermal radiation used are 0.1, 0.5, 1 and the Eckert number used are 1, 2 and 5. The cooling problem is often encountered in engineering applications; for example in the cooling of electronic components and nuclear reactors. Comparisons of the present results with previously work is performed and excellent agreement has been obtained. We obtained the results as shown in tables (1)- (2) and figures (2)-(13) below;

Table 1

A comparison of the values obtained for the local skin-friction, the local Nusselt number and the surface temperature by Makinde [a] in the absent of thermal radiation and Eckert number, i.e. $Ra = Ec = 0$.

Bi_x	Gr_x	Pr	λ_x	$f''(0)$ Makinde[24]	$\theta'(0)$ Makinde[24]	$\theta(0)$ Makinde[24]	$f''(0)$ Present	$\theta'(0)$ Present	$\theta(0)$ Present
0.1	0.1	0.72	1	-0.2000518	0.076578477	1.76578477	-0.2000519	0.076578478	1.76578478
1.0	0.1	0.72	1	-0.2459676	0.281651449	1.28165144	-0.2459676	0.281651449	1.28165145
10	0.1	0.72	1	-0.2695171	0.382952717	1.03829527	-0.2695171	0.382952718	1.038295272
0.1	0.5	0.72	1	0.4221216	0.048257030	1.48257030	0.4221217	0.048257030	1.482570304
0.1	1.0	0.72	1	0.9895493	0.034011263	1.34011263	0.9895494	0.034011263	1.340112630
0.1	0.1	3.00	1	-0.3748695	-0.023814576	0.76185423	-0.3748696	-0.02381458	0.761854232
0.1	0.1	7.10	1	-0.4138825	-0.057164001	0.42835998	-0.4138825	-0.057164002	0.428359981
0.1	0.1	0.72	5	0.3741286	0.576670381	6.76670381	0.3741286	0.5766703814	6.766703814
0.1	0.1	0.72	10	0.9010790	1.106605802	12.0660580	0.9010791	1.1066058028	12.06605803

Table 2

Computation showing $f''(0)$, $\theta'(0)$ and $\theta(0)$ for different embedded flow parameter values

Bi_x	Gr_x	Pr	λ_x	Ec	Ra	$f''(0)$	$\theta'(0)$	$\theta(0)$
0.1	0.1	0.72	1	1	0.1	-0.00559356550615563	0.216556134999959516	3.16556134999959492
1.0	0.1	0.72	1	1	0.1	-0.14433555929307157	0.734173416277385416	1.73417341627738653
10	0.1	0.72	1	1	0.1	-0.20982197423365378	0.950599829815936381	1.09505998298159346
0.1	0.5	0.72	1	1	0.1	1.94922003941417632	0.344568434522710964	4.44568434522710910
0.1	1.0	0.72	1	1	0.1	5.33467259894761092	0.551527748304988630	6.51527748304989007
0.1	0.1	3.00	1	1	0.1	-0.18765862159613869	0.178943107428566034	2.78943107428565940
0.1	0.1	7.10	1	1	0.1	-0.22883280272867598	0.198605192198536200	2.98605192198536296
0.1	0.1	0.72	5	1	0.1	0.573612212020427003	0.729613352819421856	8.29613352819422366
0.1	0.1	0.72	10	1	0.1	1.10770291495196527	1.26695594901374964	13.6695594901375036
0.1	0.1	0.72	1	2	0.1	0.241670685752199021	0.414742971040799768	5.14742971040799802
0.1	0.1	0.72	1	5	0.1	1.34977954076616834	1.44357448270189437	15.4357448270189419
0.1	0.1	0.72	1	1	0.5	-0.00758112230575465	0.181454592477094334	2.81454592477094279
0.1	0.1	0.72	1	1	1.0	-0.01813084843143244	0.150980890223783970	2.50980890223783915

