An LMI Approach to Design Dynamic Output Feedback Control for Stochastic Hybrid Systems

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Abstract: This paper deals with the stabilization of a class of uncertain stochastic hybrid systems. The uncertainties are norm bounded type. Under the complete access to the system mode a dynamic output feedback controller that makes the closed-loop dynamics of this class of systems regular, impulse-free and stochastically stable is designed. The state space matrices of this controller are the solution of some linear matrix inequalities (LMIs).
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1. Introduction

In practice, there exist some industrial systems that cannot be represented by the class of linear time-invariant model, since the behavior of the dynamics of these systems is random with some special features. As an example of such systems, we mention those with abrupt changes, breakdowns of components, etc. Such class of dynamical systems can be adequately described by the class of stochastic hybrid systems which is the subject of this paper.

This class of systems referred to also as Markovian jump systems, Systems with random structures, have attracted a lot of researchers, attention and many problems have been tackled and solved. Among these problems, we quote those of stability, stabilizability, H_∞ control problem and filtering problem. For more details on what has been done on this class of systems, we refer the reader to the recent books by Arnold (2008), Boukas (2007) and Boukas (2005) and the references therein where different results on stochastic hybrid systems with or without time-delay have been developed. These two books present a good literature review on the subject up to 2004. Particularly, the stabilization problem has attracted many researchers from control community and many results have been reported in the literature.

For the singular system which also can be used to represent a variety of practical systems like electrical circuits, mechanical systems, robotics, etc. (see Boukas (2001) for some examples), the developed results in the literature for regular systems cannot be used and new techniques need to be developed. Some attempts have been made (i) to check the stability and (ii) to stabilize the class of deterministic singular systems. For more details on these, we refer the reader to Boukas (2002) for stability and to Arnold (2008), Boukas (2007) and Boukas (2003) for the stability and the stabilization, and the references therein. Note also that other problems have been tackled among them we address in this paper H_∞ control problem (see Dai (1989), de Farias (2000) and the references therein for more details). For the singular stochastic hybrid systems, Boukas and his coauthors have attempted to tackle some problems for this class of systems when the dynamics have time-delay. For more details, we refer the reader to Ishihara (2002), Kats (2002) where LMI results on the design of stabilizing state feedback controllers have been developed. To the best of our knowledge, the stabilization of continuous-time singular stochastic hybrid systems using a dynamic output feedback controller has never been tackled and our objective in this paper is to study this problem. This technique of stabilization is, even in the deterministic case, a hard problem in general that cannot easily be formulated as an LMI problem. Our goal in this paper consists of designing a dynamic output feedback controller that makes the closed-loop dynamics of the class of systems we are studying, regular, impulse-free and stochastically stable. Under the assumption of the complete access to the system mode, a stabilizing dynamic output feedback controller is designed. The gains of such controller are determined by solving a set of LMIs. We have to note that to get the LMI setting, equality restrictive condition is used. The rest of the paper is organized as follows. In Section 2, the problem we are considering is stated and some definitions are given. Section 3 gives the main results of the paper that determines the static output feedback controller which assures the closed-loop dynamics of the stochastic hybrid system is regular, impulse-free and stochastically stable.

2. General problem statement

Let us consider a dynamical singular system defined in a fundamental probability space (Ω, Φ, P)

and assume that its dynamics is described by the following differential system:

$$\begin{cases} \tilde{E}\dot{x}(t) = A(r_t,t)x(t) + B_1(r_t,t)w(t) + B_2(r_t,t)u(t) \\ z(t) = C_1(rt)x(t) + D_{11}(rt)w(t) + D_{12}(rt)u(t) \\ y(t) = C_2(r_t)x(t) + D_{21}(rt)w(t) \\ x(0) = x_0 \end{cases} \quad (1)$$

