

## Determination of number of kanban in a cellular manufacturing system with considering rework process

Mojtaba Aghajani <sup>1\*</sup>, Abbas Keramati <sup>2</sup>

<sup>1,2</sup> Department of Industrial Engineering, Faculty of Engineering, University of Tehran, Tehran, Iran

\* Full telephone:+98-21-82095510; Fax No: +98-21-88006076; [m.aghajani@ymail.com](mailto:m.aghajani@ymail.com)

**Abstract:** The main element of just-in-time production system is kanban and the number of kanban influences the product inventory level. This research studies the cellular manufacturing system (CMS) controlled by kanban mechanism which defective items are produced in any production run of each product and rework is carried out to transform them into serviceable items. In each cycle after the normal production of each product the machine is setup for the rework process. In this paper, an mixed-integer nonlinear programming (MINLP) model in order to minimize total cost was developed to determine number of kanban, batch size, and number of batches. A simulated annealing (SA) algorithm as a meta-heuristic method to avoid the large computational time in optimal solution is developed for solving a large MINLP. Some problems are solved by SA and it is shown that the SA algorithm is an efficient method for solving the proposed model.

[Mojtaba Aghajani, Abbas Keramati. **Determination of number of kanban in a cellular manufacturing system with considering rework process.** *Rep Opinion* 2014;6(7):59-70]. (ISSN: 1553-9873). <http://www.sciencepub.net/report>. 9

**Keywords:** Kanban, Cellular manufacturing system, Mixed integer non-linear programming (MINLP), Meta-Heuristic, Simulated annealing

### 1. Introduction

Most existing production–inventory models assumes that all items produced are of perfect quality, but in real world production systems due to various reasons including process deterioration, human mistakes and other factors generation of imperfect quality items is inevitable. In this paper, the above issues will be considered and is assumed that all imperfect quality items can be repaired through rework. Rework is the transformation of imperfect quality items generated during production period into serviceable ones.

In recent years, the concepts of cellular manufacturing and just-in-time (JIT) production has received much attention and applied in many developed countries. A cellular manufacturing system (CMS) is a manufacturing philosophy that leads to achieving the JIT production system [Sridhar and Rajendran, 1996]. In this system, machines and similar parts are grouped into cells and part-families. Within each cell a particular part family with similar manufacturing characteristics such as process routing and tooling requirements are produced. JIT production system has been implemented widely in many factories to improve the productivity of the production and reduce production costs. The principle of JIT philosophy is a production of what is needed at the right time and in the right quantity [Hutchins, 1993].

Kanban system as information system is one of the major elements of JIT philosophy. Kanban is the Japanese word which means for a card. This system is the best-known pull systems in which production is triggered by the customer demand at the final stage. In

a production environment that is controlled by kanban the material flow is from preceding stage to succeeding stage, but the information flow is downward and from succeeding stage to preceding stage. At each intermediate stage, production and delivery is triggered by succeeding stage order. This process is carried all the way from the last stage to the first stage. Goods are manufactured only when workstations receive a card. Empty containers show that more parts will be needed for production. Each workstation will only produce enough components to fill containers and then stops [Shahabudeen et al., 2002].

Since the number of kanban influences the product inventory level, it is very important for managers to determine the number of kanban used in the system introductory phase [Fukukawa and Hong, 1993]. In this paper, we study the CMS controlled by kanban under a JIT philosophy with rework policy through the development of mathematical models. The main objective of the mathematical model is determining of kanban numbers and lot sizes between two stages to minimize total relevant cost including material handling, holding and setup costs.

Further, in this article we will see that the model of the problem is of MINLP type which takes a long time to achieve optimal solution. Thus, this paper presents a simulated annealing (SA) as a Meta-Heuristic algorithm to reduce computational time. The remainder of the paper is as follows:

In Section 2, literature review about kanban is introduced. The problem is described in Section 3. In Section 4, problem assumptions are presented. Section

5 discusses the mathematical model and in Section 6 this proposed model is solved. At last in Section 7 the conclusions of this study and future study are presented.

## 2. Literature review

For the first time, [Monden, 1983] stated the importance of the kanban that was the basis for many researches. He presents the summary of Toyota approach for determining the appropriate number of kanban.

In the literature, some researchers used a mathematical programming approach to determine the optimal number of Kanban. [Kimura and Terada, 1981] as first researchers in the modeling of kanban, described the operation of kanban systems and developed a model for the kanban system. [Bitran and Chang, 1987] extended Kimura and Terada serial model and offered a nonlinear integer formulation for kanban systems in a single item and multi-stage production setting. [Bard and Golany, 1991] extended Bitran and Chang's model and developed a mixed integer linear program to determine the number of kanbans in a multi-item and multi-stage by minimizing the setup, shortage and inventory holding costs. [Mitwasi and Askin, 1994] have developed a nonlinear integer mathematical model for the multi-item and single stage kanban system. [Gupta and Alturki, 1997] introduced a flexible kanban system and used an algorithm based on mathematical models to manipulate the number of kanban for stochastic processing times and variable demand environment. (Sarker and Balan, 1996, 1998, 1999) in some valuable papers determine number of kanbans and batch sizes to transport the material between two workstations for single and multi-stage production line.

Several models in the literature are developed to describe the supply chain system. [Wang and Sarker, 2006] extended the study by Sarker and Balan (1999) and develop a mixed-integer nonlinear programming model to determine the number of kanbans and transported batch size in multi-stage supply chain system. In this regard the objective was to minimize inventory holding, transportation of each container and setup costs. [Rabbani et al., 2009] extended Wang and Sarker by introducing new formula for the total cost. They developed a memetic algorithm heuristic method to determine the number of kanban and batch sizes between two plants for single product supply chain system. Also, they found that memetic algorithm is reliable to finding the optimal number of kanbans and the lot sizes in problems with large dimensions. [Widyadana et al., 2010] used optimal and meta-heuristic methods to determine the Kanban quantity and withdrawal lot sizes in a supply chain system. They used a genetic algorithm (GA) and a hybrid of genetic algorithm and simulated annealing

(GASA) method and compared the performance of GA and GASA with that of the optimal method using MIP.

