Analysis of bivariate correlated data under the Poisson-gamma model

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Abstract: This article reviews bivariate correlated count data by using mixed Poisson - Gamma. In the model presented in solidarity response variables which are counted as components of the model is unobservable heterogeneity. Hierarchical Bayesian method to estimate the model parameters have been used, Gibbs sampling algorithm for finding prior distribution model parameters is proposed. Finally, given the economic model has been cooperative sector.

[Narges Ramooz, Farzad Eskandari. **Analysis of bivariate correlated data under the Poisson-gamma model.** *Rep Opinion* 2016;8(6):82-91]. ISSN 1553-9873 (print); ISSN 2375-7205 (online). http://www.sciencepub.net/report. 2. doi:10.7537/marsroj08061602.

Keywords: Bivariate correlated count data, Poisson-gamma model, Hierarchical Bayes, Metropolis Hastings algorithm

1. Introduction

There are several techniques are used to analyze continuous data and while the limited methods for data analysis were counted. The model outlined in analyzing these data in recent years has been considered one of the main models analysis is based on Poisson regression model. The models in numerous papers, including Winkelman (2003) and in more detail by Green (2008) have been studied.

In fact Poisson regression model as an example of the generalized linear models that use of its main condition is equivalent to the average variance of the dependent variable. This condition leads to more sprawl is not established in the model generally to overcome this problem using mixed models using hidden variables is suggested. Among these methods using negative binomial (NB) is a standard method of counting used for the analysis (Miaou, 1994) NB model parameter estimation and inference ways about it in the papers Green (2007), Haibe (2007) and Cameron Trivedi (2005) is discussed. Another method used for dispersion control over the use of mixed Poisson-gamma and Poisson distributions is lognormal. It is easily proved that the marginal distribution of the negative binomial distribution is the distribution margin all of these models can be used to overcome the fragmentation of the model. According to information, if we find the relationship between the data count in the model is acceptable using multiple regression models. Several techniques for introducing this correlation exist in multiple Poisson model. In previous studies variable correlation coefficient y_1 and y_2 resulting in a hybrid model is not restricted to positive numbers and also covers negative correlation also because in this article we have used the recent models.

There are several ways to estimate the Poisson regression model parameters, Ho and Singer (2001) and Kocherlakota (2001) by Newton-Raphson method and generalized least squares applied to maximize the likelihood. Karlis and Ntzoufras (2005) EM algorithm to solve the problem of convergence in the method of Newton - Raphson created the work survived. Chip and Winkelman (2001), T-Sinus (2005) and Karlis (2005) proposed Bayesian inference. We and Coleman (2006) in the form of Markov chain Monte Carlo simulation, (MCMC) algorithms Gibbs sampling and Metropolis - Hastings took to work. Eskandari and Kazemnejad (2000) Hierarchical Bayesian model for Poisson regression model mixed - lognormal applied for correlated data. In this paper, the MCMC method for estimating the parameters in the model Poisson - gamma using hierarchical Bayes is used. In general, the structure of this paper is as follows: in section 2 model and some structural characteristics are studied. In section 3, appropriate algorithms for parameter estimation is studied in section 4, the actual data is discussed.

2. Fitting model

2.1 Bivariate correlated Poisson-Gamma model

Suppose that $y_{ij}, i = 1,2,\dots,n_j$ j=1,2, Counts individual values i^th^ is based on different responses. The desirable set of models that are not correlated between individuals, but between responses correlated.

$$cov\,(y_{ij}, y_{kl})\begin{cases} = 0 & for\,i \neq k \\ \neq 0 & for\,i = k \end{cases}$$

It is assumed y_{ij} is Poisson random variable with mean rate parameter λ_{ij}:

$$y_{ij} | x_{ij}, b_{ij} \sim poisson\,(\lambda_{ij})$$

Which λ_{ij} it is defined as follows:

$\lambda_{ij} = \exp(x'_{ij}\beta_j + b_{ij})$

x_{ij} is a K*1 vector of explanatory variables (auxiliary) and β_j is a K*1 considered a vector of parameters, including the amount β_0. Correlation between y_1 and y_2 by correlation between the components of non-observed heterogeneity b_{i1} and b_{i2} defined:

$$b_i = (b_{i1}, b_{i2}) \sim wishart(0, \Sigma)$$

Which Σ variance-covariance matrix the correlation is between b_{ij} covered. λ_{ij} Wishart Bivariate distribution and y_i Poisson-gamma mixture distribution will be two variables.

