A New Integration of Logarithmic Fuzzy Preference Programming and GTMA Method for Technology Selection

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Abstract: Selecting the right technology is always a difficult task for decision-makers. Technologies have varied strengths and weaknesses which require careful assessment by the purchasers. The purpose of this paper is applying a new integrated method to technology selection. Proposed approach is based on Logarithmic fuzzy preference programming and GTMA methods. Logarithmic fuzzy preference programming method is used in determining the weights of the criteria by decision makers and then rankings of technologies are determined by GTMA method. A numerical example demonstrates the application of the proposed method.

[Amirhossein Behrooz, Ehsan Salmani Zarchi. **A New Integration of Logarithmic Fuzzy Preference Programming and GTMA Method for Technology Selection.** *Rep Opinion* 2013;5(4):25-31]. (ISSN:1553-9873). http://www.sciencepub.net/report. 5

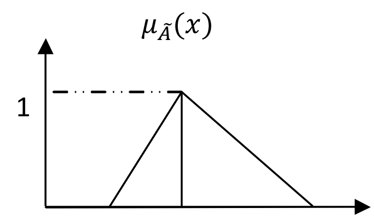
Keywords: Technology selection, graph theory and matrix approach, Fuzzy set and Logarithmic fuzzy preference programming.

1. Introduction

Selection of technologies is one of the most challenging decision making areas the management of a company encounters. It is difficult to clarify the right technology alternatives because the number of technologies is increasing and the technologies are becoming more and more complex. However, right technologies could create significant competitive advantages for a company in a complex business environment. The aim of technology selection is to obtain new know-how, components, and systems which will help the company to make more competitive products and services and more effective processes, or create completely new solutions (FarzipoorSaen, 2006).The rest of the paper is organized as follows: The following section presents a concise treatment of the basic concepts of fuzzy set theory. Section 3 presents the methodology of Logarithmic fuzzy preference programming and GTMA. The application of the proposed framework to technology selection is addressed in Section 4. Finally, conclusions are provided in Section 5.

2. Fuzzy sets and Fuzzy Numbers

Fuzzy set theory, which was introduced by Zadeh (1965) to deal with problems in which a source of vagueness is involved, has been utilized for incorporating imprecise data into the decision framework. A fuzzy set $\tilde{A}$ can be defined mathematically by a membership function $\mu_{\tilde{A}}(X)$, which assigns each element x in the universe of discourse X a real number in the interval [0,1]. A triangular fuzzy number $\tilde{A}$ can be defined by a triplet (a, b, c) as illustrated in Fig 1.



F 0 . A L ngu M fuzzy nu U r $\tilde{A}$

The membership function $\mu_{\tilde{A}}(X)$ is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a} & a \le x \le b \\ \frac{x-c}{b-c} & b \le x \le c \\ 0 & oterwise \end{cases} \quad (1)$$

Basic arithmetic operations on triangular fuzzy numbers $A_1 = (a_1,b_1,c_1)$, where $a_1 \le b_1 \le c_1$, and $A_2 = (a_2,b_2,c_2)$, where $a_2 \le b_2 \le c_2$,can be shown as follows:

Addition: $A_1 \oplus A_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$ (2)

Subtraction: $A_1 \ominus A_2 = (a_1 - c_2, b_1 - b_2, c_1 - a_2)$ (3)

Multiplication: if k is a scalar

$$k \otimes A_1 = \begin{cases} (ka_1, kb_1, kc_1), & k > 0 \\ (kc_1, kb_1, ka_1), & k < 0 \end{cases}$$

$A_1 \otimes A_2 \approx (a_1a_2, b_1b_2, c_1c_2)$, if $a_1 \geq 0$, $a_2 \geq 0$ (4)

Division: $A_1 \oslash A_2 \approx (\frac{a_1}{c_2}, \frac{b_1}{b_2}, \frac{c_1}{a_2})$, if $a_1 \geq 0$, $a_2 \geq 0$ (5)