Taking the aforesaid facts into consideration at choosing and allocating the optimal number of kanbans, most researchers do not consider production of defective (reworkable) items in the manufacturing process. Also, little attention was paid to CMS as a core element of JIT production systems. Hence, in this study the above issues will be considered to fill the gap.

## 3. Problem description

A CMS with rework process that controlled by kanban is considered here. With considering different types of cell, CMS can be divided into several types that in this study product-focused cell is considered. In product-focused cell, product family that follow the same or similar operation sequence assigned to a cell and can be produced completely from raw material to finished goods in the same cell.

In our problem, each cell consists of  $M$  stations that parts move from station  $i$  to station  $i + 1$  and this process is carried from the first stage to the final stage. To avoid inventory accumulation, production of next product in each stage starts when the inventory level of previous product becomes zero. Each station produces the required amount of next station, thus for each product, production quantity in a product cycle (sum of production and re-work cycle) is the same at all stations. But, the number of production setup for producing this production quantity with respect to setup and inventory holding cost, may be different in each station. For example, station  $i$  produces 100 units in a 4 setup times with lot size of 25 and station  $i + 1$  produces this number in a 2 setup times with lot size of 50.

In a kanban system, kanban controls the production or transportation of parts. When station  $i + 1$  takes and uses a full container, withdrawal kanban is detached from it and put in withdrawal kanban post. Then withdrawal kanban is picked up from the withdrawal kanban post and attached to the empty containers that transshipped to station  $i$  where a full container of parts with production kanban is located. In this station, the production kanban is detached and put in the production kanban post and the withdrawal kanban is attached to the full container. This transshipped to station  $i + 1$  and the cycle is repeated.

In order to minimize total cost of the system, we determined the optimal number of kanban and batch sizes through mathematical model.

## 4. Problem assumptions

The following assumptions are considered in this paper:

(1) The production rate of serviceable item is greater than the demand rate.

- (2) Demand is known. (6) All defective items are reworkable and generated only during production cycle.
- (3) All products have the same processing route. (7) Proportion of defective is constant in each cycle.
- (4) Production sequence in each station is known. (8) Scrap is not produced at any cycle.
- (5) Production and rework setups are made on the same facility.

## 5. Problem modeling

For determining the optimal number of kanban, lot-size and number of batches, from minimization of total relevant costs per unit time is used. The objective is to minimize the total cost such as holding inventory, container transshipment of each container and setup cost in production system with rework policy.

The notation used in model is:

$c$	index for cell	$c = 1, 2, \dots, z$
$k$	index for production facility (station)	$k = 1, 2, \dots, m$
$i$	index for product	$i = 1, 2, \dots, n$

### Parameters

$p_{ikc}$ ( $r_{ikc}$ )	production (rework) rate of product $i$ in station $k$ in cell $c$ , units/year
$D_{ic}$	the demand rate for product $i$ in cell $c$ , units/year
$\alpha_{ikc}$ ( $\beta_{ikc}$ )	proportion of serviceable (reworkable) product $i$ in station $k$ in cell $c$ with $\alpha_{ikc} + \beta_{ikc} = 1$
$h_{ic}$ ( $\hat{h}_{ic}$ )	inventory holding cost for unit serviceable (reworkable) product $i$ in station $k$ in cell $c$ per unit time, dollar /unit/time
$S_{ikc}$	setup cost of product $i$ in station $k$ in cell $c$ , dollar/setup
$Snr_{ikc}$	setup cost from production to rework for product $i$ in station $k$ in cell $c$ , dollar/setup
$Srn_{ikc}$	setup cost from rework to production for product $i$ in station $k$ in cell $c$ , dollar/setup
$Cm_{ikc}$	shipping cost for product $i$ in station $k$ to next station in cell $c$ , dollar/ship
$T_{ikc}$	production cycle of product $i$ in station $k$ in cell $c$ , year
$TT_{kc}$	cumulative cycles (time needed to produce all products in a round=sum of all product cycle), year
$Tp_{ikc}$	uptime of product $i$ in station $k$ in cell $c$ at production process, year
$Tr_{ikc}$	uptime of product $i$ in station $k$ in cell $c$ at rework process, year
$Td_{ikc}$	downtime of product $i$ in station $k$ in cell $c$ , year
$Ir_{ikc}$	average inventory level for reworkable product $i$ in station $k$ in cell $c$ , units/year
$Ip_{ikc}$	average inventory level for serviceable product $i$ in station $k$ in cell $c$ , units/year
$I_{(t)}$	inventory level, units
$TC$	total cost of a system, dollars/year
$TC_s$	total cost of set up, dollars/year
$TC_m$	total cost of shipping, dollars/year
$TC_h$	total cost of inventory holding, dollars/year
$TC_{hp}$	total cost of inventory holding for serviceable items, dollars/year
$TC_{hr}$	total cost of inventory holding for reworkable items, dollars/year

### Variables

$K_{ikc}$	number of kanban in production process for product $i$ in station $k$ in cell $c$
$Kr_{ikc}$	number of kanban in rework process for product $i$ in station $k$ in cell $c$
$Q_{ic}$	total quantity of finished goods $i$ produced over a one product cycle $i$ in cell $c$ , units/year
$Qo_{ikc}$	production lot quantity per production setup
$QW_{ikc}$	work-in-process shipping quantity for product $i$ in station $k$ in cell $c$ , units/shipment
$Qr_{ic}$	rework quantity for rework process $Qr_{ic} = \beta_{ikc} Q_{ic}$
$n_{ikc}$	number of production setup to produce $Q_{ic}$ in station $k$
$m_{ikc}$	number of shipments placed during the production (rework) uptime for product $i$ in station $k$ in cell $c$

#### 5.1. Cost of inventory holding

In this model we assume that all the defective items are reworked after the  $n_{ikc}$  production setups. After the  $n_{ikc}$  production cycle (relevant time to normal production), the machine is immediately set up for the rework of the defective items. It is to be noted that an inventory cycle for a product consists of  $n_{ikc}$  production and one rework run

that this control policy is only a special case of more general set-up policies. The basic assumption of this model is that the production rate of serviceable item must always be greater than or equal to the demand rate. Hence, the following condition must hold:

$$\alpha_{ikc} p_{ikc} \geq D_{ic} \text{ for each } i = 1, \dots, n \quad k = 1, \dots, m \quad c = 1, \dots, z$$