3. Bayesian analysis bivariate numerical data

According to the model likelihood function will be as follows:

$$f(y_{i1}, y_{i2}|x, b_j, \Sigma| = \int \prod_{j=1}^{2} f(y_{ij}|B_j, b_{ij})\emptyset(\overrightarrow{b_i}|\Sigma)\, d\overrightarrow{b_i}$$

Σ by selecting optional, multiple integrals above have the shape will not be closed. Bayesian MCMC method so analysis can be a way to solve the above problem. If this is the initial parameters of the prior distribution in Bayesian statistics, a statistical distribution, up, then we'll have a hierarchical Bayes model, which has been considered in this study. Press (1982) showed that former Wishart distribution as a conjugate can be used to reverse the covariance matrix. It also showed that mainly Wishart distribution is very reasonable and normal for multivariate analysis and thus on the hierarchical model for the two parameters (Σ and β) independently in the area can be studied:

First level:

$$\beta \sim N_K(\beta_\beta, V'_\beta)$$

$$\Sigma^{-1} \sim wishart(v_\Sigma, V_\Sigma)$$

Second level:

$$\beta_\beta \sim N_k(\beta_0, sig\beta)$$

$$V_\beta^{-1} \sim wishart(v_{0\beta}, R_0)$$

$$V_\Sigma^{-1} \sim wishart(v_{0\Sigma}, \Sigma_0)$$

So that $(\beta_0, sig\beta, v_{0\beta}, R_0, v_{0\Sigma}, \Sigma_0)$ Parameters are known and assume β_0 and $sig\beta$ are independent of each other.

3.1 posterior distributions

Considering the previous model structure as well as the distribution of parameters to calculate the probability distribution is just the product of $\pi(\Theta)$ in $L(\Theta)$ acquired and turned it into a probability distribution. The posterior distribution model assumes bivariate Poisson - Gama will be as follows:

$$\prod(B_j, B_\beta, v_\beta, \overrightarrow{b_i}, \Sigma|y_i, x) \propto L(B_j, \vec{b}_i|y_i, x)P(B_j, B_\beta, v_\beta, \overrightarrow{b_i}, \Sigma, y_i)$$
$$\propto L(B_j, \vec{b}_i|y_i)P(\overrightarrow{b_i}|\Sigma)\, P(\Sigma^{-1}|V_\Sigma, v_\Sigma)$$
$$\propto g(V_\Sigma^{-1})P(B_j|B_\beta, V_\beta)g(B_\beta)g(v_\beta^{1-})$$

Given the likelihood function posterior distribution will be as follows.

$$\prod(B_j, B_\beta, V_\beta, \overrightarrow{b_i}, \Sigma|y_i, x) \propto L(B_j, \vec{b}_i|y_i, x)P(B_j, B_\beta, v_\beta, \overrightarrow{b_i}, \Sigma, y_i)$$
$$\propto L(B_j, \vec{b}_i|y_i)P(\overrightarrow{b_i}|\Sigma)\, P(\Sigma^{-1}|v_\Sigma V_\Sigma)$$
$$* g(V_\Sigma^{-1})P(B_j|B_\beta, V_\beta)g(B_\beta)g(V_\beta^{-1})$$
$$\propto \prod_{t=1}^{n} exp\left(-erp\left(x'_{1j}\beta_j + b_{ij}\right)\right)\left(exp\left(X'_{tj}B_j + b_{ij}\right)\right)^{y_{ij}}$$
$$* \prod_{i=1}^{n} \frac{|\overrightarrow{b_i}|\frac{n-3}{2}}{|\Sigma|\frac{n}{2}} . \exp(-\frac{1}{2}tr(\Sigma^{-1}\overrightarrow{b_i}))$$
$$* \frac{|\Sigma^{-1}|\frac{v_{\Sigma-2}}{2}\ exp(-\frac{1}{2}tr(V_\Sigma^{-1}\Sigma^{-1}))}{|v_\Sigma|\frac{v_\Sigma}{2}}$$

$$* \frac{|V_{\Sigma}^{-1}|^{\frac{v\Sigma-2}{2}} exp(-\frac{1}{2}tr(V_{\Sigma}^{-1}\Sigma^{-1}))}{|\Sigma_0|^{\frac{v_0\Sigma}{2}}}$$

$$* \prod_{i=1}^{2} |V_B|^{-\frac{1}{2}} exp\,(-\frac{1}{2}\Big(B_j - B_B)'\big(V_B^{-1}(B_j - B_B)\big)\Big)$$

$$* exp\,(-\frac{1}{2}(B_B - B_0)'\, sig\, B^{-1}(B_B - B_0))$$

$$* |V_B^{-1}|^{\frac{v_{0\beta}-2}{1}} . \frac{exp\,(-\frac{1}{2}tr\,(R_0^{-1}V_B^{-1}}{|R_0|^{\frac{v_0 B}{1}}}$$

As can be seen posterior distribution obtained form is not closed, as a result Gibbs sampling to calculate an estimate model parameters need to sample from the posterior distribution will be obtained.