where $x(t) \in \Re^n$ is the state vector, $x_0 \in \Re^n$ is the initial state, $u(t) \in \Re^n$ is the control input, $y(t) \in \Re^n, \{r_t, t \geq 0\}$ is the continuous-time Markov process taking values in a finite space $\varphi = \{1,2,...,N\}$ and describes the evolution of the mode at time t, E is a known singular matrix with $\text{rank}(E) = n_E < n$, $A(r_t,t) \in \Re^{n\times n}$ and $B(r_t,t) \in \Re^{n\times n}$ are matrices with the following forms for every $i \in \varphi$:

$$A(i,t) = A(i) + D_A(i)F_A(i,t)E_A(i),$$
$$B_2(i,t) = B_2(i) + D_B(i)F_B(i,t)E_B(i)$$

where $A(i) \in R^{n\times n}, B(i) \in R^{n\times n}$, $C(i) \in R^{n\times n}$, $D_A(i)$, $E_A(i)$, $D_B(i), E_B(i)$ are real known matrices with appropriate dimensions, and $F_A(i,t)$ and $F_B(i,t)$ are unknown real matrices that satisfy the following:

$$F_A^T(i,t)F_A(i,t) \leq I, F_B^T(i,t)F_B(i,t) \leq I \quad (2)$$

The Markov process $\{r_t, t \geq 0\}$ beside taking values in the finite set φ, represents the switching between the different modes and its dynamics is described by the following probability transitions:

$$P[r_{t+h} = j \mid r_t = i] = \begin{cases} \lambda_{ij}h + o(h) & \text{when } r_t \text{ jumps from } i \text{ to } j \\ 1 + \lambda_{ij}h + o(h) & \text{otherwise} \end{cases} \quad (3)$$

where λ_{ij} is the transition rate from mode i to mode j with $\lambda_{ij} \geq 0$ when $i \neq j$ and $\lambda_{ii} = -\sum_{j=1, j\neq i}^{N} \lambda_{ij}$ and $o(h)$ is such that $\lim_{h\to 0} \frac{o(h)}{h} = 0$.

Throughout this paper, we assume that the system state $x(t)$ is not accessible for feedback while the system mode r_t is.

Remark 2.1. Notice that when E is not singular, system (1) can be transformed to the class of Markov jump linear systems and the results developed in the literature can be used either to check the stochastic stability, or to design the state feedback or the output feedback controllers that stochastically stabilize this class of systems. For more details on this matter we refer the reader to Arnold (2008), Boukas (2007) and the references therein.

Definition 2.1. Boukas (2001).

i. System (1) is said to be regular if the characteristic polynomial, $\det(s\tilde{E} - A(i))$ is not identically zero for each mode $i \in \varphi$.

ii. System (1) is said to be impulse-free, i.e. $\deg(\det(s\tilde{E} - A(i))) = rank(\tilde{E})$ for each mode $i \in \varphi$.

In the literature we can find different definitions for stochastic stability. Among them we quote the moment stability, the stability in probability and almost sure stability. For simplicity, we denote $x(t; x_0, r_0)$, as $x(t)$ in the sequel, the solution of system (1) when the initial conditions are, respectively, x_0 and r_0, the concept of stochastic stability, stochastic stabilizability and their robustness we will use in this paper are given by the following definitions (see Arnold (2008), Boukas (2007) or Boukas (2005)).

Definition 2.2. System (1) with $u(t) \equiv 0$ is said to be:

1. Stochastically stable if there exists a constant $M(x_0, r_0) > 0$ such that the following holds for any pair of initial conditions (x_0, r_0):

$$E\left[\int_0^\infty x^T(t)x(t)dt \mid x_0, r_0\right] \leq M(x_0, r_0); \quad (4)$$

2. robust stochastically stable if it is stochastically stable for all admissible uncertainties.

Definition 2.3. System (1) is said to be:

1. Stochastically stabilizable if there exists a controller $\tilde{K}(s)$ that

$$U(s) = \tilde{K}(s)Y(s) \quad (5)$$

Where $\tilde{K}(s) = \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix}$ and $\begin{cases} \dot{x}_k = A_k x_k + B_k y \\ u = C_k x_k + D_k y \end{cases}$

such that the closed loop system is stochastically stable.

