Foldable And Deployable Pyramidal Plated Structures
Part I: Design

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Abstract: This paper explains the various trials carried out to design the foldable pyramid, using trials on folding plane surfaces in a symmetrical way and then assembling all surfaces of the structure to form the three-dimensional model. All preliminary studies and the suggested solutions that can be used to achieve the solution for folding the pyramidal shape will be addressed.
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1 Introduction

The conditions are: firstly, that the pyramid model will have a square base, which means that the model will be symmetrical, where this will help ease in designing calculations just for one face and then applying the theories to the other faces. Secondly, the model must be well concealed without leaving gaps at the final designed model. This is to prevent water or air from entering into the shaped design while considering the availability of using the model for different purposes, such as an operation theatre unit. Finally plate structures will be used, whereby this would highlight some of the points, such as the importance of the dividing angles plus reducing the movement freedom and difficulties of imparting the model.

It is possible for these conditions to be determined at the early stage of the desirable design and then proceeding further until reaching the final design, while experiencing some difficulties during the process. The design starts by trials on models with "zero-thickness" materials which are hereby called (A) models.

2 Trials and Ideas

2.1 Flat Surface (Model A1)

De Focatiis and Guest (2002) employed tree leaves to benefit from them in the design of foldable structures. This experiment was on a flat square surface divided into four symmetrically equal leaves, provided that the shape should fold towards the centre of the square. For the sake of simplicity of this folding, the Miura-Ori's Map (Bain, 1980), illustrated in Figure 1, has been taken into consideration. In Figure 2, the parts of the four square angles must be less than 90°; the reason behind that is to prevent and reduce strain during the deployment of the model. This trial for obtaining the pyramid shape of the three dimensions indicates that small changes in the right angle as well as removing some part in the middle of each side of the square should be made (Figure 2).

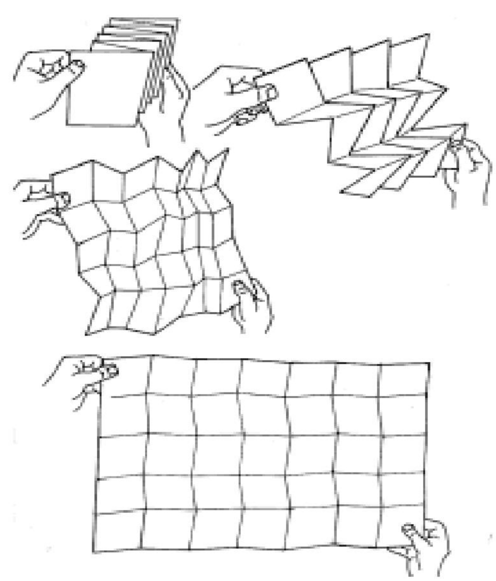

Figure 1: Miura-Ori Map (Bain, 1980)

In fact, this pyramid does not provide good reduction in height, width and depth at the final fold, and it would not be a good idea even if it does in one of them, because the fold in this model would not be very compact.

Although the trial was not successful, it has benefited from the same main points in design. The top point of the model should remain and move perpendicularly during the deploying and folding process. Furthermore, dividing the shape into main parts and then dividing each main part into more small parts is good. As a matter of fact, the division for the main parts was at the middle of the square side. Nevertheless, the next trial of dividing will be in the square's corner.

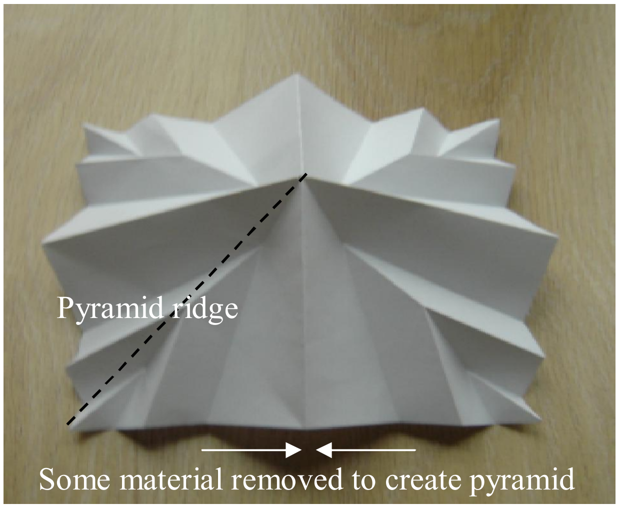

Figure 2. Model A1 (pyramid shape from a flat surface)

2.2 Folding One Face of the Pyramid (model A2, A3)

We benefit from Model A1 by working on the flat model in order to remove some of the parts so as to obtain a 3-D pyramid shape. This means that in Model A2 we start from the final shape, that is, the pyramid, which is divided into four main parts that forms four identical triangles which meet at one point, that is, the top point of the pyramid. This is in comparison with four squares in Model A1. In order to be capable of making this shape foldable then we must divide every triangle into smaller parts. When the parts are divided to the tree leaves, the final shape shows high reduction in width and some increase in height (Figure 1.3).