Figure. 1 shows inventory levels of serviceable and reworkable products when  $n_{ikc} = 2$ . From Figure. 1, it can be seen that during the interval  $T_p$  the product  $i$  is produced at rate  $p_i$ , but the serviceable quantity of this product is produced at rate  $p_i \alpha_i$  and in the same interval reworkable quantity of this product is produced at rate  $p_i \beta_{ikc}$ . Reworkable items stay in the recoverable inventory until the rework process begins. Hence, the inventory of serviceable and reworkable units of product  $i$  increases at rate  $p_i \alpha_i - D_i$  and  $p_i \beta_{ikc}$ . As the inventory level for serviceable (in production or rework process) items reaches the batch size,  $Q_{wi}$ , the parts are carried by containers from station  $i$  to station  $i + 1$ . Production continues until the number of items produced in the uptime  $T_p$  reaches  $Q_{o_{ikc}}$ . After  $n_{ikc}$  production setups, the setup starts for rework process on defective items produced in production cycle. Thus, during the rework process, the inventory of reworkable items decreases at rate  $r_i$  and inventory of serviceable items increases at rate  $p_i - D_i$ . It is assumed that no defective occurs during the rework process time.

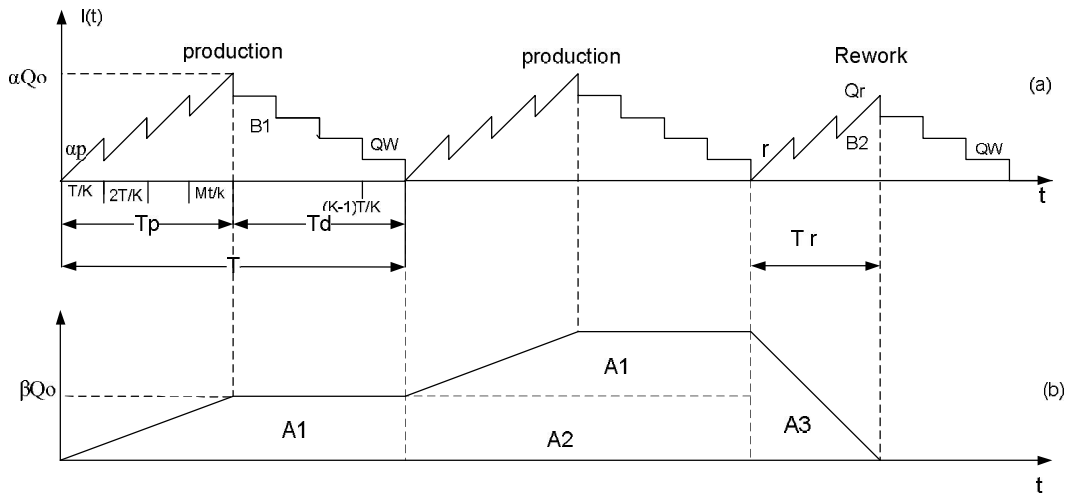


Figure 1. On-hand Inventory of serviceable (a) and reworkable (b) items

Total inventory holding cost is sum of serviceable and reworkable items inventory holding costs:

$$TC_h = TC_{hp} + TC_{hr}$$

For serviceable product we can write:

$$TC_{hp} = \sum_{c=1}^z \sum_{k=1}^m \sum_{i=1}^n h_{ic} I p_{ikc} \tag{1}$$

Inventory for serviceable items at production and rework cycle occurs. In the production cycle the following identities can be found from Figure. 1

$$QW_{ikc} = \frac{Q_{o_{ikc}} \alpha_{ikc}}{K_{ikc}}, T_p = \frac{Q_{o_{ikc}}}{p_{ikc}}, m_{ikc} = \frac{K_{ikc} Q_{o_{ikc}}}{p_{ikc} T_{ikc}}$$

and also in the rework process the relations:

$$T_{ikc} = \frac{Q_{r_{ikc}}}{D_{ic}}, QW_{ikc} = \frac{Q_{r_{ikc}}}{K_{r_{ikc}}}, T_{r_{ikc}} = \frac{Q_{r_{ikc}}}{r_{ikc}}$$

are obtained.

Production and rework setups can be decomposed into 3 types of polygons, c1, c2 and c3 as are shown in Figure. 2 where in production cycle total sum of surrounded area for above regions are as follows:

$$\begin{aligned} \text{Area}(C1) &= \frac{\alpha_{ikc} F}{2} \\ \text{Area}(C2) &= \frac{T_{ikc}}{2K_{ikc}} [(\alpha_{ikc} p_{ikc} T_{ikc} - D_{ic} T_{ikc})] \end{aligned}$$

$$\text{Area}(C3) = \frac{(m_{ikc} - 1)(m_{ikc} + 1) + Q_{o_{ikc}}\alpha_{ikc}(m_{ikc} - 1)](K_{ikc} - m_{ikc})(K_{ikc} - m_{ikc} + 1)Q_{o_{ikc}}\alpha_{ikc}T_{ikc}}{2K_{ikc}^2}$$

After substitution  $m_{ikc} = \frac{K_{ikc}Q_{o_{ikc}}}{p_{ikc}T_{ikc}}$  and simplification, the total serviceable inventory during a  $n_{ikc}$  production cycle equals

$$\frac{\alpha_{ikc}Q_{o_{ikc}}^2}{2} \left( \frac{(1 + K_{ikc})\alpha_{ikc}}{D_{ic}K_{ikc}} - \frac{1}{p_{ikc}} \right) n_{ikc} \quad (2)$$

Since in the rework cycle (relevant time to rework process) no defective occurs, therefore in this cycle  $\alpha_{ikc} = 1$  and in an equivalent manner the serviceable inventory related to a rework cycle can be described as follows:

$$\frac{Qr_{ikc}^2}{2} \left( \frac{Kr_{ikc} + 1}{Kr_{ikc}D_{ic}} - \frac{1}{r_{ikc}} \right) \quad (3)$$