Table 1 shows the comparison of Makinde [24] work with the present work for Ec = Ra = 0 and it noteworthy that there is a perfect agreement. Table 2, shows the values of the skin-friction coefficient, the Nusselt number and the surface temperature in terms of $f''(0)$, $\theta'(0)$, $\theta(0)$ respectively for various values embedded flow parameters. From Table 2, it is understood that the skin-friction, the rate of heat transfer and the wall surface temperature at the plate surface increases with an increase in local Grashof number, the internal heat generation and the Eckert number. However, an increase in the convective surface heat transfer parameter, the local Biot number and the radiation parameter decreases the skin-friction but increases the rate of heat transfer and the wall temperature at the plate surface except that increasing radiation parameter decreases the rate of heat transfer at the wall plate and the wall temperature. Similarly, from figure2, it was clearly seen that increases in the Prandtl number increases the skin-friction at the plate surface but increases the rate of heat transfer and the surface wall temperature at the plate wall surface.

A. Velocity profiles

Figures 2-7 depicts the effects of various thermophysical parameters on the fluid velocity profile. It was observed that generally, the fluid velocity increases gradually away from the plate, attain its peak value within the boundary layer and the decreases to the free stream zero value satisfying the boundary conditions. From figure 2, it was observed that increases the local Grashof number thickens the velocity boundary layer thickness across the flow channel. Figure 3 depicts the effects of convective surface heat transfer parameter on the fluid velocity and it was observed that increases in the convective surface heat transfer or local Biot number thinning the velocity boundary layer thickness. In figure 4, the influence of Prandtl number on the fluid velocity was displayed and it is interesting to note that velocity boundary layer thickness decreases with an increase in the Prandtl number. Figure 5 depicts the effects of local internal heat generation parameter on the fluid velocity. An increase in the exponentially decaying internal heat generation causes a further increase in the velocity boundary layer thickness. Similarly, figure 6 depicts the influence of thermal radiation on the fluid velocity and it is interesting to note that increases the radiation parameter thickens the velocity boundary layer thickness away from the plate surface. Figure 7 represents the curve of fluid velocity against spanwise coordinate η for various values of Eckert number which shows that increase in Eckert number leads to a sudden increase in the fluid velocity immediately away from the wall plate before satisfying the boundary conditions. It is interesting to note that it thickens the velocity boundary layer thickness close to the wall plate when the velocity profile attains its maximum value point.

B. Temperature profiles

Figures 8-13 illustrate the fluid temperature profiles within the boundary layer. Generally, the fluid temperature is maximum at the plate surface and decreases exponentially to zero value far away from the plate satisfying the boundary conditions. From this figures, it is noteworthy that the thermal boundary layer thickness increases with an increases with an increase in exponentially decaying internal heat generation and the Eckert number and decreases with an increase in the values of local Grashof number, local Biot number, Prandtl number and thermal radiation. At high Prandtl fluid has low velocity, which in turn also implies that that at lower fluid velocity the specie diffusion is comparatively lower and hence higher specie concentration is observed at high Prandtl number. It is interesting to note that as thermal radiation increases, the thermal boundary layer thickness decreases at the wall surface plate but a little away from the wall surface plate it thickens the thermal boundary layer thickness satisfying the boundary conditions.

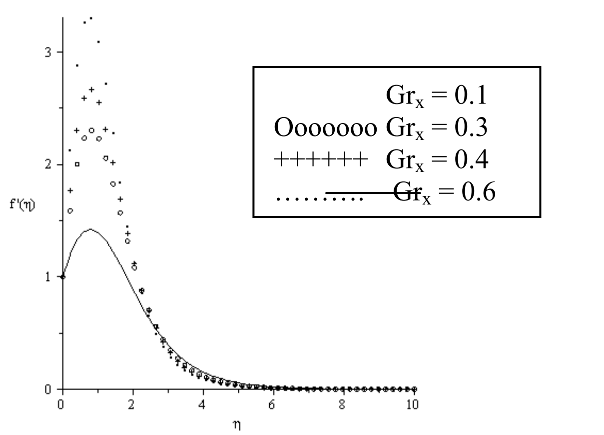

Figure 2: Effects of local Grashof number on the velocity profile for Pr = 0.72, Bi_x = 0.1, λ_x =10, Ra = 0.1, Ec = 1.