2. Robust stochastically stabilizable if there exists a control of the form (5) such that the closed-loop system is stochastically stable for all admissible uncertainties.

The aim of this paper is to (i) develop LMI-based conditions for system (1) with $u(t) \equiv 0$ to check if a given system is regular, impulse-free and stochastically stable; and (ii) design a dynamic output feedback controller of the form (5) that makes the closed-loop dynamics of the class of systems under study regular, impulse-free and stochastically stable.

Before closing this section, let us give some Lemmas that we will use in the rest of the paper.

Lemma 2.1 (Boukas (2007)). Let H, F and G be real matrices of appropriate dimensions, then, for any scalar $\varepsilon \succ 0$ and a matrix F satisfying $F^T F \leq I$, we have

$$HFG + G^T F^T H^T \leq \varepsilon HH^T + \varepsilon^{-1} G^T G \qquad (6)$$

Lemma 2.2 Arnold (2008), Boukas (2007). The linear matrix inequality $\begin{bmatrix} H & S^T \\ S & R \end{bmatrix} > 0$ is equivalent to $R > 0$, $H - S^T R^{-1} S > 0$, where $H = H^T$, $R = R^T$ and S is a matrix with appropriate dimension.

Lemma 2.3 Arnold (2008), Boukas (2007). For any matrix u, and $v \in \Re^{n\times n}$ with $v > 0$, we have $uv^{-1}u^T \geq u + u^T - v$.

Lemma 2.4 (Boukas (2007)) System (1) is regular, impulse-free and stochastically stable if there exists a set of nonsingular matrices $X = (X(1),...,X(N))$, such that the following coupled LMIs hold for every $i \in \varphi$:

$$\begin{cases} \widetilde{E}^T X(i) = X^T(i)\widetilde{E} \geq 0 \\ X^T(i)A(i) + A^T(i)X(i) + \sum_{j=1}^{N} \lambda_{ij} \widetilde{E}^T X(j) < 0 \end{cases} \qquad (7)$$

Lemma 2.5 consider matrices P, Q and symmetric matrix H, the N_Q and N_P matrices with full rank have the below specification:

$\mathrm{Im}\,(N_P) = Ker\ P$, $\mathrm{Im}\,(N_Q) = Ker\ Q$, where $Ker(.)$ is null space of the matrix and the Im(.) is the rang of the matrix. Then there exists a matrix J such that: $H + P^T J^T Q + Q^T JP < 0$ if and only if $N_P^T HN_P < 0, N_Q^T HN_Q < 0$.

3. Main results

Before developing the design procedure for the dynamic output feedback controller, let us assume that $u(t) = 0$, for $t \geq 0$ and study the stochastic stability of the nominal system (1). Our concern is to establish LMI conditions to check if a given dynamical system of this class is regular, impulse-free and stochastically stable. Lemma (2.4) states the desired results on stochastic stability of such class of systems.

Let us now concentrate on the design of the dynamic output feedback controller of form (5). Plugging the controller expression in the dynamical system (1) gives $E\dot{x}_{cl} = A_{cl}x_{cl} + B_{cl}w$ with $x_{cl} = \begin{bmatrix} x \\ x_{cl} \end{bmatrix}$, $E = \begin{bmatrix} \widetilde{E} & 0 \\ 0 & I \end{bmatrix}$

$A_{cl} = \begin{bmatrix} A(i) + B_2(i)D_k C_2(i) & B_2(i)C_k \\ B_k C_2(i) & A_k \end{bmatrix}$,

$B_{cl} = \begin{bmatrix} B_1(i) + B_2(i)D_k D_{21}(i) \\ B_k D_{21}(i) \end{bmatrix}$,

$C_{cl} = \begin{bmatrix} C_1(i) + D_{12}(i)D_k C_2(i) & D_{12}(i)C_k \end{bmatrix}$,