3.2 Gibbs sampling on the hierarchical model

It was observed that the posterior distribution obtained in the pre-specified distribution and is not closed. So for Bayesian analysis and to obtain an estimate of the distribution parameter cannot be used in combination uncertain. But using simulation methods, including Mont-Carlo method, Markov chain MCMC sampling to examine this complex posterior distribution was considered. Now we want at this stage and, if necessary, using Gibbs sampling algorithm Metropolis - Hastings sampling to take action. In univariate Gibbs sampling distribution of the sample to do this sampling was important. First, be sure that if any of the parameters provided distribution parameters and other data (posterior distribution for each of the parameters on the condition of other parameters) colloquially it is called the conditional distribution calculated. Then, in iteration of each sample our full conditional distributions. If the distribution is all provided with specific distribution form the sample length is taken directly from the distribution. If the full conditional distribution package is not specific form, the algorithm we use Metropolis- Hastings. And so the sample after n repeat in this way obtain a sample of the posterior distribution will be combined.

3.2.1 b_i evaluation

Late density functions $\overrightarrow{b_i}$ include:

$$\prod(\overrightarrow{b_i}|y_i, B_j, \Sigma) = \prod_{i=1}^{n}\prod(b_i|y_i, b_j, \Sigma)$$

So for sampling from the posterior distribution b_i function of the i^{th} to the posterior distribution density of the sample

Conditional distribution parameter $\overrightarrow{b_i}$:

$$\prod(\overrightarrow{b_i}|y_i, X, B_i) \propto L\,(B_i, \overrightarrow{b_i}|y_i, X).\emptyset_j(\overrightarrow{b_i}|\Sigma)$$

$$\propto \prod_{i=1}^{n}\prod_{j=1}^{2}\frac{\exp(-\lambda_{ij})\lambda_{ij}{}^{y_{ij}}}{\Pi(y_{ij}+1)}.\frac{|\overrightarrow{b_i}|^{\frac{n-3}{2}}}{|\Sigma|^{\frac{n}{2}}}\exp(-\tfrac{1}{2}tr\,(\Sigma^{-1}\overrightarrow{b_i})$$

$$\propto \exp(\sum_{i=1}^{n}\sum_{i=1}^{2}(-\lambda_{ij} + y_{ij}\ln(\lambda_{ij}))\frac{|\overrightarrow{b_i}|^{\frac{n-3}{2}}}{|\Sigma|^{\frac{n}{2}}}\exp(-\tfrac{1}{2}tr\,(\Sigma^{-1}\overrightarrow{b_i}))$$

$$\propto \exp(-\sum_{i=1}^{n}\sum_{j=1}^{2}\exp\big(x'_{ij}\beta_j + b_{ij}\big))$$

$$\propto \exp(\sum_{i=1}^{n}\sum_{i=1}^{2}(y_{ij}\vec{b_\iota})\ \frac{|\vec{b_\iota}|^{\frac{n-3}{2}}}{|\Sigma|^{\frac{n}{2}}}\exp(-\tfrac{1}{2}\,tr\ (\Sigma^{-1}\vec{b_\iota}))$$

$$\propto \exp(\sum_{i=1}^{n}\sum_{i=1}^{2}[y_{ij}\vec{b_\iota} - \exp(x'_{ij}\beta_j + b_{ij})]\frac{|\vec{b_\iota}|^{\frac{n-3}{2}}}{|\Sigma|^{\frac{n}{2}}}\exp(-\tfrac{1}{2}\,tr\ (\Sigma^{-1}\vec{b_\iota}))$$

As can be seen the full Conditional distribution obtained for the parameter $\vec{b_\iota}$ has been closed form. So for the sampling of algorithms Metroplois - Hastings conditional $\vec{b_\iota}$ density can be used. Pockets (1998) bivariate t distribution was used as the proposed distribution. It is assumed that:

$$\vec{b_\iota} = \text{argmar}\ (\ln\Pi\left(\vec{b_\iota}|y_{ij}, x_i, B_j, \Sigma\right))$$
$$V_{\vec{b_\iota}} = (-H_{b_i})^{-1}$$

Which $V_{\vec{b_\iota}}$ in fact, the reverse Hessin $\ln\Pi\left(\vec{b_\iota}|y_{ij}, x_i, B_J, \Sigma\right)$ in $\vec{b_\iota}$ point; Mode $\vec{b_\iota}$ and variance – covariance $V_{\vec{b_\iota}}$ can use the algorithm Newton - Raphson with gradient vector is calculated.