Although multiplication and division operations on triangular fuzzy numbers do not necessarily yield a triangular fuzzy number, triangular fuzzy number approximations can be used for many practical applications (Kaufmann& Gupta, 1988). Triangular fuzzy numbers are appropriate for quantifying the vague information about most decision problems including personnel selection (e.g. rating for creativity, personality, leadership, etc.). The primary reason for using triangular fuzzy numbers can be stated as their intuitive and computational-efficient representation (Karsak, 2002). A linguistic variable is defined as a variable whose values are not numbers, but words or sentences in natural or artificial language. The concept of a linguistic variable appears as a useful means for providing approximate characterization of phenomena that are too complex or ill-defined to be described in conventional quantitative terms (Zadeh, 1975).

3. Research Methodology

In this paper, the weights of each criterion are calculated using of Logarithmic fuzzy preference programming. After that, GTMA is utilized to rank the alternatives. Finally, we select the best technology based on these results.

3.1. The LFPP-based nonlinear priority method

In this method for the fuzzy pairwise comparison matrix, Wang et al (2011) took its logarithm by the following approximate equation:

$$\ln \tilde{a} = (\ln l_{ij}, \ln m_{ij}, \ln lu_{ij}),\ i,j = 1\ldots,n \qquad (6)$$

That is, the logarithm of a triangular fuzzy judgment a_{ij} can still be seen as an approximate triangular fuzzy number, whose membership function can accordingly be defined as

$$\mu_{ij}\left(\ln\left(\frac{w_i}{w_j}\right)\right) = \begin{cases} \dfrac{\ln\left(\frac{w_i}{w_j}\right) - \ln l_{ij}}{\ln m_{ij} - \ln l_{ij}}, & \ln\left(\frac{w_i}{w_j}\right) \leq \ln m_{ij}, \\ \dfrac{\ln u_{ij} - \ln\left(\frac{w_i}{w_j}\right)}{\ln u_{ij} - \ln m_{ij}}, & \ln\left(\frac{w_i}{w_j}\right) \geq \ln m_{ij}, \end{cases} \qquad (7)$$

Where $\mu_{ij}\left(\ln\left(\frac{w_i}{w_j}\right)\right)$ is the membership degree of $\ln\left(\frac{w_i}{w_j}\right)$ belonging to the approximate triangular fuzzy judgment $\ln \tilde{a} = (\ln l_{ij}, \ln m_{ij}, \ln lu_{ij})$. It is very natural that we hope to find a crisp priority vector to maximize the minimum membership degree $\lambda = \min \{\mu_{ij}\left(\ln\left(\frac{w_i}{w_j}\right)\right) \mid i=1,\ldots,n-1;\ j=i+1,\ldots,n\}$. The resultant model can be constructed (Wang et al, 2011) as

Maximize λ

Subject to

$$\begin{cases} \mu_{ij}\left(\ln\left(\frac{w_i}{w_j}\right)\right) \geq \lambda, i = 1,\ldots,n-1; j = i+1,\ldots,n, \\ w_i \geq 0, i = 1,\ldots,n, \end{cases} \qquad (8)$$

Or as

Maximize $1-\lambda$

Subject to

$$\begin{cases} \ln w_i - \ln w_j - \lambda \ln\left(\frac{m_{ij}}{l_{ij}}\right) \geq \ln l_{ij}, \\ i = 1,\ldots,n-1; j = i+1,\ldots,n, \\ -\ln w_i + \ln w_j - \lambda \ln\left(\frac{u_{ij}}{m_{ij}}\right) \geq -\ln u_{ij}, \\ i = 1,\ldots,n; j = i+1,\ldots,n, \end{cases} \qquad (9)$$