After substitutions  $Kr_{ikc} = \frac{Qr_{ikc}}{QW_{ikc}} = \frac{n_{ikc}\beta_{ikc}Q_{o_{ikc}}}{QW_{ikc}}$

$Qr_{ikc} = n_{ikc}\beta_{ikc}Q_{o_{ikc}}$  in Eq. (3) and simplification, Eq. (4) obtained.

$$\frac{n_{ikc}^2\beta_{ikc}^2Q_{o_{ikc}}^2}{2} \left( \frac{1}{D_{ic}} + \frac{\alpha_{ikc}}{\beta_{ikc}D_{ic}K_{ikc}n_{ikc}} - \frac{1}{r_{ikc}} \right) \quad (4)$$

By adding Eqs. (2) and (4) the total serviceable inventory of one cumulative cycle can be obtained. Multiplying this expression by the number of cumulative inventory cycles  $\left(\frac{1}{\sum_{i=1}^n \frac{Q_{ic}}{D_{ic}}}\right)$  provides the total serviceable inventory in a

year.

$$Ip_{ikc} = \frac{1}{\left(\sum_{i=1}^n \frac{Q_{ic}}{D_{ic}}\right)} \left[ \frac{\alpha_{ikc}Q_{o_{ikc}}^2}{2} \left( \frac{(1 + K_{ikc})\alpha_{ikc}}{D_{ic}K_{ikc}} - \frac{1}{p_{ikc}} \right) n_{ikc} + \frac{n_{ikc}^2\beta_{ikc}^2Q_{o_{ikc}}^2}{2} \left( \frac{1}{D_{ic}} + \frac{\alpha_{ikc}}{\beta_{ikc}D_{ic}K_{ikc}n_{ikc}} - \frac{1}{r_{ikc}} \right) \right] \quad (5)$$

Substituting Eq. (5) into Eq. (1) resulted:

$$TC_{hp} = \sum_{c=1}^z \frac{1}{\sum_{i=1}^n \frac{Q_{ic}}{D_{ic}}} \sum_{k=1}^m \sum_{i=1}^n h_{ic} \left[ \frac{\alpha_{ikc}Q_{o_{ikc}}^2}{2} \left( \frac{(1 + K_{ikc})\alpha_{ikc}}{D_{ic}K_{ikc}} - \frac{1}{p_{ikc}} \right) n_{ikc} + \frac{n_{ikc}^2\beta_{ikc}^2Q_{o_{ikc}}^2}{2} \left( \frac{1}{D_{ic}} + \frac{\alpha_{ikc}}{\beta_{ikc}D_{ic}K_{ikc}n_{ikc}} - \frac{1}{r_{ikc}} \right) \right] \quad (6)$$

As described in section 3 for each product, production quantity in a product cycle ( $n_{ikc}Q_{o_{ikc}}$ ) is the same value at all stations. Thus  $n_{ikc}Q_{o_{ikc}} = Q_{ic}$ . After substitution  $n_{ikc}Q_{o_{ikc}} = Q_{ic}$  in Eq. (6), serviceable inventory cost can be expressed as follows:

$$TC_{hp} = \sum_{c=1}^z \frac{1}{\sum_{i=1}^n \frac{Q_{ic}}{D_{ic}}} \sum_{k=1}^m \sum_{i=1}^n h_{ic} \left[ \frac{\alpha_{ikc}Q_{ic}^2}{2n_{ikc}} \left( \frac{(1 + K_{ikc})\alpha_{ikc}}{D_{ic}K_{ikc}} - \frac{1}{p_{ikc}} \right) + \frac{\beta_{ikc}^2Q_{ic}^2}{2} \left( \frac{1}{D_{ic}} + \frac{\alpha_{ikc}}{n_{ikc}\beta_{ikc}D_{ic}K_{ikc}} - \frac{1}{r_{ikc}} \right) \right] \quad (7)$$

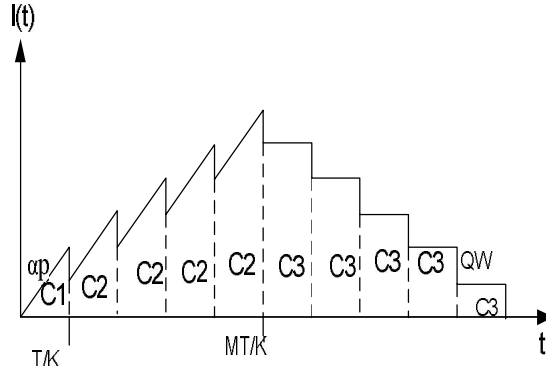


Figure 2. Decomposition of serviceable inventory

For reworkable product it can be written:

$$TC_{hr} = \sum_{c=1}^z \sum_{k=1}^m \sum_{i=1}^n \hat{h}_{ic} Ir_{ikc} \quad (8)$$

The reworkable inventory quantities can be decomposed into 3 types of polygons, A1, A2 and A3 as shown in Figure. 1. [Liu et al., 2009] presented a formula for production system with rework in which the inventory cost for reworkable and serviceable items is to be minimized. In this research, the same approach for inventory of reworkable items ( $Ir_{ikc}$ ) can be used. Accordingly, total reworkable inventory during a  $n_{ikc}$  production cycle equals  $n_{ikc} \cdot Area(A1) + \frac{(n_{ikc})(n_{ikc}-1)}{2} \cdot Area(A2) + Area(A3)$ . Where the total areas for A1, A2 and A3 can be expressed as follows:

$$Area(A1) = \frac{1}{2} \left( \frac{2\alpha_{ikc}}{D_{ic}} - \frac{1}{p_{ikc}} \right) Qo_{ikc}^2 \beta_{ikc}$$

$$Area(A2) = \frac{\alpha_{ikc} \beta_{ikc} Qo_{ikc}^2}{D_{ic}}$$

$$Area(A3) = \frac{1}{2} \frac{n_{ikc}^2 \beta_{ikc}^2 Qo_{ikc}^2}{r_{ikc}}$$

After multiplying the reworkable inventory during a one cumulative cycle with number of cumulative cycle in the year, total reworkable inventory is obtained:

$$Ir_{ikc} = \left( \frac{n_{ikc}}{2} \left( \frac{2\alpha_{ikc}}{D_{ic}} - \frac{1}{p_{ikc}} \right) Qo_{ikc}^2 \beta_{ikc} + \frac{1}{2} \alpha_{ikc} \beta_{ikc} \frac{1}{D_{ic}} (n_{ikc})(n_{ikc} - 1) Qo_{ikc}^2 + \frac{1}{2} \frac{n_{ikc}^2 \beta_{ikc}^2 Qo_{ikc}^2}{r_{ikc}} \right) / \left( \sum_{i=1}^n \frac{Q_{ic}}{D_{ic}} \right) \quad (9)$$