$$g_{\vec{b_i}} = -\Sigma^{-1}\vec{b_\iota} + [y_i - exp(x'_{ij}\beta_j + b_{ij})]$$

And Hessian matrix will be equal to:

$$H_{\vec{b_\iota}} = -\Sigma^{-1} - diag\ (exp(x'_{ij}\beta_j + b_{ij}))$$

Where in:

$$\beta = (\beta_1, \beta_2),\quad X_i = \begin{bmatrix} X^1 i1 & 0 \\ 0 & X^1 i2 \end{bmatrix}$$

Density function $fr\left(\vec{b_\iota}\middle|\vec{b_\iota}, v_{\vec{b_\iota}}, v\right)$ Consider that a bivariate t distribution with degrees of freedom is v which v as a parameter in the algorithm is Metropolis- Hastings. In this section, a suggested value $\overrightarrow{b*_\iota}$ of distribution $fr\left(\vec{b_\iota}\middle|\vec{b_\iota}, V_{\vec{b_\iota}}, v\right)$. Consider the circuits in the state of $\vec{b_\iota}$ the proposed amount $\vec{b_\iota}$ or is likely to move below:

$$\alpha\left(\vec{b_\iota}, \overrightarrow{b*_\iota}\ |y_i, \beta_i, \Sigma\right) = min\ \left\{\frac{\Pi\left(\overrightarrow{b*_\iota}\ |y_i, \beta_j, \Sigma\right) ft\ \left(\vec{b_\iota}|y_i, \beta_j, \Sigma\right)}{\Pi\left(\vec{b_\iota}|y_i, \beta_j, \Sigma\right) ft\ \left(\overrightarrow{b*_\iota}\ |y_i, \beta_j, \Sigma\right)}.1\right\}$$

If value $\alpha\left(\vec{b_\iota}, \vec{b_\iota}|y_i, \beta_j, \Sigma\right)$ is greater than u (u random numbers are generated from a uniform distribution $[0,1]$ is suggested value $\overrightarrow{b*_\iota}$ will be accepted. Otherwise, the next item in the chain as suggested value $\overrightarrow{b*_\iota}$ accepted $\vec{b_\iota}$.

3.2.2 Evaluation $^{1-}\sum$

$^{1-}\sum$ Conditional distribution parameters are as follows:

$$\prod(\Sigma^{-1}|b)\ \propto fw\ (\Sigma^{-1}|v_\Sigma, v_\Sigma).\prod_{i=1}^{n}\emptyset_i\ (b_i|\Sigma)$$

$$\propto \frac{|\Sigma^{-1}|\ \frac{v_\Sigma - 2}{2}\exp\left(-\frac{1}{2}tr(v_\Sigma{}^{-1}\Sigma^{-1})\right)}{|v_\Sigma|\ \frac{v_\Sigma}{2}} \times \frac{|\vec{b_\iota}|^{\frac{n-3}{2}}}{|\Sigma|\ \frac{n}{2}}\exp(-\frac{1}{2}tr(\Sigma^{-1}b_i))$$

$$\propto \frac{|\Sigma^{-1}|\ \frac{v_\Sigma + n - 2}{2}\exp\left(-\frac{1}{2}tr\left(\Sigma^{-1}(V_\Sigma^{-1} + \sum\vec{b_\iota'}\,\vec{b_\iota})\right)\right)}{|V_\Sigma|\ \frac{v_\Sigma}{2}}$$

The full Conditional distribution Σ a distribution and evangelism with degrees of freedom $(V_\Sigma + n)$ and matrix location $|V_\Sigma^{-1} + \sum \overrightarrow{b_\iota'}\,\overrightarrow{b_\iota}|^{-1}$. This distribution, the distribution had been unknown. Sampling can be quantified using Σ at each stage of the distribution and sampling the good news.