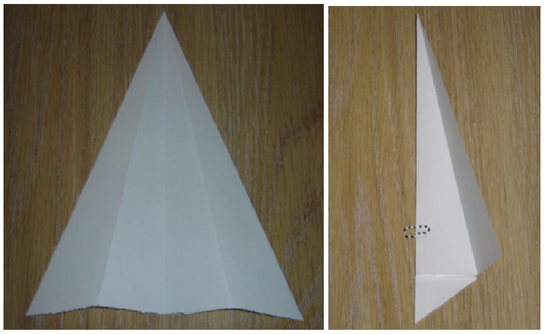

Figure 3. Model A2, One pyramid face with tree leaves division

Due to height increase in Model A2, the idea of dividing perpendicularly and horizontally appears more logical. This would support the notion that more division gives more reduction in area.

Miura Map uses the zigzag divisions rather than straight vertical and horizontal divisions. This makes the fold easier and reduces the strain. Zigzag fold changes the structure of plate shape from square to rectangular. In other words, changing the right angle in a foldable surface leads to strain reduction during folding.

Miura Map is rectangular or square surface while the pyramid face is triangular. This makes vertical fold in triangle shape not the same as in a four edged-shape. It is better to add zigzag horizontal folds to model A2 to get a high reduction in both width and height (Figure 4, Model A3).

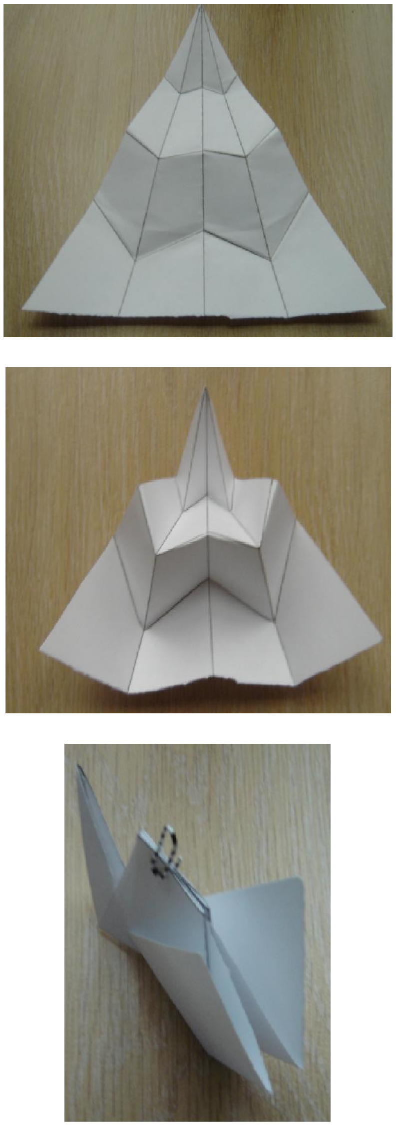

Figure 4. Model A3. A pyramid face with vertical and zigzag horizontal divisions

In this way the structure's 3-D volume is reduced, thus achieving a very compact model. However, this is so far working on one flat surface only. In the following section, joining those faces to form a complete pyramid will be dealt with.

2.3 Assembling the Pyramid Faces

Important points should be taken in account to achieve a closed 3D foldable structure. At the beginning, folding and deploying mechanism should be with no flexible material and as strain free as possible. On the other hand, a structure with less freedom will occur, which means that there would be neither free ends, nor flexible joints between segments, especially those on the critical position like a pyramid ridge line. Moreover, symmetrical movement should be applied throughout the folding and deploying process. In a pyramid case, the head point of the pyramid would be the central point of the symmetrical movement. In other words, during folding every point in the structure moves in a particular path to the central point and uses the same path with opposite direction during deploying.