Thus, the inventory cost for reworkable items can be written as follows:

$$TC_{hr} = \sum_{c=1}^z \frac{1}{\left( \sum_{i=1}^n \frac{Q_{ic}}{D_{ic}} \right)} \sum_{k=1}^m \sum_{i=1}^n \hat{h}_{ic} \left[ \frac{n_{ikc}}{2} \left( \frac{2\alpha_{ikc}}{D_{ic}} - \frac{1}{p_{ikc}} \right) Qo_{ikc}^2 \beta_{ikc} + \frac{n_{ikc}^2 \beta_{ikc}^2 Qo_{ikc}^2}{2r_{ikc}} + \frac{\alpha_{ikc} \beta_{ikc} (n_{ikc})(n_{ikc} - 1) Qo_{ikc}^2}{2D_{ic}} \right] \quad (10)$$

After substitution  $n_{ikc} Qo_{ikc} = Q_{ic}$  in the above equation the following expression is achieved.

$$TC_{hr} = \sum_{c=1}^z \frac{1}{\sum_{i=1}^n \frac{Q_{ic}}{D_{ic}}} \sum_{k=1}^m \sum_{i=1}^n \hat{h}_{ic} \left[ \frac{Q_{ic}^2 \beta_{ikc}}{2n_{ikc}} \left( \frac{2\alpha_{ikc}}{D_{ic}} - \frac{1}{p_{ikc}} \right) \right]$$

$$+ \frac{\beta_{ikc}^2 Q_{ic}^2}{2r_{ikc}} + \frac{\alpha_{ikc} \beta_{ikc} (n_{ikc} - 1) Q_{ic}^2}{2D_{ic} n_{ikc}} \quad (11)$$

By adding Eqs. (7) and (11) total inventory holding cost is obtained .

$$TC_h = \sum_{c=1}^z \frac{1}{(\sum_{i=1}^n \frac{Q_{ic}}{D_{ic}})} \left( \sum_{k=1}^m \sum_{i=1}^n h_{ic} \left[ \frac{\alpha_{ikc} Q_{ic}^2}{2n_{ikc}} \left( \frac{(1 + K_{ikc}) \alpha_{ikc}}{D_i K_{ikc}} - \frac{1}{p_{ikc}} \right) \right. \right. \\ \left. \left. + \frac{\beta_{ikc}^2 Q_{ic}^2}{2} \left( \frac{1}{D_{ic}} + \frac{\alpha_{ikc}}{n_{ikc} \beta_{ikc} D_i K_{ikc}} - \frac{1}{r_{ikc}} \right) \right] \right. \\ \left. + h_{ic} \left[ \frac{Q_{ic}^2 \beta_{ikc}}{2n_{ikc}} \left( \frac{2\alpha_{ikc}}{D_{ic}} - \frac{1}{p_{ikc}} \right) + \frac{\beta_{ikc}^2 Q_{ic}^2}{2r_{ikc}} + \frac{\alpha_{ikc} \beta_{ikc} (n_{ikc} - 1) Q_{ic}^2}{2D_{ic} n_{ikc}} \right] \right) \quad (12)$$

### 5.2. Setup cost

Total setup cost includes setup costs for switching between production and rework and setup cost for each product. Each changeover from production to rework and vice versa causes a fixed setup cost. Also, production setup for producing any product causes a fixed setup cost. The relevant set-up cost of one cell in cumulative cycles can be demonstrated by following equation:

$$\sum_{k=1}^m \sum_{i=1}^n [n_{ikc} S_{ikc} + Snr_{ikc} + Srn_{ikc}] \quad (13)$$

Multiplying this equation by the number of cumulative inventory cycles  $\left(\frac{1}{(\sum_{i=1}^n \frac{Q_{ic}}{D_{ic}})}\right)$  provides the total setup cost of one cell.

$$\frac{1}{(\sum_{i=1}^n \frac{Q_{ic}}{D_{ic}})} \sum_{k=1}^m \sum_{i=1}^n [n_{ikc} S_{ikc} + Snr_{ikc} + Srn_{ikc}] \quad (14)$$

Thus, the total setup cost can be written as

$$TC_s = \sum_{c=1}^z \frac{1}{\sum_{i=1}^n \frac{Q_{ic}}{D_{ic}}} \sum_{k=1}^m \sum_{i=1}^n [n_{ikc} S_{ikc} + Snr_{ikc} + Srn_{ikc}] \quad (15)$$

### 5.3. Transportation cost

Transport cost is related to transport pallet (container) from one station to another station. This cost has a direct relationship with pallet size. The transported quantity in each shipping is  $QW_{ikc}$  (container size). As the serviceable stock level reaches to the batch size,  $QW_{ikc}$ , the parts are carried by containers from station  $i$  to station  $i + 1$ . The number of batches placed for each product per production and rework cycle is  $\frac{n_{ikc} Q_{ikc}}{QW_{ikc}}$ . Considering this relation for all products, stations and cells and then multiplying by the number of cumulative cycles, the total transportation cost is

$$TC_m = \sum_{c=1}^z \frac{1}{\sum_{i=1}^n \frac{Q_{ic}}{D_{ic}}} \sum_{k=1}^m \sum_{i=1}^n \left[ \frac{n_{ikc} Q_{ikc}}{QW_{ikc}} (Cm_{ikc}) \right] \quad (16)$$