3.2.3 β estimation

Conditional distribution parameter β can be represented as follows:

$$\prod(\beta|y,x,\overrightarrow{b_\iota},\Sigma) = \prod_{j=1}^{2} \Pi\left(\beta_j\,|y,x,\overrightarrow{b_\iota},\Sigma\right)$$

The Conditional distribution β parameter can be defined in terms of the individual components of the β_J. So we have:

$$\Pi\left(\beta_j\,|y,x,\overrightarrow{b_\iota},\Sigma\right) \propto \emptyset_j\left(\beta_j\,|\beta_\beta,V_\beta\right).L(\beta_j,\overrightarrow{b_\iota}|\,y,x)$$

$$\propto \prod_{i=1}^{n}\prod_{j=1}^{2}\frac{exp\,(-\lambda_{ij})\lambda_{ij}^{yij}}{T\,(Y_{ij}+1)}\,erp\,(-\frac{1}{2}(\beta_j-\beta_\beta)'V_\beta^{-1}(\beta_j-\beta_\beta)$$

$$\propto erp\,(\sum_{i=1}^{n}\sum_{j=1}^{2}(-\lambda_{ij}+\,y_{ij}ln\lambda_{ij}))$$

$$\propto erp\,\left[\frac{1}{2}\beta_\beta' v_\beta^{-1}\beta_\beta+\frac{1}{2}\beta_\beta'\beta-\frac{1}{2}\beta' v_\beta^{-1}\beta_\beta\right]$$

$$\propto erp\,(\sum_{i=1}^{n}\sum_{j=1}^{2}-erp\,(x_{ij}'\beta_j+b_{ij})+y_{ij}Ln\left(exp\,(x_{ij}'\beta_j+b_{ij})\right)$$

$$\propto exp\,(-\sum_{i=1}^{n}\sum_{j=1}^{2}exp\,(x_{ij}'\beta_j+b_{ij})$$

$$.\,exp(\sum_{i=1}^{n}\sum_{j=1}^{2}y_{ij}x_{IJ}'\beta_j).\,exp\,(-\frac{1}{2}\beta_j'V_\beta^{-1}\beta_j)$$

$$\propto exp\,(-\sum_{i=1}^{n}\sum_{j=1}^{2}exp\,(x_{ij}'\beta_j+b_{ij})$$

$$\propto exp\left[-\frac{1}{2}\beta_j'V_\beta^{-1}\beta_j+(\sum_{i=1}^{n}\sum_{j=1}^{2}y_{ij}x_{ij}'+\beta_\beta'V_\beta^{-1})\beta_j\right]$$

As it can be seen that the function has no defined shape, so had to use the Metropolis-Hastings algorithm; in this case the same as before we used multivariate t distribution. We assume:

$$\widehat{\beta_J} = \text{argmar}\,(\text{Ln}\prod(\beta_j|y_j,X,b_i,\Sigma))$$

Be considered as a distributed fashion and $v_{\beta i} = (-H_{\beta i})^{-1}$. In the other words, $v_{\beta i}$ reverse Hassin $ln\,\prod(\beta_j|y_j,X,b_i,\Sigma)(Hessian)$ at the point in the function β_j considered.

Value β_J and variance-covariance matrix $v_{\beta i}$ Newton-Raphson algorithm can be estimated with Gradient following:

$$g_{\beta j} = -V_{\beta j}^{-1}(\beta_j-\widehat{\beta_J})+\sum_{i=1}^{n}(y_{ij}-exp\,(x_{ij}'\beta_j+b_{ij}))x_{ij}$$

And Hessian matrix will be equal to:

$$H_{\beta j} = -V_{\beta j}^{-1}-\sum_{i=1}^{n}(erp(x_{ij}'\beta_j+b_{ij}))x_{ij}x_{ij}'$$

Therefore, the proposed distribution $fT\left(\beta_j|\widetilde{\beta_j},V_\beta,v_\beta\right)$ is a multivariate t distribution with degrees of freedom v_β. At this stage, similar to a proposed value β_j^* for parameter β_j of distribution $fT\left(\beta_j|\widetilde{\beta_j},V_\beta,v_\beta\right)$ we will consider. With regard to the proposed value chain from β_j the amount proposed β_j^* with probability

$$\alpha\left(\beta_j\beta_j|Y,X,\overrightarrow{b_\iota},\Sigma\right)=\min\left\{\frac{\prod\left(\beta*_j|Y,X,\overrightarrow{b_\iota},\Sigma\right)fT\left(\beta_j|\widetilde{\beta_j},V_\beta,v_\beta\right)}{\prod\left(\beta_j|Y,X,\overrightarrow{b_\iota},\Sigma\right)fT\left(\beta*_j|\widetilde{\beta_j},V_\beta,v_\beta\right)},1\right\}$$

Will move if the value is greater than the probability If this probability is greater than U will move (U uniformly distributed in the interval [$[0,1]$) the proposed amount will be accepted β_j^* Otherwise, the next value for β_j^* considered and the process is done again.

$$\alpha\left(\beta_j\beta_j|Y,X,\overrightarrow{b_\iota},\Sigma\right)=\min\left\{\frac{\prod\left(\beta*_j|Y,X,\overrightarrow{b_\iota},\Sigma\right)fT\left(\beta_j|\widetilde{\beta_j},V_\beta,v_\beta\right)}{\prod\left(\beta_j|Y,X,\overrightarrow{b_\iota},\Sigma\right)fT\left(\beta*_j|\widetilde{\beta_j},V_\beta,v_\beta\right)},1\right\}$$

If this probability is greater than U will move (U uniformly distributed in the interval [$[0,1]$) the proposed amount will be accepted β_j^* Otherwise, the next value for β_j^* considered and the process is done again.