When assembling the four faces of the pyramid for folding, we should make sure that the joints between plates provide free movement for all plates during deploying and folding based on the perpendicular movement of pyramid's top point. Thus, at this stage cutting will be made according to the theory of Miura Maps which follows zigzag line in horizontal cutting and straight lines stretching from the top point in the pyramid down to the base side forming vertical cutting. Lengths at base side are equidistant (Figure 4).

In model A4 (Figure 5), the base side will be cut into eight equal lengths, i.e., the slope angle will not be equal in each cross between vertical and horizontal division. Horizontal fold is made into five levels, while the vertical fold is made into eight main pieces. The height of the pyramid equals one quarter of the square base side. The assessment in this model requires review of each stage of deploying and folding. Therefore, stages should be divided into A, B, and C (see Figure 5). The most significant remarks on this model is that the folding process needs more force at points located in contact areas between the four faces of the pyramid ridges. It is also noticed that in this model the movement is a sequence, i.e., stress is made on the contact point located at the pyramid ridge. Subsequently, the next point starts moving but with less stress until shifting from A to B in a sequence movement. (See Figure 6).

In this trial, there is no device to tackle the problem of "high strain" except by cutting the material at the pyramid ridge before the deploying process. Then, deploying of the model in a sequence is made, starting from the divergent contact point to the next one until reaching the same point on the next face. So, connection starts from here to form the final shape of the pyramid.

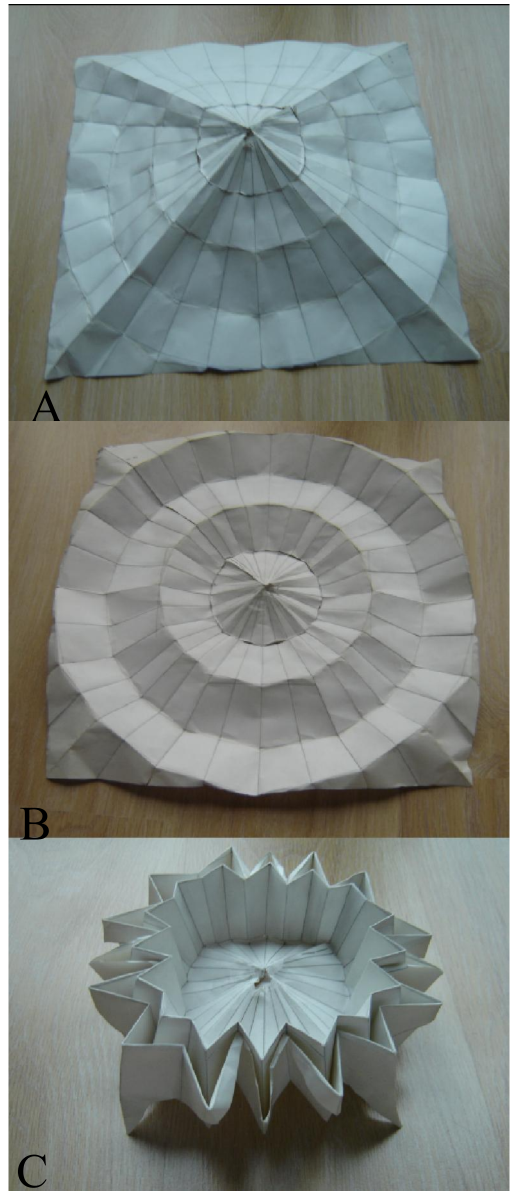

Figure 5. Model A4 (Folding stages)

Although this trial is applied on a relatively low-height pyramid, and needs high strain to fold, the suggested solution for this problem is to detach the main faces of the pyramid in just one corner in order to provide flexible deploying and folding process. The basic problem was in deploying and folding plates on the pyramid ridge. This assumes that the main problem is the right angle situated between the pyramid faces which should be tackled so as to have a perfect shape of foldable pyramid.

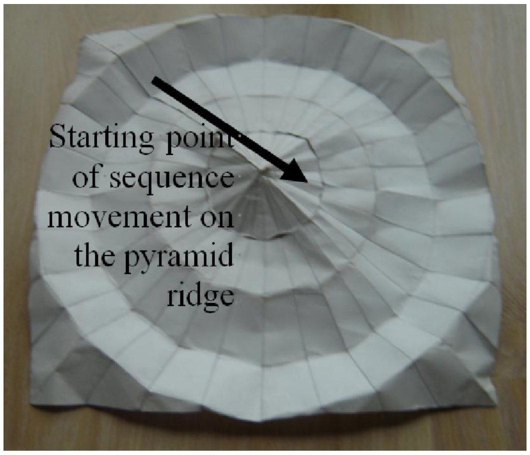

Figure 6. Model A4 (Sequence movement)