Substituting  $\frac{Q_{ikc}}{QW_{ikc}} = \frac{K_{ikc}}{\alpha_{ikc}}$  in Eq. (10)

$$TC_m = \sum_{c=1}^z \frac{1}{\sum_{i=1}^n \frac{Q_{ic}}{D_{ic}}} \sum_{k=1}^m \sum_{i=1}^n \left[ \frac{n_{ikc} K_{ikc}}{\alpha_{ikc}} (Cm_{ikc}) \right] \quad (17)$$

By adding Eqs. (12), (15), and (17), the total cost of a system is obtained.

$$TC = TC_h + TC_m + TC_s$$



$$\begin{aligned}
TC = \sum_{c=1}^z \frac{1}{\left(\sum_{i=1}^n \frac{Q_{ic}}{D_{ic}}\right)} & \left( \sum_{k=1}^m \sum_{i=1}^n h_{ic} \left[ \frac{\alpha_{ikc} Q_{ic}^2}{2n_{ikc}} \left( \frac{(1+K_{ikc})\alpha_{ikc}}{D_i K_{ikc}} - \frac{1}{p_{ikc}} \right) \right. \right. \\
& + \left. \frac{\beta_{ikc}^2 Q_{ic}^2}{2} \left( \frac{1}{D_{ic}} + \frac{\alpha_{ikc}}{n_{ikc} \beta_{ikc} D_i K_{ikc}} - \frac{1}{r_{ikc}} \right) \right] \\
& + \left. h_{ic} \left[ \frac{Q_{ic}^2 \beta_{ikc}}{2n_{ikc}} \left( \frac{2\alpha_{ikc}}{D_{ic}} - \frac{1}{p_{ikc}} \right) + \frac{\beta_{ikc}^2 Q_{ic}^2}{2r_{ikc}} + \frac{\alpha_{ikc} \beta_{ikc} (n_{ikc} - 1) Q_{ic}^2}{2D_{ic} n_{ikc}} \right] \right. \\
& \left. + [n_{ikc} S_{ikc} + Snr_{ikc} + Srn_{ikc}] + \left[ \frac{n_{ikc} K_{ikc}}{\alpha_{ikc}} (Cm_{ikc}) \right] \right) \quad (18)
\end{aligned}$$

$n_{ikc}, K_{ikc} \geq 1$  and integer  $\forall i = 1, \dots, n \quad k = 1, \dots, m \quad c = 1, \dots, z$

## 6. Solving the problem

Many of combinatorial optimization problems are complex and hard to be solved. Therefore, due to the complexity of such problems some heuristic methods are useful to find near optimal solutions in less time than exact methods. Meta-heuristic search techniques in comparison with heuristic techniques have the advantage of having a specific framework and can be hybridized with heuristic techniques in any of their steps. The discussed problem in this study as a combinatorial problem is solved via SA as a meta-heuristic technique. Further, GAMS software is used as an optimal method to evaluate the efficiency of proposed SA algorithm by comparing their outputs.

### 6.1. Simulated annealing

Simulated annealing is a stochastic search method which was initially used by [Kirkpatrick et al., 1983]. This algorithm, like other meta-heuristic algorithms, attempts to solve hard combinatorial optimization problems through a controlled randomization. Its' ease of use and ability to provide a good solution for real-world problems make SA one of the most powerful and popular metaheuristic to solve many optimization problems [Arkat et al., 2007]. The most important feature of this algorithm is to avoid entrapment in a local optimum by accepting non-improver solutions with certain probability in each temperature. With reducing the temperature, probability of accepting the worst solution decreases in each iteration.

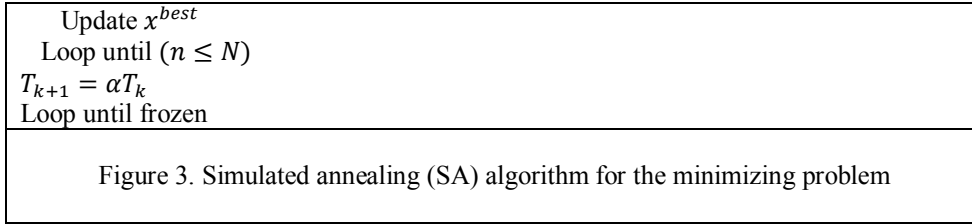
The basic structure of SA algorithms for a minimization problem is shown in Figure. 3 where the following parameters are used:

$x^n$	Current solution
$x^{best}$	Best solution
$f(x^n)$	Value of object function in solution $x^n$
$n$	Repetition counter
$T_0$	Initial temperature
$N$	Number of repetitions allowed at each temperature level
$\alpha$	Cooling rate
$T_f$	Final temperature

```

Initialize the SA parameters  $T_0, N, K, \alpha, T_f$ 
Initialized counter  $n = 0, K = 0$ 
Do(outside loop)
  Set  $n = 0$ 
  Generate initial solution set  $x^0, x^{best} = x^0$ 
  Do (inside loop)
    Generate neighbouring solution
     $x^{n+1}$  by operation ( $x^n \rightarrow x^{n+1}$ )
    Calculate  $\Delta = f(x^{n+1}) - f(x^n)$ 
    If  $\Delta \leq 0$  then
      set :  $n = n + 1 \quad x^n = x^{n+1}$ 
    Else
      Generate random  $y \rightarrow u(0,1)$ 
      If  $y < e^{-\frac{\Delta}{T_k}}$  then  $x^n = x^{n+1}$  set:  $n = n + 1$ 
    End if
  
```





6.2. Determination of the control parameters

The efficiency of the SA algorithm depends upon the definition of following control parameters:

(1). Initial temperature ( $T_0$ ): the acceptance probability of non-improver solutions is determined by an initial temperature. As stated in [ Safaei et al., 2008] the initial temperature ( $T_0$ ) should be high enough so that the probability of accepting the non-improver solutions in the primary iterations is at least 95%.that is displayed in Figure. 4.