3.2.3 β_β Estimation

$$\prod\left(\beta_\beta|\beta,V_\beta\right)\propto\prod_{j=1}^{2}\emptyset\left(\beta_j|\beta_\beta,V_\beta\right)\emptyset\left(\beta_\beta|\beta_0,sig\beta\right)$$
$$\propto\prod_{j=1}^{2}|V_\beta|^{-\frac{1}{2}}erp\ (\beta_j-\beta_\beta)'V_\beta^{-1}(\beta_j-\beta_\beta)$$
$$\times|sig\beta|^{-\frac{1}{2}}erp\ (-\frac{1}{2}(\beta_\beta-\beta_.)'sig\beta(\beta_\beta-\beta_.))$$
$$\propto|V_\beta|^{-\frac{1}{2}}erp\ (-\frac{1}{2}\sum_{j=1}^{2}(\beta_\beta-\beta_.)'sig\beta^{-1}(\beta_\beta-\beta_.))$$
$$\times|sig\beta|^{-\frac{1}{2}}erp\ (-\frac{1}{2}(\beta_\beta-\beta_.)'sig\beta^{-1}(\beta_\beta-\beta_.))$$
$$\propto\left[\sum_{j=1}^{2}\beta_j'V_\beta^{-1}+\beta_.'sig\beta^{-1}\right]\beta_\beta+\beta_\beta'(-V_\beta^{-1}-\frac{1}{2}sig\beta^{-1})\beta_\beta$$

Here also observed that the distribution obtained form is not known. So, for example, of the need to Metropolis- Hastings algorithms with multivariate t distribution will be proposed. So similar to the previous section we have:

$$\widetilde{\beta_\beta}=argmar\ \left(Ln\prod(\beta_\beta|\beta,v_\beta)\right)$$
$$V_{\beta_\beta}=(-H_{\beta_\beta})^{-1}$$

Where in $\widetilde{\beta_\beta}$ distribution mode $V_{\beta\beta}$ Reverse Hessein $Ln\prod(\beta_\beta|\beta,v_\beta$ in $\widetilde{\beta_\beta}$. Here, too, $\widetilde{\beta_\beta}$ and variance-covariance matrix V_{β_β} the algorithm can be made by Newton - Raphson with Gradient g_{β_β} approximated.

$$g_{\beta_\beta}=\sum_{j=1}^{2}\beta_jV_\beta^{-1}+\beta_.sig\beta^{-1}+\beta_\beta\left(-V_\beta^{-1}-\frac{1}{2}sig\beta^{-1}\right)$$

And Hessian matrix will be equal to:

$$H_{\beta_\beta} = -V_\beta^{-1} - \frac{1}{2} sig\beta^{-1}$$

Therefore, the proposed distribution $fT(\beta_\beta|\overline{\beta_\beta}, V_{\beta\beta}, v_{\beta\beta})$ a multivariate t distribution with degrees of freedom $v_{\beta\beta}$ here too similar to the proposed amount equal to a variable β_β functions as β_j^* distribution function $fT(\beta_j|\widehat{\beta_j}, V_\beta, v_\beta)$ considered.

In this case the value chain β_β following with probability will move to β_j^*.

$$\propto = \min\left\{\frac{\prod(\beta_\beta^*|\beta, v_\beta) fT(\beta_\beta|\hat{\beta}_j, V_\beta, v_\beta)}{\prod(\beta_\beta|\beta, v_\beta) fT(\beta_\beta|\hat{\beta}_\beta, V_{\beta\beta}, v_{\beta-\beta})}, 1\right\}$$

If the value of probability on the basis of the proposed β_β^* is larger than U (U has a uniform distribution on the interval $[0,1]$) the proposed amount is an accepted β_β^*.