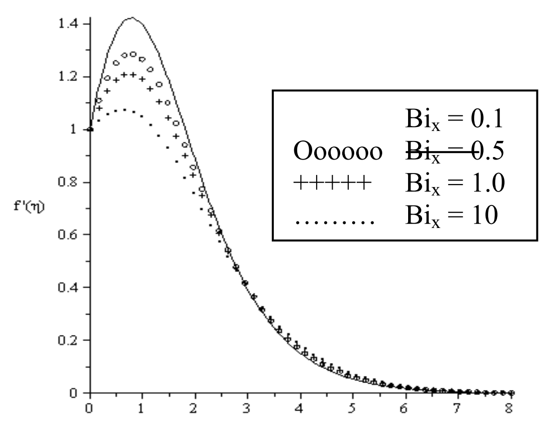

Figure 3: Effects of local Biot number on the velocity profile for Pr = 0.72, , λ_x =10, Ra = 0.1, Ec = 1, Ra = 0.1.

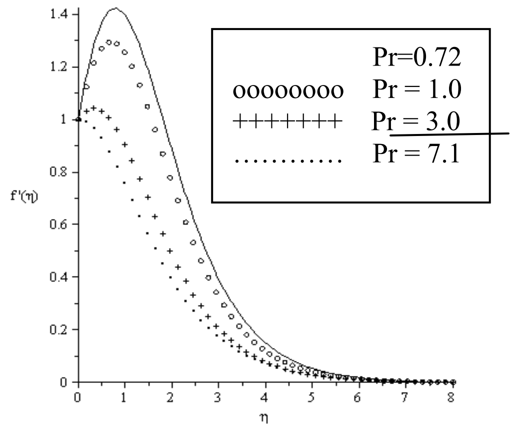

Figure 4: Effects of Prandtl number on the velocity profile for Bi_x = 0.1, λ_x =10, Ra = 0.1, Ec = 1, Ra = 0.1.

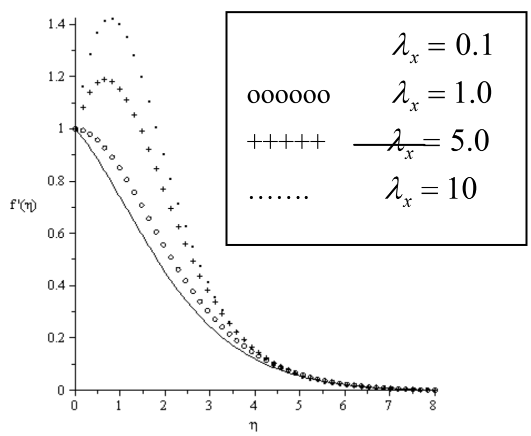

Figure 5: Effects of internal heat generation on the velocity profile for Bi_x = 0.1, Pr =0.72, Ra = 0.1, Ec = 1, Ra = 0.1.

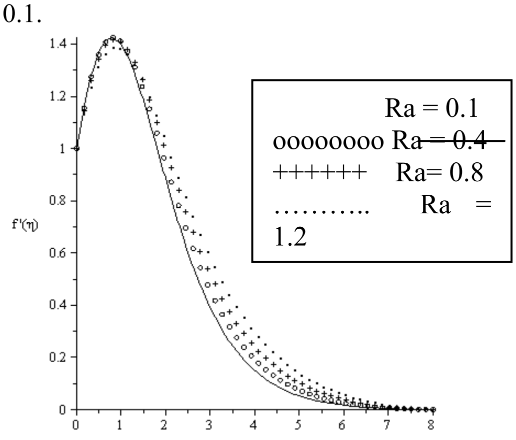

Figure 6: Effects of Radiation parameter on the velocity profile for Bi_x = 0.1, Pr =0.72, λ_x = 10, Ec = 1, Gr_x = 0.1.