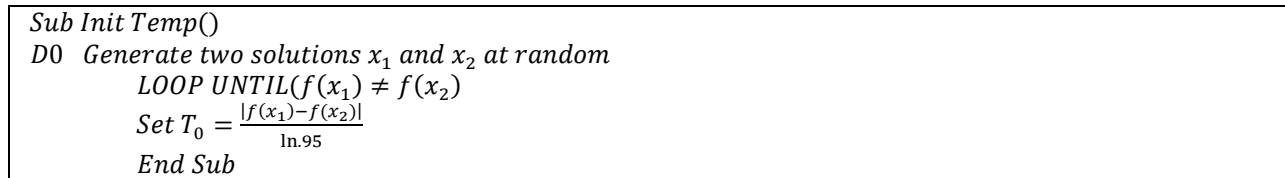


Figure 4. Pseudo code of the initial temperature

(2). Cooling rate ( $\alpha$ ): the most common way to reduce temperature is  $T_{k+1} = \alpha T_k$  where  $\alpha$  is the cooling rate and its' value is between 0 and 1 but it is close to 1. In this paper, cooling rate ( $\alpha$ ) is: 0.98

(3). Number of iterations before a temperature change ( $N$ ): number of solutions generated in each temperature is defined with  $N$  which is dependent on the problem size. In this work  $N = 50$ .

(4). The stopping criterion:

- a). When the algorithm reaches a certain number of iteration
- b). When the algorithm reaches the final temperature

In this paper, the search is ended when the final temperature was:  $T_f = .1$

6.3. Initial solution

The first step for solving the problem is to generate an initial solution. Generally, in SA algorithms the initial solution is produced through randomly or heuristic. In this work, the former is used to generate initial solution. It is necessary to calculate the number of kanban ( $K_{ikc}$ ) and number of production setup ( $n_{ikc}$ ) for every product, station and cell, while lot size ( $Q_{ic}$ ) should be determined for every product and cell. Accordingly, the former variables are shown as three- dimensional vectors and the later is displayed as two- dimensional vector.

$k(1,k,c)=$

	1	2	...	z
1	$k(1,1,1)$	$k(1,1,2)$	...	$k(1,1,z)$
2				
.	.			
.	.			
.	.			
m	$k(1,m,1)$	$k(1,m,2)$	...	$k(1,m,z)$

$k(n,k,c)=$

	1	2	...	z
1	$k(n,1,1)$	$k(n,1,2)$	...	$k(n,1,z)$
2				
.	.			
.	.			
.	.			
m	$k(n,m,1)$	$k(n,m,2)$	...	$k(n,m,z)$

$Q(i,c)=$

	1	2	...	z
1	$Q(1,1)$	$Q(1,2)$	...	$Q(1,z)$
2				
.	.			
.	.			
.	.			
m	$Q(n,1)$	$Q(n,2)$	...	$Q(n,z)$

In three dimensional vector ( $K_{ikc}$  and  $n_{ikc}$ ) each row represents a station and each column depicts a cell and while in two dimensional vector ( $Q_{ic}$ ) rows and columns represent products and cells respectively.

6.4. Moving to a neighboring solution

Neighborhood solutions are generated with heuristic method in each iteration:

(1) For two dimensional vector ( $Q_{ic}$ ):

a) Select a cell (column) randomly and determine two different products (rows) and swap the values in these locations.

b) Select a one component of the vector randomly and change the values in the component selected. Changing the component of the vector may also be applied to more than one component.

(2) For three dimensional vector ( $K_{ikc}$  and  $n_{ikc}$ ):

a) Select a one or several products randomly and for each product, select a cell (column) and two different stations (rows) randomly. Then swap the values in these locations.

b) Select a product randomly and then for each one, select cells (columns) and stations (rows) randomly and change the values in these locations.

7. Computational results

To evaluate the performance of the proposed algorithm the computational results are presented in this section. In order to solve the problems, production data are generated as follows:

- Production rate (rework rate) ~ uniform [20000, 25000];
- Setup cost (for all type) ~ uniform [25, 50];
- Holding cost (for serviceable and reworkable) ~ uniform [20, 55];
- Demand rate ~ uniform [5000, 7500];
- Shipping cost ~ uniform [200, 400];
- Proportion of serviceable products ~ uniform [0.75, 0.84];

To illustrate the validity of the model a simple example problem with following given parameters in Table 1 is solved by GAMS software. The optimal solution such as number of kanban, lot size, number of production setup and objective function value for resolved simple problem are shown in Table 2.

Table 1. Production data for the numerical example

cell number	Product number	Station number	D							S	S	
1	1	1	5500	20000	20000	.82	20	35	30	40	25	330
		2		22500	20500	.76		25	40	50	340	
	2	1	5000	22500	20000	.79	35	30	30	40	30	310
		2		24000	21500	.82		50	25	25	230	
	3	1	5500	20500	22000	.77	45	25	35	45	45	200
		2		23000	20000	.76		30	35	35	270	
2	1	1	6000	24000	22500	.84	30	35	35	45	30	230
		2		24500	22500	.75		45	45	25	300	
	2	1	5500	21500	24500	.80	35	20	45	40	40	280
		2		21500	24000	.80		25	50	50	400	
	3	1	5000	24500	20500	.75	20	40	50	30	30	320
		2		24000	21000	.77		40	40	25	50	220

Table 2. The solution result for the numerical example

Cell number	Production number	station 1			station 2		
		Q	K	n	Q	K	n
1	1	894	4	1	-	3	2
	2	551	3	1	-	2	1
	3	510	2	1	-	1	1
2	1	898	2	2	-	4	2
	2	606	4	2	-	2	1
	3	502	3	1	-	2	1

objective function value=89059

Table 3. The solution result

Problem name	SA outputs	Optimal Solution (opt)	% of SA deviation from optimal solution
P1: 4 product,3 station,3cell	223270	216520	3.11
P2: 5 product,3 station,3cell	248550	237840	4.50
P3: 6 product,4 station,4cell	383200	378580	1.22
P4: 7 product,4 station,4cell	414460	400700	3.43
P5: 8 product,5 station,5cell	612410	604250	1.35
P6: 9 product,6 station,6cell	897620	872940	2.82
P7: 10 product,7 station,7cell	1209000	1191400	1.47
P8: 10 product,8 station,8cell	1676400	1634000	2.59
P9: 10 product,9 station,9cell	2001200	1971300	1.52
P10: 10 product,10 station,10cell	2480600	2433100	1.95