$D_{cl} = D_{11}(i) + D_{12}(i)D_k D_{21}(i)$

The closed loop sate space matrices can be written based on $\widetilde{K}$ as:

$$\begin{aligned} A_{cl} &= \overline{A} + \underline{B}\widetilde{K}\underline{C} \\ B_{cl} &= \overline{B} + \underline{B}\widetilde{K}\underline{D}_{21} \\ C_{cl} &= \overline{C} + \underline{D}_{12}\widetilde{K}\underline{C} \\ D_{cl} &= D_{11} + \underline{D}_{12}\widetilde{K}\underline{D}_{21} \end{aligned} \qquad (8)$$

where $\overline{A}(i) = \begin{bmatrix} A(i) & 0 \\ 0 & 0 \end{bmatrix}$, $\underline{B}(i) = \begin{bmatrix} 0 & B_2(i) \\ I & 0 \end{bmatrix}$, $\underline{C}(i) = \begin{bmatrix} 0 & I \\ C_2(i) & 0 \end{bmatrix}$, $\overline{C}(i) = \begin{bmatrix} C_1(i) & 0 \end{bmatrix}$, $\overline{B}(i) = \begin{bmatrix} B_1(i) \\ 0 \end{bmatrix}$, $\underline{D}_{12}(i) = \begin{bmatrix} 0 & D_{12}(i) \end{bmatrix}$, $\underline{D}_{21}(i) = \begin{bmatrix} 0 \\ D_{21}(i) \end{bmatrix}$, and the $0, I$ are zero and identity matrices with appropriate dimensions. The objective is to obtain the state space form representation matrices of controller.

As seen in (5) the closed loop state space matrices are linear function of the controller matrix $\widetilde{K}$. The Lemma 2.5 has basic role in our theoretical derivations.

- The nominal stability criteria using LMI:

The stability of the closed loop system is the most important issue in the controller design.

Based on Lemma 2.4, the closed-loop system is regular, impulse-free and stochastically stable if there exists a set of nonsingular matrices $\mathrm{X} = (\mathrm{X}(1),\ldots,\mathrm{X}(\mathrm{N}))$ such that the following holds for every $i \in \varphi$:

$$\begin{cases} E^T X(i) = X^T(i)E \geq 0 \\ X^T(i)A_{cl}(i) + A_{cl}^T X(i) + \sum_{j=1}^{N} \lambda_{ij} E^T X(j) < 0 \end{cases} \qquad (9)$$

Using the expression of $\mathrm{A}_{cl}(\mathrm{i})$, the second matrix inequality in equation (9) will be:

$$X^T(i)(\overline{A}(i) + \underline{B}(i)\widetilde{K}\underline{C}(i)) + (\overline{A}(i) + \underline{B}(i)\widetilde{K}\underline{C}(i))^T X(i) + \sum_{j=1}^{N} \lambda_{ij} E^T X(j) < 0$$

Now defining matrices $P_{x_{cl}}$, Q and $H_{x_{cl}}$ as:

$$Q(i) = \underline{C}(i)$$
$$P_{x_{cl}}(i) = \underline{B}^T(i)X(i)$$
$$H_{x_{cl}}(i) = \bar{A}^T(i)X(i) + X^T(i)\bar{A}(i) + \sum_{j=1}^{N}\lambda_{ij}E^T X(j) \tag{10}$$

The equation (11) is rewritten as following:

$$H_{x_{cl}}(i) + Q^T(i)\tilde{K}^T P_{x_{cl}}(i) + P_{x_{cl}}^T(i)\tilde{K}Q(i) < 0 \tag{11}$$

According to Lemma 2.5, the inequality (11) is equivalent to:

$$N_{P_{x_{cl}}}^T(i)H_{x_{x_{cl}}}(i)N_{P_{x_{cl}}}(i) < 0 \tag{12.a}$$
$$N_Q^T(i)H_{x_{x_{cl}}}(i)N_Q(i) < 0 \tag{12.b}$$

The inequality (12.a) is not an LMI of $X(i)$ because $X(i)$ appears in both $H_{x_{cl}}(i)$ and $N_{P_{x_{cl}}}(i)$. Defining $T_{x_{cl}}$ and P as

$$P(i) := \underline{B}^T(i) \tag{13}$$
$$T_{x_{cl}}(i) := (X^{-1})^T(i)\bar{A}^T(i) + \bar{A}(i)X^{-1}(i) + \sum_{j=1}^{N} X^{-T}(i)\lambda_{ij}E^T X(j)X^{-1}(i) \tag{14}$$

The inequality (12.a) converts to an LMI set.