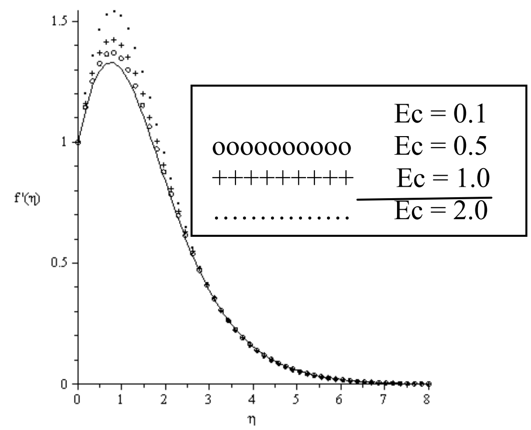

Figure 7: Effects of Eckert number on the velocity profile for Bi_x = 0.1, Pr =0.72, λ_x = 10, Gr_x = 0.1, Ra = 0.1.

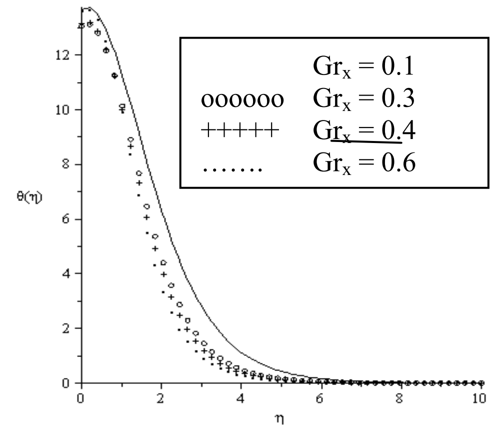

Figure 8: Effects of local Grashof number on the temperature profile for Pr = 0.72, Bi_x = 0.1, λ_x =10, Ra = 0.1, Ec = 1.

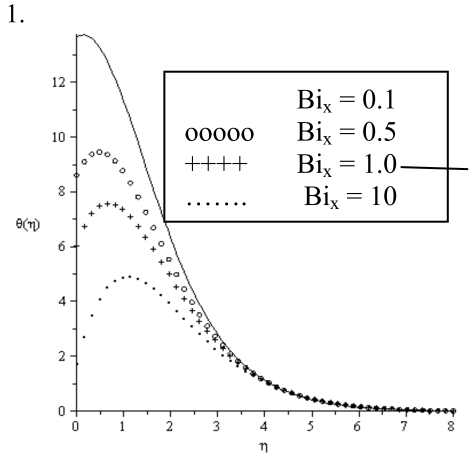

Figure 9: Effects of local Biot number on the temperature profile for Pr = 0.72, λ_x =10, Ra = 0.1, Ec = 1, Ra = 0.1.

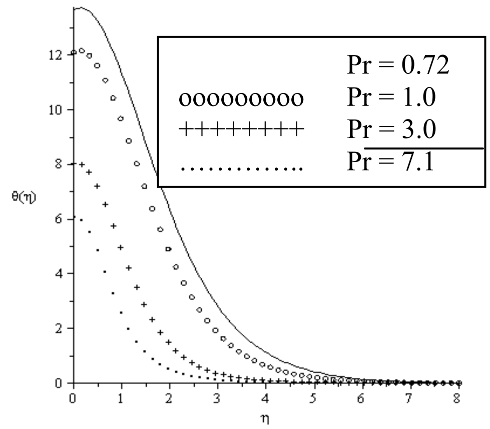

Figure 10: Effects of Prandtl number on the temperature profile for $Bi_x = 0.1$, $\lambda_x = 10$, Ra = 0.1, Ec = 1, Ra = 0.1.