3.2.4 V_β^{-1} Estimation

$$\prod(V_\beta^{-1}|\beta) \propto \int \omega\left(V_\beta^{-1}|U *_\beta R_\circ\right) \prod_{j=1}^{2} \emptyset_j\left(\beta_\beta V_\beta^{-1}\right)$$

$$\propto \frac{|V_\beta^{-1}| \frac{V_\circ\beta - 2}{2} erp\left(-\frac{1}{2} tr\left(R_\circ^{-1} V_\beta^{-1}\right)\right)}{|R_\circ|^{\frac{1}{2}}} . |V_\beta^{-1}|$$

$$\propto erp\left(-\frac{1}{2}\sum_{j=1}^{2}\left[(\beta_j - \beta_\beta)' V_\beta^{-1}(\beta_j - \beta_\beta)\right]\right)$$

$$\propto \left[V_\beta^{-1}\right]^{v_{\circ\Sigma}} erp\left(-\frac{1}{2} tr\left(V_\beta^{-1}\left\{R_\circ^{-1} + \sum_{j=1}^{2}[\beta_j - \beta_\beta)'(\beta_j - \beta_\beta)]\right\}\right)\right)$$

Distribution or degrees of freedom are the same kernel distribution and evangelism $(v_{\circ\Sigma} + 2)$ and matrix location

$$\left[R_\circ^{-1} + \sum_{j=1}^{2}\left[(\beta_j - \beta_\beta)'(\beta_j - \beta_\beta)\right]\right]^{-1}$$

In other words, there is:

$$V_\beta^{-1}|\beta \sim wishart\ \left((V_{0\Sigma} + 2), (R_\circ^{-1} + \sum_{j=1}^{2}[(\beta_j - \beta_\beta)'(\beta_j - \beta_\beta)^{-1}]\right)$$

Can be directly using the Gibbs sampler and sample at each stage of the distribution and evangelism V_β^{-1} will be updated

3.2.5 V_Σ^{-1} Estimation

Conditional distribution V_Σ^{-1} is in the following form:

$$\prod(V_\Sigma^{-1}|\Sigma) \propto\ f\omega\ (v_{\circ\Sigma}, \Sigma)\ f\omega(v_\Sigma, V_\Sigma)$$

$$\propto \frac{|\Sigma^{-1}|^{\frac{v\Sigma-2}{2}}\ \ exp\left(-\frac{1}{2} tr\ (\Sigma\Sigma^{-1})\right)}{|V_\Sigma|^{\frac{v_\Sigma}{2}}}$$

$$\times \frac{|V_\Sigma^{-1}|^{\frac{v_{\circ\Sigma}-2}{1}} exp\left(-\frac{1}{2} tr\ (\Sigma_\circ^{-1} V_\Sigma^{-1})\right)}{|\Sigma_\circ|^{\frac{v_{\circ\Sigma}}{2}}}$$

$$\propto exp\left(-\frac{1}{2} tr(V_\Sigma^{-1}(\Sigma^{-1} + \Sigma_\circ^{-1}))\right)|V_\Sigma^{-1}| \frac{v_{\circ\Sigma} + v_\Sigma - 2}{2}$$

As can be seen, distribution and evangelism are the same kernel distribution with degrees of freedom $(v_{\circ\Sigma} + v_{\Sigma})$ and matrix location $(\sigma^{-1} + \Sigma_{\circ}^{-1})^{-1}$ so using Gibbs sampling can be updated to include this parameter.

4. Check the actual data model

Small and medium enterprises in terms of the capability of creating many job opportunities and a considerable impact on the economy, special attention and policy makers have been strongly supportive programs follow them. Technological advances and the advent of globalization, the importance of this group of companies has doubled. Paving the way for the growth of small businesses is the issue seriously by the leading countries in this matter is followed. Gamma Poisson regression model in this field by the use of Bayesian hierarchical method, factors to prioritize cooperatives as small and medium businesses was considered.

4.1 Model Diagnosis

In this section, taking into account the actual data from firms quick to study the suggestions have been discussed. The first response variables used in this model is the frequency of use of a company early returns from loan firms and the response variable number of payments the right to education (4% education) that both variables are numerical. It should be noted that the right to education annual rate of 4% of the total profit is a company that is deposited in educating members of cooperatives to the ministry.

The explanatory variables in this study, based on feasibility studies have been provided by companies such employment rate is desired. Second, third and fourth explanatory variables in the model are taken from the company's capital, profits anticipated from the project and amount requested for the project is completed. As seen in the above-mentioned four variables appropriate benchmark to assess the financial performance of a company, including payment of four per cent, and the training is repeatedly receiving loans.

The overall review data on response variables (Table 1) will be seen that these two variables is acceptable assuming Poisson. On the other hand, according to Table 2, we see that the variance of observations is higher than the average in other words, the model is more scattered. Therefore, they have proposed models that controls the distribution of this over used. On the other hand, by examining the correlation between two variables y_1 and y_2 call concluded, given the correlation between them is confirmed.