Theorem 3.1. For $X \geq 0$, the inequality $N_{P_{x_{cl}}}^T H_{x_{cl}} N_{P_{x_{cl}}} < 0$ is equivalent to:

$$N_P^T T_{x_{cl}} N_P < 0 \tag{15}$$

Proof: the matrices $P_{x_{cl}}$ and P are related to each other as following:

$$P_{x_{cl}} = PS \tag{16}$$
$$S = X \tag{17}$$

therefore we have

$$N_{P_{x_{cl}}} = S^{-1}N_P \tag{18}$$

inserting $N_{P_{x_{cl}}}$ from equation (18) in the inequality $N_{P_{x_{cl}}}^T H_{x_{cl}} N_{P_{x_{cl}}} < 0$ we have

$$N_P^T(S^{-1})^T H_{x_{cl}} S^{-1} N_P < 0 \tag{19}$$

According to definition of $H_{x_{cl}}$ in equation (10) and $T_{x_{cl}}$ in equation (14), the inequality (19) is equivalent to equation (15).

Now, referring to equation (12) Theorem 3.1, the sufficient condition to exist a stabilizing controller is obtained as:

$$N_P^T T_{x_{cl}} N_P < 0 \tag{20.a}$$
$$N_Q^T H_{x_{cl}} N_Q < 0 \tag{20.b}$$

the inequality (20.a) is an LMI of X^{-1} and inequality (20.b) is an LMI of X. Therefore, the inequalities set (20) is not an LMI of X. To overcome this difficulty, it is assumed that the matrices X and X^{-1} have a structure as following:

$$X(i) = \begin{bmatrix} X_1(i) & X_2(i) \\ X_2^T(i) & X_3(i) \end{bmatrix} \tag{21.a}$$
$$X^{-1}(i) = \begin{bmatrix} Y_1(i) & Y_2(i) \\ Y_2^T(i) & Y_3(i) \end{bmatrix} \tag{21.b}$$

where X is a symmetric positive definite with dimension of $(n+n_k)\times(n+n_k)$ and the sub matrices X_1 and Y_1 are of $n \times n$ dimension. n and n_k are open loop system ($G(s)$) and the controller ($K(s)$) dimension, respectively. The following Theorem shows how to express the inequalities (20) using X_1 and Y_1 in an LMI framework.

The following Theorem shows how to describe the equation (20) utilizing $X_1(i)$ and $Y_1(i)$.

Theorem 3.2: the inequalities set

$$N_P^T(i)T_{x_{cl}}(i)N_P(i) < 0$$
$$N_Q^T(i)\,H_{x_{cl}}(i)\,N_Q(i) < 0$$

holds if and only if:

$$N_O^T(i)(A^T(i)X_1(i) + X_1(i)A(i) + \sum \lambda_{ij}\tilde{E}X_1(i))N_O(i) < 0$$
$$N_C^T(A(i)Y_1(i) + Y_1^T(i)A^T(i) + \sum_{j=1}^{N}\lambda_{ij}Y_1^T(i)\tilde{E}^T Y_1^{-1}(j)Y_1(i))N_C < 0$$

where $N_C(i)$ and $N_O(i)$ are full rank matrices such that:

$$\operatorname{Im} N_o(i) = \ker C_2(i)$$
$$\operatorname{Im} N_C(i) = \ker B_2^T$$

Proof: first, we show that $N_P^T T_{x_{cl}} N_P < 0$ is equivalent to

$$N_C^T(A(i)Y_1(i) + Y_1^T(i)A^T(i) + \sum_{j=1}^{N}\lambda_{ij}Y_1^T(i)\tilde{E}^T Y_1^{-1}(j)Y_1(i))N_C < 0$$

To do this, inserting $\bar{A}$ from equation (8) and $X^{-1}(i)$ from equation (21.b) in the equation (14) and inserting the matrix $\underline{B}(i)$ from equation (8) in equation (13).