From previous discussion, it is required now to change the base shape of the pyramid from square into octagonal shape, in order to change the right angle located in four sides of the pyramid. This model (Figure 7) is extended from the notion of dividing the four faces of the pyramid into small plates. Thus, deployment and folding is made according to consequent steps through changing the square- based pyramid into multiple shapes, in which each shape is formed separately. In this process, there is a replacement of small parts from one plane to another. In other words, changing coordinates of plates on plane X, Y, Z, whereas, the plate located in the enclosed area in each coordinate is to be shifted to another area in different plane. For instance, if difficulty is experienced in deploying one plate or has no place in the plane (X1, Y1, Z1), it can be shifted in later deploying stages to another plane (X2, Y2, Z2) until this step is complete, taking into account, connecting this part with other parts and its motional impact on the linked plates. If this technique is good for deploying, the problem of linking the four faces of the pyramid would disappear at the right angle as well as removing the strain problem of the structure.

In the process of changing the shape of the pyramid from square into octagonal shape, the symmetrical movement of the typical four faces of the pyramid should be taken into account. This means, cutting of faces should be made on the basis of multiples of four. In addition, symmetry of all lengths and angles should be maintained for all pyramid parts. For example, if dividing is based on shifting the base from the square shape into the octagonal shape, the vertical division would be based upon dividing the head angle of the triangle (one face of the pyramid) equally. Horizontal dividing determines the same lengths along the vertical division, i.e., each level in the horizontal division will form an octagonal shape. All these changes will be applied on the model, now referred to as Model A5.

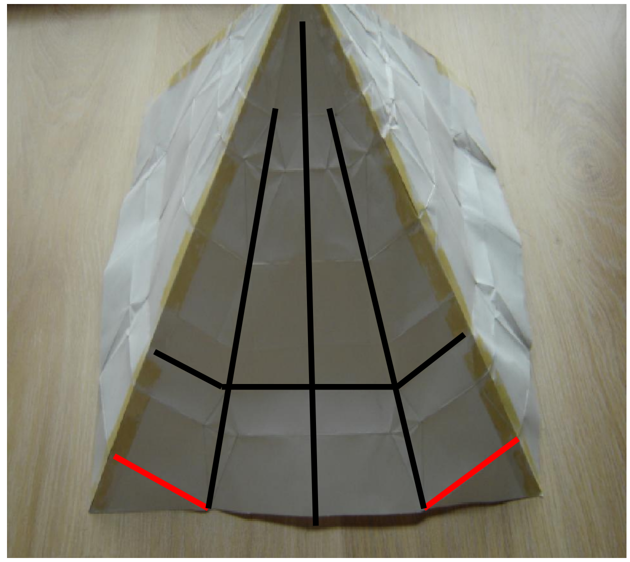
Figure 7. Model A5 showing additional levels on the pyramid ridges

This trial modelling is to be applied on the equilateral triangles. The head of each one will be divided into four equal parts, each one forming 15 degrees, thus maintaining the longitudinal division. The first division will be vertical on the triangle base from the top point. This will divide the triangle into two right angle triangles. The second division will be made on the two triangles that have thus yielded from the first division but with an angle 15 degrees. The horizontal division will be perpendicular to the vertical lines of the height in the four triangles, with 75 degrees angle on the second vertical division. The pyramid ridges would be regarded as highest lines of the original triangle and thus the horizontal division will be perpendicular to it which makes additional level only on the pyramid ridges that resulted from the variant of length between the highest line of the triangle and the triangle side (see Figure 7).

Deploying and folding in this trial will follow subsequent steps until reaching the complete folding. The complete deploying of the pyramid is regarded as stage A, while the next step B would be shifting the pyramid to an octagonal base pyramid. This step is done after changing the four angles located between the original four faces from 90° to 180° and then folding the four extra edges at the end of the pyramid corners to the top to form the octagonal base. (Figure 8) demonstrates.

To minimise the height of the pyramid we need to tuck the horizontal levels into each other. From the top view of the model a number of octagonal intermingled shapes can be seen, whereas the first smaller level will be inserted in the second level and second into the third and so on. The deployment process from B to C of Figure 3.8 requires force on all parts of the model because of the lack of adequate spaces between parts of the first and second level. As a result, there are curves and bending or defects in joints (Figure 9), thus presenting the same high strain problem. However, moving from stage (C) to stage (D), the force will be smaller compared with the fold between stages B and C. At the end of stage D, the model would turn into very small size compared to previous trials. This would support the notion of finding solution out of abandoning one of the conditions mentioned above, or by finding compromising solution without altering the structure or damaging part of the plates and joints.