Table 1: spss output of the test Kolmogorov – Smirnov

The second response	The first response	
0.288	0.540	Test statistics
1.00	0.930	p-value

Table 2. Descriptive statistics table

Var.	Mean	Max	Min	Variations range	No.	Index
2.182	2	5	1	4	12	Number of for taking loans
4.992	3.92	8	1	7	12	The number of pay four percent
0.629	1.58	3	1	2	12	Type of activity
283.27	31	60	14	46	12	Employment rate
66.62	13.42	30	4	26	12	amount of capital
19.061	8.83	16	4	12	12	rate of profit
13.174	6.08	13	2	11	12	Amount received facilities

The following table also assumes the existence of a significant correlation between two variables call at 0.01 is confirmed.

Table 3: Correlation between two response variations

Second answer variable	First answer variable	canonical name	Variable name
0.903 0.00	1 1	Correlation p-value	The first response
1 1	0.903 0.00	Correlation p-value	The second response

According to the above-knit bivariate Poisson-gamma mixture model for data in this study seems appropriate.

4.2 Data Model Based on Bayesian

Bayesian analysis in recent decades due to lack of some unnecessary assumptions of the classical analysis as well as its efficiency due to low sample size has been Statisticians. In this way, by combining prior information and data obtained from observations can be achieved on an accurate estimation parameters. In this study, taking into account the quadratic loss function, the mean posterior distribution of parameters i.e. a complete sampling of the distribution conditional parameters can be used as estimate parameters.

If the primary parameter is also considered a statistical distribution, then, a Bayesian hierarchical model, as we have noted in the introduction of this model parameter estimation method is used. In this study, using software WinBUGS to estimate the model parameters and to determine the distance estimation is reliable for acceptance or rejection.

Table 4: Statistical summaries of parameters Poisson model - log bivariate normal for first response

High level (97.5% quantiles)	Low level (2.5% quantiles)	S.D.	Mean	Model parameters	Number explanatory variable
0.0325	0.01789	0.01277	0.1044	Employment plan	1
0.07831	0.04783	0.03183	0.01148	company's capital	2
24.71	-5.047	9.243	8.645	company's earnings forecast	3
10.28	-4.723	3.63	3.047	amount of reception facilities	4
0.143	0.0123	0.051	0.0209	Type of activity 1. Industrial 2. Agriculture 3. Services	5

Table 5: Statistical summaries of parameters Poisson model - log bivariate normal response to the second

High level (97.5% quantiles)	Low level (2.5% quantiles)	S.D.	Mean	Model parameters	Number explanatory variable
0.2285	0.01388	0.08704	0.06145	Employment plan	1
0.3461	0.01631	0.1191	0.06536	company's capital	2
24.69	-5.057	9.243	8.652	company's earnings forecast	3
10.29	-4.723	3.623	3.046	amount of reception facilities	4
0.07651	0.0132	0.0511	0.0210	Type of activity 1. Industrial 2. Agriculture 3. Services	5

5. Conclusion

Based on the results obtained in this study and the results listed in Tables estimates obtained (Tables 4 and 5) and some believe they Poisson-gamma model was considered in this study achieved the following results:

1. Feasibility Project employment rate mentioned in cooperative direct impact on the number of companies for taking loans from local businesses to return soon. However, employment in the cooperative sector are important components of this sector and its significant relationship indicates this is important to facilitate distribution between small business and cooperatives in position to return soon, their employment rate is affected. The employment rate paid by the company on the frequency of 4% is effective right to education.

2. Capital of both companies on the response variable in terms of economic impact this seems quite reasonable, because the power company's high capital firm as its financial resources and high financial strength implies a guarantee of payment is reception facilities as well as payments scheduled taxes.

3. Series of cooperative activities (industry, agriculture and services) also call on both variables involved. If it is concluded extracted from feasibility studies, according to the company's activities will be different numbers of facilities and industrial companies by the Ministry of Cooperatives, Labor and Social Welfare have priority over the rest of cooperatives. On the other hand, these types of co mutually to pay 4% more than others have provided the right to education.

4. Based on the findings of the company's anticipated profit in the financial year no significant effect on the numbers of facilities the Ministry of Health in order to achieve one of the goals of economic cooperation is 25% shares and the equitable distribution of wealth among cooperative companies, excluding their annual profits.

5. Grants received by the company as a firm on these two variables have not been effective response, in other words, we cannot say necessarily a company that has used the most of the facilities used to a greater number and vice versa, if you use company facilities and less of a reason for the low number of facilities this suggests there is a significant inconsistency in providing the local firms is quick; the factors affecting these inconsistencies to be a prelude to start new research.

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4/27/2016