Now, to calculate N_P, we have

$$\left.\begin{array}{r} \operatorname{Im} N_P = \ker P \\ \ker P = \{x \mid Px = 0\} \\ \operatorname{Im} N_P = \{y \mid N_P z = y\} \end{array}\right\} \Rightarrow N_P z = x \overset{\times P}{\Rightarrow} PN_P z = Px = 0$$

For some z:

$$PN_P z = 0$$

Choosing $PN_P = 0$ we obtain $N_P = \ker P$, we can derive $\ker P$ as

$$\begin{bmatrix} 0 & I \\ B_2^T(i) & 0 \end{bmatrix}\begin{bmatrix} V_1(i) & V_2(i) \\ V_3(i) & V_4(i) \end{bmatrix} = 0$$

So, $\begin{cases} V_3(i) = V_4(i) = 0 \\ B_2^T(i)V_1(i) = 0 \\ B_2^T(i)V_2(i) = 0 \end{cases}$ and

$\ker P(i) = \begin{bmatrix} V_1(i) & V_2(i) \\ 0 & 0 \end{bmatrix}$. For simplicity, assume $V_2 = 0$ then

$$\ker P(i) = \begin{bmatrix} V_1(i) & 0 \\ 0 & 0 \end{bmatrix}; \quad B_2^T(i)V_1(i) = 0 \tag{23}$$

Therefore, $N_P(i) = \begin{bmatrix} V_1(i) & 0 \\ 0 & 0 \end{bmatrix}$ where $V_1(i)$ is a member of the null space of $B_2^T(i)$. Since the second row of the matrix $N_P(i)$ is zero, the second row and column of the matrix $T_{x_{cl}}$ has no effect on the condition $N_P^T T_{x_{cl}} N_P < 0$, and we can eliminate the both row and column. Hence, choosing $N_C = V_1$, the inequality $N_P^T T_{x_{cl}} N_P < 0$ can be rewritten as

$$\begin{aligned} &N_C^T(A(i)Y_1(i) + Y_1^T(i)A^T(i) + Y_1^T(i)\tilde{E}X_1(j)Y_1(i) + \\ &+ Y_2^T(i)X_2^T(j)Y_1(i) + Y_1^T(i)\tilde{E}X_2(j)Y_2^T(i) \\ &+ Y_2^T(i)X_3(j)Y_2^T(i))N_C < 0 \end{aligned} \tag{24}$$

so, the equivalency of two inequalities is proved.

In the same way it can be shown that $N_Q^T H_{x_{cl}} N_Q < 0$ and

$$N_O^T(i)(A^T(i)X_1(i) + X_1^T(i)A(i) + \sum_{j=1}^{N} \lambda_{ij}\tilde{E}X_1(j))N_O(i) < 0 \tag{25}$$

are equivalent.

Selecting X as $X = diag(X_1, X_3)$, we have $X^{-1} = diag(Y_1, Y_3)$, where $X_1^{-1} = Y_1, X_3^{-1} = Y_3$. then the equations (24) is simplified to

(26)

$$\begin{aligned} &N_C^T(A(i)Y_1(i) + Y_1^T(i)A^T(i) + \\ &+ Y_1^T(i)\tilde{E}Y_1^{-1}(j)Y_1(i))N_C < 0 \end{aligned} \tag{26}$$

Pre- and post multiplying $E^T X(i) = X^T(i)E \geq 0$, respectively, by $X^{-T}(i)$ and $X^{-1}(i)$, one has $X^{-T}(i)E^T = EX^{-1}(i) \geq 0$.