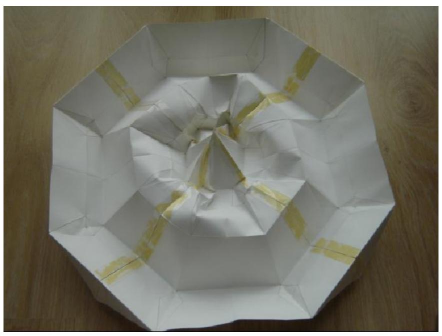

Figure 8. Model A5 – Folding steps.

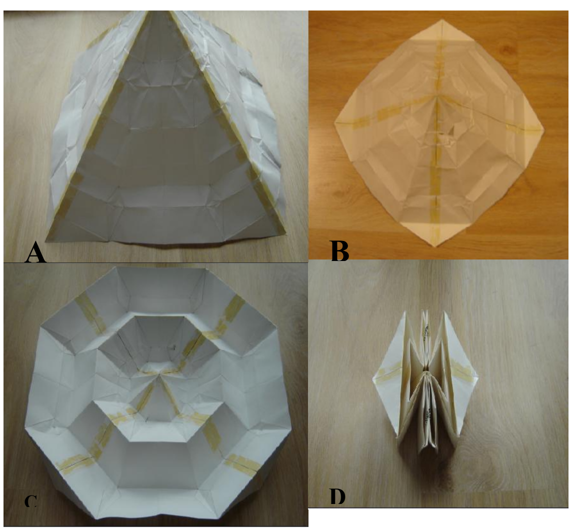

Figure 9. Damage on segments and joints caused by lack of spaces

3 Deploying Mechanism

The recurrence of the high strain problem in the previous trials might lead to a “deadlock point” during the deployment process or alternatively, the idea of providing some sort of more freedom to the main structure. It may be better to provide a freedom in certain deploying stages, for example, between B and C in Model A5, and thus a small size as well as easy control over it can be achieved after full deployment. This might suggest the eventual solution for the pyramid application.

3.1 Deploying Obstacles

In fact, there are two basic obstacles that prevent deployment without increasing strain. The first is illustrated in Figure 10 where the red configuration is the semi-deployed configuration. The structure should move from the red configuration to blue configuration in a strain-free manner to make the final pyramid shape. The problem is that the horizontal distance between *a* and *c* is fixed, because *c* is unable to move to the right or to the left, where *c* is the top point of the pyramid which only moves up and down. To get a pyramid shape *abc* should end up as *ab'c'*. Since *ab* is a fixed length, then a rotation of *ab* to *ab'* cannot be obtained first without some movement of *c* to the right, which is not allowed.

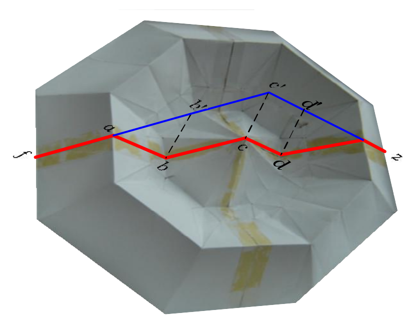

Figure 10. A cross section along a sector (semi-deployed configuration)

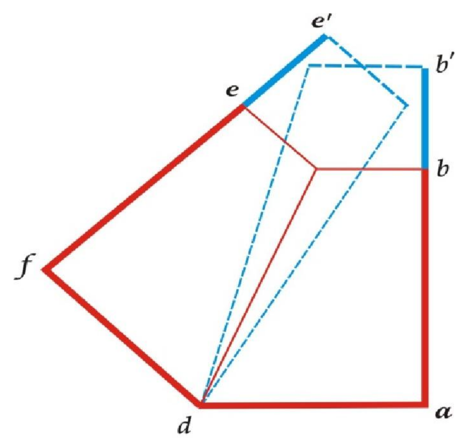

Figure 11. Over-lap of level two segments during fold

In the second obstacle, illustrated in Figure 11, the red shape is the fully deployed position while, the blue shape shows locations associated with high strain. The main obstacle here is the distance *e-b*, which is enough for two segments in that stage, which is required to move to *e'-b'* in the next blue stage. Since, *e'-b'* is shorter than *e-b*, then the corresponding segments are required to over-lap which cannot happen.