It is seen that the normalization constraint $\sum_{i=1}^{n} w_i = 1$ is not included in the above two equivalent models. This is because the models will become computationally complicated if the normalization constraint is included. Before normalization, without loss of generality, we can assume $w_i \geq 1$ for all $i = 1,\ldots,n$ such that $\ln w_i \geq 0$ for $i = 1,\ldots,n$. Note that the nonnegative assumption for $\ln w_i \geq 0$ (i = 1,. . . ,n) is not essential. The reason for producing a negative value for λ is that there are no weights that can meet all the fuzzy judgments in $\tilde{A}$ within their support intervals. That is to say, not all the inequalities $\ln w_i - \ln w_j - \lambda \ln\left(\frac{m_{ij}}{l_{ij}}\right) \geq \ln l_{ij}$ or $-\ln w_i + \ln w_j - \lambda \ln\left(\frac{u_{ij}}{m_{ij}}\right) \geq -\ln u_{ij}$ can hold at the same time. To avoid k from taking a negative value, Wang et al (2011) introduced nonnegative deviation variables δ_{ij} and η_{ij} for $i = 1,\ldots,n-1; j = i+1,\ldots,n,$ such that they meet the following inequalities:

$$\ln w_i - \ln w_j - \lambda \ln\left(\frac{m_{ij}}{l_{ij}}\right) \geq \ln l_{ij}, i = 1,\ldots,n-1; j = i+1,\ldots,n$$

$$-\ln w_i + \ln w_j - \lambda \ln\left(\frac{u_{ij}}{m_{ij}}\right) \geq -\ln u_{ij}, i = 1, \ldots, n; j = i+1, \ldots, n \quad (10)$$

It is the most desirable that the values of the deviation variables are the smaller the better. Wang et al (2011) thus proposed the following LFPP-based nonlinear priority model for fuzzy AHP weight derivation:

Minimize $J = (1-\lambda)^2 + M.\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}(\delta_{ij}^2 + \eta_{ij}^2)$

Subject to

$$\left\{\begin{array}{c} x_i - x_j - \lambda \ln\left(\frac{m_{ij}}{l_{ij}}\right) + \delta_{ij} \geq \ln l_{ij}, \\ i = 1, \ldots, n-1; j = i+1, \ldots, n, \\ -x_i + x_j - \lambda \ln\left(\frac{u_{ij}}{m_{ij}}\right) + \eta_{ij} \geq -\ln u_{ij}, \\ i = 1, \ldots, n; j = i+1, \ldots, n, \\ \lambda, x_i \geq 0, i = 1, \ldots, n \\ \delta_{ij}, \eta_{ij} \geq 0, i = 1, \ldots, n-1; j = i+1, \ldots, n \end{array}\right\} \quad (11)$$

Where $x_i = \ln w_i$ for i = 1,. . . ,n and M is a specified sufficiently large constant such as $M = 10^3$. The main purpose of introducing a big constant M into the above model is to find the weights within the support intervals of fuzzy judgments without violations or with as little violations as possible.

3.2. Graph Theory and Matrix Approach (GTMA)

A graph G = (V, E) consists of a set of objects V = {v_1, v_2,….} called vertices or nodes, and another set E = {e_1, e_2, ….}, of which the elements are called edges, such that each edge e_k is identified with a pair of vertices. The vertices v_i and v_j associated with edge e_k are called the end vertices of e_k. The most common representation of a graph is by means of a diagram, in which the vertices are represented by small points or circles, and each edge as a line segment joining its end vertices. The application of graph theory was known centuries ago, when the longstanding problem of the Konigsberg bridge was solved by Leonhard Euler in 1736 by means of a graph. Since then, graph theory has proved its mettle in various fields of science and technology such as physics, chemistry, mathematics, communication science, computer technology, electrical engineering, sociology, economics, operations research, linguistics, internet, etc. Graph theory has served an important purpose in the modeling of systems, network analysis, functional representation, conceptual modeling, diagnosis, etc. Graph theory is not only effective in dealing with the structure (physical or abstract) of the system, explicitly or implicitly, but also useful in handling problems of structural relationship. The theory is intimately related to many branches of mathematics including group theory, matrix theory, numerical analysis, probability, topology, and combinatory. The advanced theory of graphs and their applications are well documented (Harary, 1985; Wilson and Watkins, 1990; Chen, 1997; Deo, 2000; Jense and Gutin, 2000; Liu and Lai, 2001; Tutte, 2001; Pemmaraju and Skiena, 2003; Gross and Yellen, 2005; Biswal, 2005).