Replacing E and $X^{-1}(i)$ give

$$Y_1^T(i)\tilde{E}^T = \tilde{E}Y_1(i) \geq 0$$

$$Y_3(i) \geq 0$$

Now, let us assume the existence of $W(i) = W^T(i)$ such that $Y_1^T(j)\tilde{E}^T < W(j)$ holds for every $j \in \varphi$. If we define $\Gamma_i(Y_1)$ and $\Pi_i(Y_1)$ as

$$\Gamma_i(Y_1) := \left[\sqrt{\lambda_{i1}}Y_1^T(i) \quad \cdots \quad \sqrt{\lambda_{ii-1}}Y_1^T(i) \quad \sqrt{\lambda_{ii+1}}Y_1^T(i) \quad \cdots \quad \sqrt{\lambda_{iN}}Y_1^T(i)\right]$$

$$\begin{aligned} \Pi_i(Y_1) := diag[&Y_1(1) + Y_1^T(1) - \tilde{W}(1), \cdots, \\ &Y_1(i-1) + Y_1^T(i-1) - \tilde{W}(i-1), \\ &, Y_1(i+1) + Y_1^T(i+1) - \tilde{W}(i+1), \cdots, \\ &, Y_1(N) + Y_1^T(N) - \tilde{W}(N)] \end{aligned} \tag{27}$$

and using Lemma 2.3, we obtain

$$\begin{aligned} &\sum_{j=1}^{N} \lambda_{ij} Y_1^T(i)Y_1^{-1}(j)Y_1(i) \leq \\ &\lambda_{ii}Y_1^T(i) + \Gamma_i(Y_1)\Pi_i^{-1}(Y_1)\Gamma_i^{-1}(Y_1) \end{aligned} \tag{28}$$

and the following sufficient conditions:

$$W(j) > Y_1^T(j)\tilde{E} = \tilde{E}^T Y_1(i) \geq 0$$

$$\begin{bmatrix} J(i) & \Gamma_i(Y_1) \\ \Gamma_i^T(Y_1) & -\Pi_i(Y_1) \end{bmatrix} < 0 \tag{29}$$

with

$$\begin{aligned} J(i) = &N_C^T(i)\bar{A}(i)Y_1(i)N_C(i) + N_C^T(i)Y_1^T(i)\bar{A}^T(i)N_C(i) \\ &+ \lambda_{ii}N_C^T(i)Y_1^T(i)N_C(i) \end{aligned}$$

The following Theorem summarizes the results of this development.

Theorem 3.2. If there exist sets of nonsingular matrices $X_1 = (X_1(1), X_1(2), \ldots\ldots, X_1(N))$ and $Y_1 = (Y_1(1), Y_1(2), \ldots\ldots, Y_1(N))$ and a set of symmetric and positive-definite matrices $W = (W(1), \ldots, W(N))$, such that the following set of LMI s holds for each $i \in \varphi$:

$$\begin{cases} Y_1^T(i)\tilde{E}^T = \tilde{E}Y_1(i) \\ (25) \\ (29) \end{cases}$$

then system (1) is regular, impulse-free and stochastically stable and the state space matrices of the controller can be computed through

$$H_{x_{cl}}(i) + Q^T(i)\tilde{K}^T P_{x_{cl}}(i) + P_{x_{cl}}^T(i)\tilde{K}Q(i) < 0 \tag{30}$$

Remark 3.1. Notice that the conditions we developed are only sufficient and since the matrix X was assumed diagonal, the LMIs may be conservative. But we have to notice that without this assumption the solution cannot be put in the LMI setting.

7. Conclusion

This paper dealt with the class of singular stochastic hybrid systems. LMI results on stochastic stability and stochastic stabilizability are developed. Under the assumption that the state vector is not available for feedback a dynamic output feedback controller is designed to make the closed-loop dynamics of this class of systems regular, impulse-free and stochastically stable. The controller state space matrices are determined by solving a set of coupled LMIs for the nominal system.

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