Convergent Criterion for the Numerical Solution of the Duffin's Equation by the One Step Euler Scheme.

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Abstract: The criterion for the convergence of the Duffin's equation with an approximate expression for the numerical solution by a one step or Euler scheme has been established. The Euler scheme is a one step numerical scheme of first order accuracy.

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1.Introduction

We are interested in the approximate solution of the system,

$\dot{u}(t) = f(t, u(t \quad)), \quad \forall_t \in [t_0, T]$, ………………………………………..(i)

with the initial condition given on

with the initial condition $u(t_0) = u_0$, ……………………………………………...(ii)

We will always suppose that f satisfies the Cauchy-Lipschitz conditions.

Definition.

The approximation of equations (i)and (ii) is defined by a one step scheme.

$U_{k+1} = U_k + F(U_{k+1}, U_k, h)$, …………………………………(iii)

is said to be convergent if for any initial U_0,

$lim_{h\to 0\, max}|U(t_k) - U_k| = 0$, ………………………………………………(iv)

Indeed, there could be a truncation error in the initial condition as we do not suppose that the scheme has the exact initial condition of the ODE.

It's extremely important to sought for veritable numerical schemes in diverse applications spanning across physical sciences, applied mathematics, etc, thus there grows huge urge to explore a viable numerical scheme for each application that arises.

Diverse numerical methods are iterative in nature, and thus it is tremendously important to sought for swift convergence. While at present, we centered on convergence, future investigations would be expanded to elucidate more other interesting features and mathematical properties.

2. Discussion

Cauchy-Lipschitz Existence Theorem

Theorem. (Cauchy-Lipschitz).

Suppose that $[t_1, t_2]$ is a compact interval and f is a continous function from $[t_1, t_2] \times |R^d$ which satisfies the following property: there exists a constant L such that;

$|f(t, v) - f(t, w)| \leq |v - w|, \forall_t \in [t_1, t_2], \forall_{v,w} \in |R^d$ …..(v)

Here [.]denotes some norms on $|R^d$. then for any t_0 in $[t_1, t_2]$, and u_0 in $|R^d$, there exists a unique continuously differentiable function u from $[t_1, t_2]$ to $|R^d$ which satisfies equations (i) and (ii).

Euler's Scheme to a Model Equation

For the model equation $y' = \lambda y$; ……………………………………………(vi)

$y_{n+1} = y_n + \lambda h y_n = (1 + \lambda h) y_n$ ……………………………………….(vii)

This equation indicates obviously that one step or Euler scheme is of first order accuracy.

For stability;

$|\sigma| \ll l$, where $\sigma = (1 + \lambda h)$, ………………………………………….(viii)

Duffin's Equation.

The Duffin's equation arises out of the motion of a pendulum, it is expressed as;

$\check{\phi} + a sin\phi = cost$, $\phi(0) = \phi(\pi) = 0$, ……………………………………………..(ix)

The differential equation has an integral equivalent or equation representation viz;

$\phi(t) - \int_0^t (t - y)\left\{a(\phi(y)) - \frac{1}{6}\phi^3(y)\right\} dy = ct$, ………………………………………….. (x)

where c is the unknown value of $\phi'(0)$which is called the shape parameter.

Based on the Taylor's series approximation, sinϕ is replaced by; $\phi - \frac{1}{6}\phi^3$ for the first few terms.

Adopting the Euler or one step scheme (iii), an expression for an approximate numerical solution of the Duffin's equation is given by;

$$\Phi_{n+1} = \Phi_n - h(asin\boldsymbol{\Phi} - cost) \quad , \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(xi)a}$$

$$\Phi_{n+1} - \Phi_n = -h(asin\Phi - cost) \quad , \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(xi)b}$$

The approximate expression for the numerical solution of the Duffin's equation would converge if equations (xi)b satisfies the scheme in (iv), and obviously this would be practically achievable by choosing a relatively small step size, h such enough, could be close to zero and thus (xi)b would approach zero faster.

3. Conclusion

The criterion for the convergence of the Duffin's equation with an approximate expression for the numerical solution by a one step or Euler scheme has been established. The Euler scheme is a one step numerical scheme of first order accuracy. Despite the availability of known higher order numerical schemes like the Runge-Kutta method, etc, the one step scheme could still be adopted by choosing appropriate relatively small step sizes. Convergence is often crucial in the field of numerical methods, as these methods are often iterative in nature and thus the need for convergence among other pertinent factors comprising computation efficiency and costs.

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