Also 10 test problems (Table 3) are studied to evaluate the performance of the proposed SA. The results obtained from the proposed SA algorithm and GAMS software are compared in this table. The difference between the output values of two methods is shown in Figure 5. To evaluate the quality of obtained outputs from proposed algorithm, the percentage of deviation is defined as follow:

$$\frac{TC(SA) - TC(EXACT)}{TC(EXACT)} \times 100$$

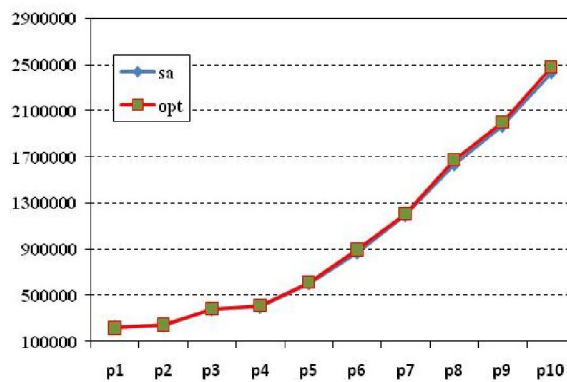


Figure 5. The solution result

As is seen, there is no significant difference between the quality of the outputs. Average percent deviation is about 2.39 percent which shows the ability of algorithms to achieve high-quality, near-optimal solutions in less time. Hence, SA algorithm for solving large size problems, that computational time for

achieving a solution in exact method is high, can be used.

### 8. Conclusion

Kanban system is one of the basic tools for just in time production system that leading to lower

inventory, holding, delivery and set up cost. It also increases the flexibility of the system.

In this study to determine number of Kanban, lot sizes and number of batches in CMS with rework process a model which minimizes the total cost, has been developed. The proposed model is useful for managerial decisions in real life situations. In order to solve the model, SA algorithm is proposed. For evaluate the performance of SA algorithm, ten problems are solved by both the SA algorithm and GAMS software (as the optimal method). By comparing the outputs of these methods, it is found that SA is reliable and effective for finding near-optimal solutions in a less time and consequently can be used for large dimension problems. For future research, other meta-heuristic methods (particle swarm, tabu search, ant colony algorithm, etc.) can be used to perhaps give better results. Additionally, in the case of dynamic, fuzzy and stochastic data and products with different processing routes this work can be extended.

#### Corresponding Author:

Mojtaba Aghajani  
Department of Industrial Engineering  
College of Engineering  
University of Tehran, Tehran, Iran  
E-mail: [m.aghajani@ymail.com](mailto:m.aghajani@ymail.com)

#### References

1. Arkat J, Saidi M, Abbasi B. Applying simulated annealing to cellular manufacturing system design. *International Journal of Advanced Manufacturing Technology* 2007; 32 (5–6): 531–536.
2. Bard JF, Golany B. Determining the number of kanban in a multiproduct multistage production system. *International Journal of Production Research*. 1991;29 (5): 881–895.
3. Bitran GR, Chang L. A mathematical programming approach to a deterministic kanban system. *Management Science*. 1987; 33 (4): 427–441.
4. Fukukawa T, Hong SC. The determination of the optimal number of kanbans in a JIT production system. *Computers and Industrial Engineering*. 1993; 24 (4): 551–559.
5. Gupta SM, AlTurki YAY. An algorithm to dynamically adjust the number of kanbans in stochastic processing times and variable demand environment. *Production Planning and Control*. 1997; 8 (2): 133–141
6. Hutchins D. *Just in time*. Madras: Productivity Press (India) (P) Ltd, 1993.
7. Kimura O, Terada H. Design and analysis of pull system, a method of multi stage production control. *International Journal of Production Research*. 1981; 19 (3): 241–253.
8. Kirkpatrick S, Gelatt CD, Veechi M P. Optimization by simulated annealing. *Science* 1983; 220 (4598): 671–679.
9. Liu N, Kim Y, Hwang H. An optimal operating policy for the production system with rework. *Computers and Industrial Engineering*. 2009; 56 (3): 874–887.
10. Mitwasi MG, Askin RG. Production planning for a multi-item, multi-stage kanban system. *International Journal of Production Research*. 1994; 32 (5): 1173–1195.
11. Monden Y. *The Toyota production system*. Industrial Engineering and Management Press 1983; Norcross, GA.
12. Rabbani M, Layegh J, Mohammad Ebrahim R. Determination of number of kanbans in a supply chain system via Memetic algorithm. *Advances in Engineering Software*. 2009; 40 (6): 431–437.
13. Safaei N, Saidi-Mehrabad M, Jabal-Ameli MS. A hybrid simulated annealing for solving an extended model of dynamic cellular manufacturing system. *European Journal of Operational Research*. 2008; 185 (2): 563–592.
14. Sarker BR, Balan CV. Operations planning for kanbans between two adjacent workstations. *Computers and Industrial Engineering*. 1996; 31 (1): 221–224.
15. Sarker BR, Balan CV. Operations planning for a single-stage kanban system operating under linear demand. *International Journal of Production Research*. 1998; 36 (2): 357–375.
16. Sarker BR, Balan CV. Operations planning for a multi-stage kanban system. *European Journal of Operational Research*. 1999; 112 (2): 284–303.
17. Shahabudeen P, Gopinath R, krishnaiah K. Design of bi-criteria kanban system using simulated annealing technique. *Computers and Industrial Engineering*. 2002; 41 (4): 355–70.
18. Sridhar J, Rajendran C. Scheduling in flowshop and cellular manufacturing systems with multiple objectives—a genetic algorithmic approach. *Production Planning and Control*. 1996; 7 (4):374–382.
19. Wang S, Sarker BR. Optimal model for supply chain system controlled by kanban under JIT philosophy. *European Journal of Operational Research*. 2006; 172 (1): 179–200.
20. Widyadana GA, Wee HM, Chang JY. Determining the optimal number of Kanban in multi-products supply chain system. *International Journal of Systems Science*. 2010; 41 (2): 189-201.