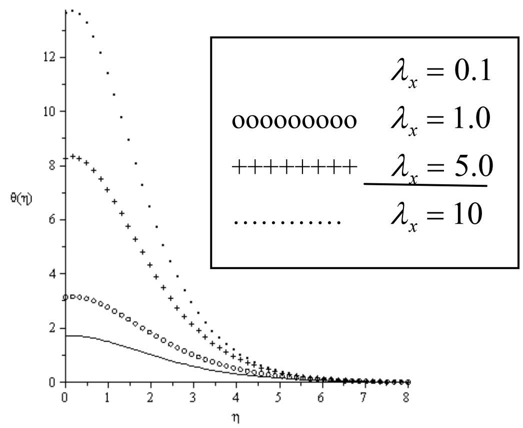

Figure 11: Effects of internal heat generation on the temperature profile for $Bi_x = 0.1$, Pr =0.72, Ra = 0.1, Ec = 1, Ra = 0.1.

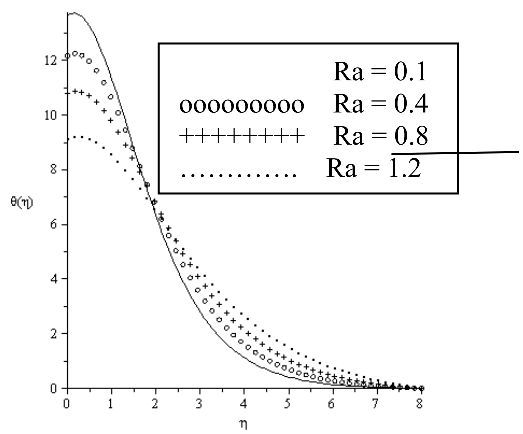

Figure 12: Effects of Radiation parameter on the temperature profile for $Bi_x = 0.1$, Pr =0.72, $\lambda_x = 10$, Ec = 1, $Gr_x = 0.1$.

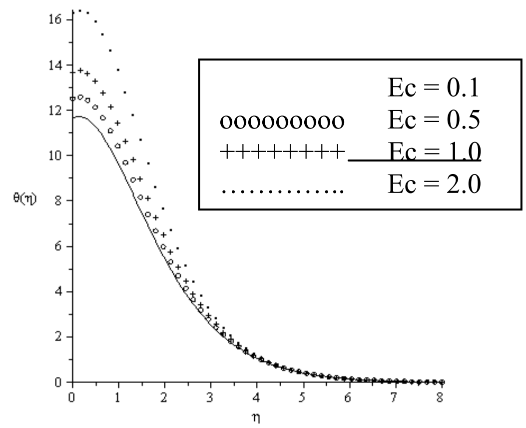

Figure 13: Effects of Eckert number on the temperature profile for $Bi_x = 0.1$, Pr =0.72, $\lambda_x = 10$, $Gr_x = 0.1$, Ra = 0.1.

5. Conclusions

The Similarity solution for natural convection from a moving vertical plate with internal heat generation and a convective boundary condition in the presence of thermal radiation and viscous dissipation is studied. A set of non-linear coupled differential equations governing the fluid velocity and temperature is solved numerically for various material parameters. A comprehensive set of graphical results for the velocity and temperature is presented and discussed. Our results reveal among others, that the internal heat generation, thermal radiation and the Eckert number prevents the flow of heat from the left surface to the right surface of the plate unless the local Grashof number is strong enough to convert away both the internally generated heat in the fluid. Generally, the fluid velocity increases gradually away from the plate, attain its peak value within the boundary layer and the decreases to the free stream zero value satisfying the boundary conditions. It is interesting to note that the fluid velocity within the boundary layer increases with increasing values of exponentially decaying internal heat generation, thermal radiation and the Eckert number a little away from the wall plate and attain its peak before obeying the boundary conditions.

Acknowledgements

The author thanks the Covenant University Management for their financial support for the work and the effort of Professor Jacob Gbadeyan for his effort in completing the research work on time.