The obstacle of strain build-up has resulted from lack of sufficient spaces that permit the stiff plates to move from stage to stage. The stiffness of plates hinders the complete formation during deploying process in the enclosed area. It is thus necessary here to increase the divisions in the plates in the second level because their sizes are bigger than the available spaces. In addition, the plates in the first level can easily be deployed and folded without having any strain build-up, if some of them are not connected to some of the plates of the second level. Thus, the outcome of the greater division forms typical plates similar to those of the first level but in reverse manner, besides other plates that formulate other triangles of bigger sizes but not in contact with the plates of the first level. Although, adding further plate division is essential to solve the second problem, the deployment may still be not entirely strain free.

With reference to Figure 10, C can be moved vertically to C' provided that B can also move firstly to the right and then to the left, corresponding to the upward movement of C, so long as it remains in the space between A and C. The application of this motion on the three dimensional model means folding the plates of the first level is necessary in order to shrink their size while going through point of high strain before deployment continues. Connecting the plates of the first level with their counterpart plates in the second level obstructs the process of folding. Therefore, it is necessary to give the model some sort of freedom at this point, then this freedom should be controlled after full deployment. This technique can be effected in two aspects. The first possibility is the replacement of some of the stiff material of the parts that are in high strain with more flexible material. The second possibility is the removal of the connection between the edges of some plates in the first level and opposite plates in the second level during the folding process.

3.2 Soft Material Hinges Solution

Figure 12 shows the use of flexible materials and their locations as well as full deployment process in this model, referred to as Model A6. Some of the materials of the second level have been replaced by flexible ones. These materials were originally small triangles taken out of all plates of the second level.

With these modifications the model is capable of having a full deployment process without any strain build-up. Although this model is regarded as a step forward in the solution of deploying the pyramid, the connecting points between plates of the first and second levels are only eight contact points and these may be insufficient to endure the weight of the first level. In other words, it is possibly and easily affecting the rigidity of the final shape and its stiffness, whether through natural factors or sudden vibrations or even plates weight. In addition, the flexible material that has been used could easily tear during folding or deploying, compared to the plates made of stiff materials. Thus, this particular model should not be regarded as a full lasting solution for the foldable pyramid.

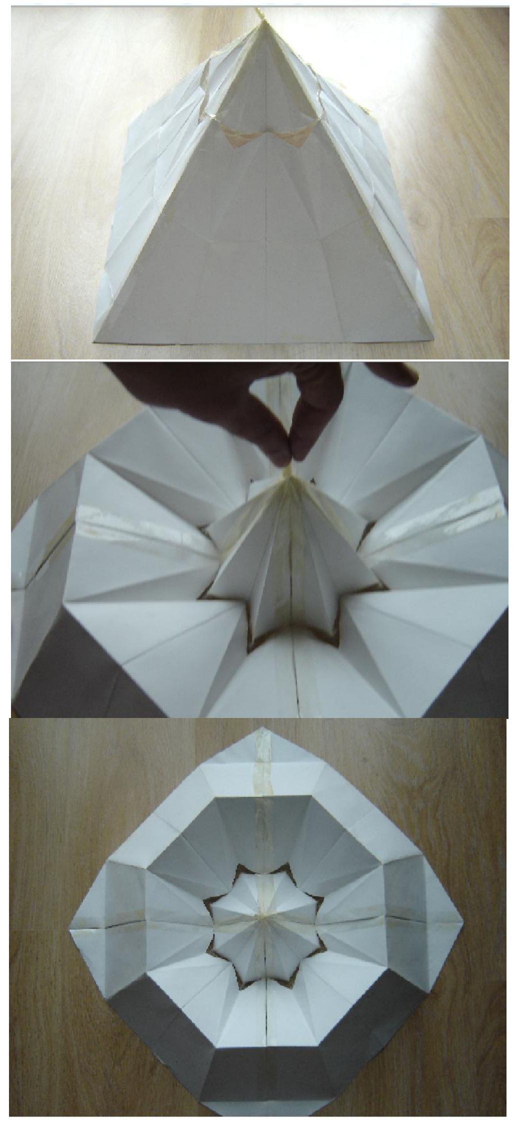

Figure 12. Model A6 with flexible materials

3.3 Free Connection Points Solution

From the discussion in previous sections, it can be seen that it is necessary to deal with the problem of high strain carefully. If the suggested solution of the high strain is to give the structure more degrees of freedom during the deploying process (for example, by inserting cuts in the hinges), then it is essential to control or remove the freedom after full deployment. The other solution is to initially disconnect some of the plates in the first level from the counterparts in the second level until the deployment process has passed the high strain point in Model A7. Then after fully deployment, these plates can then be connected to each other. This might be an acceptable temporary solution for a fully foldable pyramid structure.