3.2.1. Methodology of GTMA

The main steps are given below:

Step 1: Identify the pertinent attributes and the alternatives involved in the decision-making problem under consideration. Obtain the values of the attributes (A_i) and their relative importance (a_{ij}). An objective or subjective value, or its range, may be assigned to each identified attribute as a limiting value or threshold value for its acceptance for the considered decision-making problem. An alternative with each of its selection attributes, meeting the acceptance value, may be short-listed. After short-listing the alternatives, the main task in choosing the alternative is to see how it serves the considered attributes.

Step 2:

1. Develop the attributes digraph considering the identified pertinent attributes and their relative importance. The number of nodes shall be equal to the number of attributes considered in Step 1 above. The edges and their directions will be decided upon based on the interrelations among the attributes (a_{ij}).
2. Develop the attributes matrix for the attributes digraph. This will be the M*M matrix with diagonal elements as A_i and off-diagonal elements as a_{ij}.
3. Obtain the permanent function for the attributes matrix.
4. Substitute the values of A_i and a_{ij}, obtained in step 1.
5. Arrange the alternatives in the descending order of the index. The alternative having the highest value of index is the best choice for the decision-making problem under consideration.
6. Obtain the identification set for each alternative.
7. Evaluate the coefficients of dissimilarity and similarity. List also the values of the coefficients for all possible combinations.

8. Document the results for future analysis/reference.

Step 3: Take a final decision, keeping practical considerations in mind. All possible constraints likely to be experienced by the user are looked into during this stage. These include constraints such as: availability or assured supply, management constraints, political constraints, economic constraints, environmental constraints, etc. However, compromise may be made in favor of an alternative with a higher index.

4. A Numerical Application of Proposed Approach

This paper, the proposed methodology that may be applied to a wide range of technology selection problems is used for robot selection. We considered cost as a non-beneficial attribute and Vendor reputation, Load capacity and Velocity and as beneficial attributes for Technology selection. These attributes are taken from Farzipoorsaen (2006). These attributes are shown in Table 1.

Table 1. Attributes for robot selection

criteria	Attributes
C_1	Cost (10000$)
C_2	Vendor reputation
C_3	Load capacity(kg)
C_4	Velocity(m/s)

In this paper, the weights of criteria are calculated using of LFPP, and these calculated weight values are used as GTMA inputs. Then, after GTMA calculations, evaluation of the alternatives and selection of technology is realized.

Logarithmic Fuzzy Preference Programming:

In LFPP, firstly, we should determine the weights of each criterion by utilizing pair-wise comparison matrices. We compare each criterion with respect to other criteria. You can see the pair-wise comparison matrix for Flexible Manufacturing System criteria in Table 2.

Table 2. Inter-criteria comparison matrix

	C_1	C_2	C_3	C_4
C_1	(1.00,1.00,1.00)	(3.67,4.50,5.67)	(2.00,2.73,4.38)	(0.25,1.84,3.66)
C_2	(0.18,0.22,0.28)	(1.00,1.00,1.00)	(1.78,3.28,4.30)	(0.62,0.89,1.30)
C_3	(0.25,0.40,0.61)	(0.23,0.31,0.56)	(1.00,1.00,1.00)	(0.84,2.07,2.96)
C_4	(0.28,0.55,4.59)	(0.81,1.15,2.18)	(0.46,0.71,3.38)	(1.00,1.00,1.00)

After forming the model (11) for the comparison matrix and solving this model using of Genetic algorithms, the weight vector is obtained as follow:

$w^t = (0.3970, 0.2171, 0.1329, 0.2529)^T$

Then, weighted normalized matrix is formed by multiplying each value with their weights. All weighted values are aggregated to form Table 3.