Nomenclature

Bi_x - local Biot number, [-]
c_p - specific heat at constant pressure, [-]
c, m, l - positive constants, [-]
f - dimensionless stream function, [-]
Gr_x - local Grashof number, [-]

Ra - thermal radiation parameter, [-]
Ec - Eckert number, [-]
g - gravitational acceleration, [Ls^{-2}]
k - thermal conductivity, [$Wm^{-1}K^{-1}$]
Pr - Prandtl number, [-]
Re_x - local Reynolds number, [-]
T - fluid temperature, [K]
T_∞ - free stream temperature, [K]
T_f - hot fluid temperature, [K]
U_∞ - free stream velocity, [Ls^{-1}]
u, v - Cartesian co-ordinates, [m]

Greek letters

α thermal diffusivity, [m^2s^{-1}]
β thermal expansion coefficient, [K^{-1}]
υ - kinematic viscosity, [m^2s^{-1}]
λ - internal heat generation parameter, [-]
η - similarity variable, [m]
ρ - fluid density, [kgm^{-3}]
θ - dimensionless temperature, [-]

Correspondence: Olanrewaju, Philip Oladapo (PhD)
Department of Mathematics
Covenant University, Ota, Nigeria
P. M. B. 1023.
oladapo_anu@yahoo.ie,
philip.olanrewaju@covenantuniversity.edu.ng

References

[1] Incropera FP et al. Fundamentals of heat and mass transfer. 6th ed. New York: John Wiley; 2007.
[2] Kays WM, Crawford ME. Convective heat and mass transfer. New York: McGraw Hill; 1980. p. 51–4.
[3] Bejan A. Convective heat transfer. 3rd ed. New York: John Wiley; 2004. p. 84.
[4] Rogers David F. Laminar flow analysis. New York: Cambridge University Press; 1992. p. 13–139.
[5] Shu J-J, Pop I. On thermal boundary layers on a flat plate subjected to a variable heat flux. Int J Heat Fluid Flow 1988; 19:79–84.
[6] Sakiadis BC. Boundary-layer behavior on continuous solid surfaces. I. Boundary-layer equations for two-dimensional and axisymmetric flow. AIChE J. 1961; 7:26–28. doi:10.1002/aic.690070108
[7] Crane IJ. Flow past a stretching plate. Z Angew Math Phys. 1970;21(56):1– 37.
[8] Gupta PS, Gupta AS. Heat and mass transfer on a stretching sheet with suction and blowing. Can J Chem Eng. 1977; 55:744–746. doi: 10.1002/ cjce.5450550619
[9] Carragher P, Crane IJ. Heat transfer on a continuous stretching sheet. Z Angew Math Mech. 1982; 62:564–565. doi:10.1002/zamm.19820621009
[10] Danberg JE, Fansler KS. A nonsimilar moving wall boundary-layer problem. Quart Appl Math. 1976; 34:305–309.
[11] Chakrabarti A, Gupta AS. Hydromagnetic flow and heat transfer over a stretching sheet. Quart Appl Math. 1979; 37:73–78.
[12] Vajravelu K. Hydromagnetic flow and heat transfer over a continuous moving porous, flat surface. Acta Mech. 1986; 64:179–185. doi: 10.1007/ BF01450393
[13] Dutta BK. Heat transfer from a stretching sheet with uniform suction and blowing. Acta Mech. 1986; 78:255–262. doi: 10.1007/BF01179221
[14] Lee SL, Tsai JS. Cooling of a continuous moving sheet of finite thickness in the presence of natural convection. Int J Heat Mass Transfer. 1990; 33:457– 464. doi: 10.1016/0017-9310(90)90181-S
[15] Wanous KJ, Sparrow EM. Heat transfer for flow longitudinal to a cylinder with surface mass transfer. J Heat Transf ASME Ser C. 1965; 87(1):317–319.
[16] Catherall D, Stewartson K, Williams PG. Viscous flow past a flat plate with uniform injection. Proc R Soc A. 1965; 284:370–396. doi: 10.1098/ rspa.1965.0069
[17] Sparrow EM, Quack H, Boerner CJ. Local non-similarity boundary layer solutions. J AIAA 1970; 8(11):1936–1942.
[18] Sparrow EM, Yu HS. Local non-similarity thermal boundary-layer solutions. J Heat Transf ASME. 1971; 93:328–334.
[19] Raptis A, Perdikis C, Takhar HS. Effects of thermal radiation on MHD flow. Appl Math Comput. 2004; 153:645–649. doi: 10.1016/S0096-3003(03)00657-X
[20] Hayat T, Abbas Z, Sajid M, Asghar S, The influence of thermal radiation on MHD flow of a second grade fluid. Int J Heat Mass Transfer. 2007; 50:931– 941. doi:10.1016/j.ijheatmasstransfer.2006.08.014
[21] Ishak A. Similarity solutions for flow and heat transfer over a permeable surface with convective boundary condition. Appl. Math. Comput. 2010; 217:837–842. doi:10.1016/j.amc.2010.06.026
[22] Aziz A. A similarity solution for laminar thermal boundary layer over a flat plate with a convective surface boundary condition. Commun Nonlinear Sci Numer Simulat. 2009; 14:1064–1068.
[23] Makinde OD, Olanrewaju PO. Buoyancy effects on thermal boundary layer over a vertical plate with a convective surface boundary condition. J Fluids Eng. 2010; 132:044502–0044505. doi:10.1115/1.4001386.