The additional divisions of the plates on the second level in this model were made in half of the plates, and not in all the plates. As compared with Model A6, divisions were there made on only two parts on the left side of each face of the pyramid, shown in blue (see Figure 13). The resulting parts shown on Figure 13 in red were connected with the corresponding ones in the same level, which were disconnected with the parts of the first level. The purpose of the disconnection is to provide free movement on the edges. These parts were folded downward; meanwhile, the corresponding parts in the first level were folded upward which yield enough space that permits the structure to pass through a point of high strain without increasing the strain.

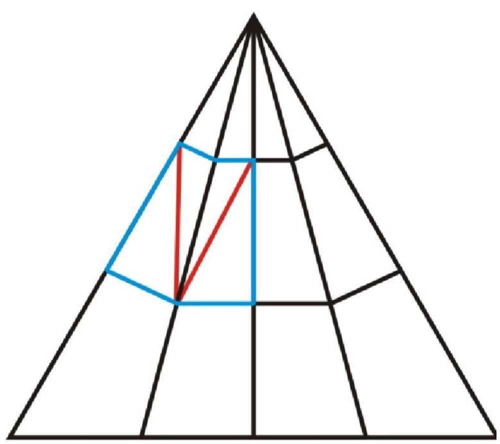

Figure 13. Additional divisions resulting in more parts in level two

Figure 14 illustrates various stages in the deployment process. In Stage A, the edges of all the plates in the first level form the first octagonal shape. Edges of the second level plates form the second octagonal shape. When the top point of the pyramid is pulled downward for folding the plates in the first level and those in the second level which are not connected start folding to form another octagonal pyramid with four protruding fins which is smaller than first one. This smaller octagonal pyramid is formed from only half of the first level plates. Since it is smaller, it provides additional spaces enough for lengths of the second level plates to move downward passing what would have been a point of high strain without actually increasing the high strain (Step C). A full foldable pyramid is in step D.