Table 3: Total weighted values of criteria

	C1	C2	C3	C4
A1	0.913023	1	0.931198	0.326592903
A2	1	0.779032	0.496737	0.531095002
A3	0.812753	0.806452	0.807816	0.772987409
A4	0.55288	0.98871	1	1
A5	1.264689	1.677419	2.698079	0.78214422
A6	0.679927	0.508065	0.747882	2.80847005

Then, according to GTMA method, we carry out pair-wise comparison with respect to their weight that shows from Table 4 to Table 10.

Table 4: pair-wise comparison of criteria with respect to each other

	C1	C2	C3	C4
C1		0.644544	0.734455	0.610324
C2	0.355456		0.60401	0.463448
C3	0.265545	0.39599		0.361543
C4	0.389676	0.536552	0.638457	
wj	0.391924	0.21614	0.141702	0.250234

Table 5: pair-wise comparison of criteria with respect to A_1

A1	C1	C2	C3	C4
C1	0.913023	0.644544	0.734455	0.610324
C2	0.355456	1	0.60401	0.463448
C3	0.265545	0.39599	0.931198	0.361543
C4	0.389676	0.536552	0.638457	0.326593

Table 6: pair-wise comparison of criteria with respect to A_2

A2	C1	C2	C3	C4
C1	1	0.644544	0.734455	0.610324
C2	0.355456	0.779032	0.60401	0.463448
C3	0.265545	0.39599	0.496737	0.361543
C4	0.389676	0.536552	0.638457	0.531095

Table 7: pair-wise comparison of criteria with respect to A_3

A3	C1	C2	C3	C4
C1	0.812753	0.644544	0.734455	0.610324
C2	0.355456	0.806452	0.60401	0.463448
C3	0.265545	0.39599	0.807816	0.361543
C4	0.389676	0.536552	0.638457	0.772987

Table 8: pair-wise comparison of criteria with respect to A_4

A4	C1	C2	C3	C4
C1	0.55288	0.644544	0.734455	0.610324
C2	0.355456	0.98871	0.60401	0.463448
C3	0.265545	0.39599	1	0.361543
C4	0.389676	0.536552	0.638457	1

Table 9: pair-wise comparison of criteria with respect to A_5

A5	C1	C2	C3	C4
C1	1.264689	0.644544	0.734455	0.610324
C2	0.355456	1.677419	0.60401	0.463448
C3	0.265545	0.39599	2.698079	0.361543
C4	0.389676	0.536552	0.638457	0.782144

Table 10: pair-wise comparison of criteria with respect to A_6

A6	C1	C2	C3	C4
C1	0.679927	0.644544	0.734455	0.610324
C2	0.355456	0.508065	0.60401	0.463448
C3	0.265545	0.39599	0.747882	0.361543
C4	0.389676	0.536552	0.638457	2.80847

After that we calculate the permanent matrix using of MATLAB software. The permanent matrix of each alternative is indicated in Table 11.

Table 11: ranking alternative

alternative	Permanent matrix	rank
A1	1.881	5
A2	1.653	6
A3	2.320	3
A4	2.219	4
A5	2.652	2
A6	3.231	1

According to Table 11, A_6 is the best alternative among other.

5. Conclusions

Selection of technologies is one of the most challenging decision making areas the management of a company encounters. It is difficult to clarify the right technology alternatives because the number of technologies is increasing and the technologies are becoming more and more complex. This paper illustrates an application of LFPP along with GTMA in selecting best technology. Fuzzy set theory is incorporated to overcome the vagueness in the preferences. A two-step LFPP and GTMA methodology is structured here that GTMA uses LFPP result weights as input weights. Then a numerical example is presented to show applicability and performance of the methodology.

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Acknowledgement
The authors wish to thank an anonymous referee for the valuable suggestions which considerably improve the quality of the paper.

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5/6/2013