[24] Makinde, DO, Similarity solution for natural convection from a moving vertical plate with internal heat generation and a convective boundary condition. Thermal Science, 2011, vol. 15,Suppl. 1. Pp. S137-S143.
[25] Olanrewaju, PO, Gbadeyan, JA, Tasawar, H, Hendi, AA, Effects of internal heat generation, thermal radiation and buoyancy force on a boundary layer over a vertical plate with a convective surface boundary condition. S. Afr. J. Sci. 2011; 107(9/10), Art. #476, 6 pages. Doi:10.41o2/sajs.v107i9/10.476.
[26] Makinde, DO, Olanrewaju, PO, Combined effects of internal heat generation and buoyancy force on boundary layer flow over a vertical plate with a convective surface boundary condition. The Canadian Journal of Chemical Engineering. Doi: 10.1002/cjce.20614
[27] R.M. Sonth, S.K. Khan, M. S. Abel, K.V. Prasad, Heat and mass transfer in a visco-elastic fluid over an accelerating surface with heat source/sink and viscous dissipation, *Heat Mass Transfer*, **38**, pp. 213–220, 2002.
[28] C.H. Chen, Combined heat and mass transfer in MHD free convection from a vertical surface with ohmic heating and viscous dissipation, *Int. J. Eng. Sci.*, **42**, pp. 699–713, 2004.
[29] E.M. Abo-Eldahab, M.A. El Aziz, Viscous dissipation and Joule heating effects on MHD-free convection from a vertical plate with power-law variation in surface temperature in the presence of Hall and ios-slips currents, *Appl. Math. Model.* **29**, pp. 579–595, 2005.
[30] Pal, D and Mondal, H, Effects of Soret Dufour, chemical reaction and thermal radiation on MHD non-Darcy unsteady mixed convective heat and mass transfer over a stretching sheet," Communications in Nonlinear Science and Numerical Simulation, Volume 16, Issue 4, April 2011, Pages 1942-1958.
[31] A. J. Chamkha (1997). Hydromagnetic natural convection from an isothermal inclined surface adjacent to a thermally stratified porous medium," International Journal of Engineering Science, vol. 37, no. 10-11, pp. 975-986.
[32] A. Heck, Introduction to Maple, 3rd Edition, Springer-Verlag, 2003

7/31/2012