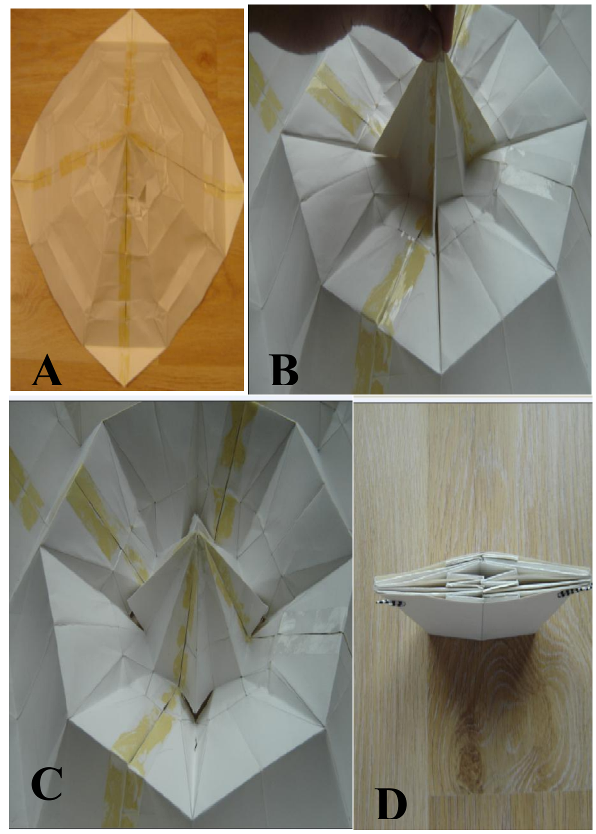

Figure 14. Model A7 – Folding steps

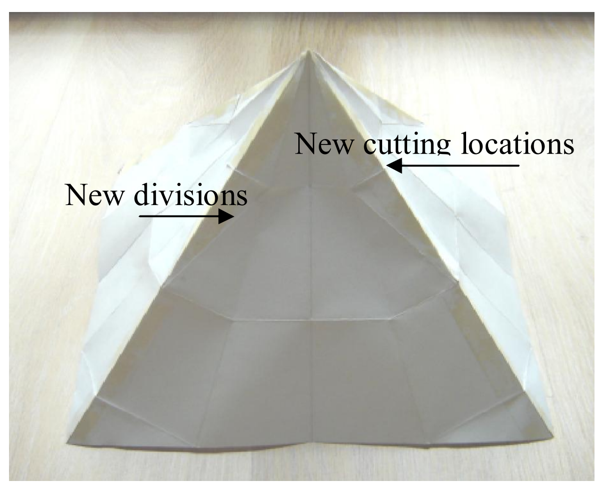

Figure 15. Model A8 –Showing differnet cuttings and divisions

Model A8 with different divisions can give the same outcome. Figure 15 shows different locations for the additional divisions on the model as well as locations of connected points between the first level and second level. The different intermediate shape is shown in Fig 16 when the top point of the pyramid is pulled upwards. In this model, the plates in the first level and those in the second level which are not connected fold to form a small square-based pyramid and the protruding fins are this time fold as four upward fins in the second level (Figure 14-step B).

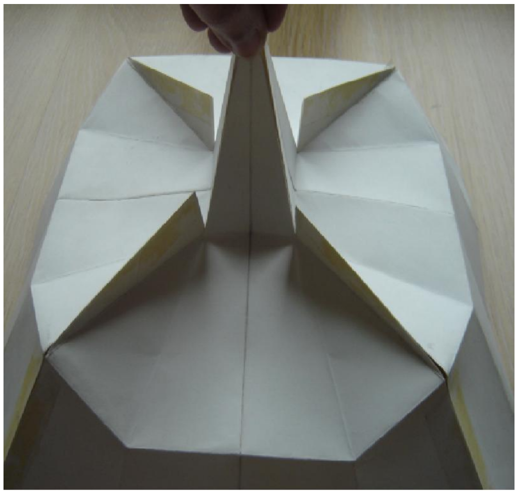

Figure 16. Model A8 –Step C - Showing different segment movement

Both models have provided the required strain-free deployment. However, in Model A8 (Figure 16) additional divisions were on the plates of the main faces or at the pyramid ridges. To obtain a foldable pyramid, these folding divisions should be made inward folds, whereas they are actually outward folds to form the ridges of the final pyramid (Figure 17).

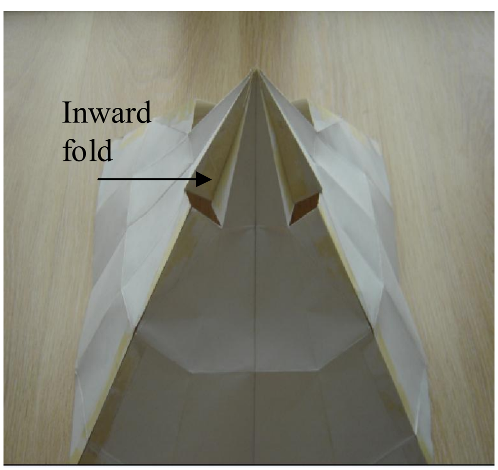

Figure 17. Model A8. Step B – Showing difference in movement

To avoid complication of joints design in Model A8, Model A7 Figure 14 would be used in the following design stages. This model meets most of the necessary conditions, especially the rigidity after deployment, as well as lacking any strain build-up during deploying and folding at all stages. Hence, Model A7 is considered the most appropriate folding solution for the pyramid shape. However, all trials, tests, and studies done so far were only on models that have "no thickness" or very thin material. Clearly, when thickness of the plates is taken into account, there will be some necessary adjustments to the fold lines and folding process. These adjustments are indicated in Figure 18 which shows the final proposed design.

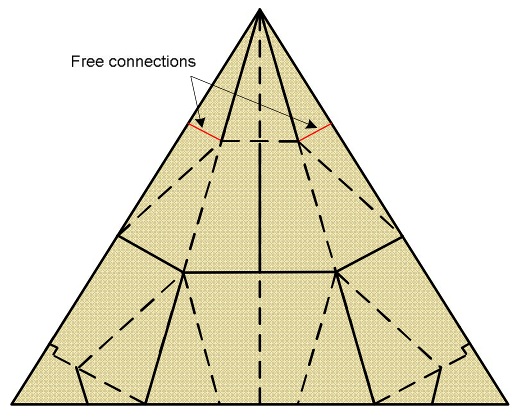

Figure 18. The final proposed design

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