Study of oscillatory motion of Oberbeck pendulum and definition of rolling friction coefficient

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Abstract: Annotation" A method of studying oscillatory motion and defining the rolling friction coefficient with the help of Oberbeck pendulum has been presented.

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The Oberbeck pendulum is a universal physical setup that allows studying and testing the general law of dynamics for rotary motion. The didactic possibilities of this pendulum were described in [1]. It was shown that with this pendulum it is possible to test the general law of dynamics for rotary motion, define the moment of inertia and the moment of friction and resistance forces and study the regularities of pendulum oscillations.

Consider how to define the inertia moment of the Oberbeck pendulum using the laws of rotary motion. In this case at one of the pendulum rods we set the weight m_0 at a distance d from the pendulum axis and let him move. The equation of its motion will have the form

$$\left(I_0 + m_0 d^2\right)\ddot{\varphi} = -mgd \sin\varphi .$$

Here $\ddot{\varphi} = \beta$ is the angular acceleration, φ is the angle of deviation from the equilibrium position.

For small angles ($\varphi < 10^0$), $\sin\varphi \approx \varphi$ and the equation will have the form of the homogeneous differential equation of the second order

$$\left(I_0 + m_0 d^2\right)\ddot{\varphi} + mgd\varphi = 0$$

or

$$\ddot{\varphi} + \frac{mgd}{I_0 + m_0 d^2}\varphi = 0 .$$

It follows from that equation that

$$\omega^2 = \frac{4\pi^2}{T^2} = \frac{mgd}{I_0 + m_0 d^2} .$$

Defining T from experiment, we can find I_0

$$I_0 = m_0 d\left(\frac{gT^2}{4\pi^2} - d\right)$$

Calculated in such a way, the values of I_0 will differ from those found with other ways by several percents since the friction force moment M was neglected.

The effect of the friction forces on the Oberbeck pendulum motion can be studied by considering how the pendulum oscillations damp.

At one of the rods we set the weight m_0 at a distance d from the pendulum center. With a goniometric protractor fixed near the pendulum axis we will study the damped oscillations of pendulum. The initial angle of deviation is usually $\varphi_0 = 10^0 - 12^0$, the number of oscillations is designated as N. From the equation of damped oscillations we define the damping coefficient δ.

The damping factor is defined by the following expression

$$\theta = \delta T = \frac{1}{N}\ln\frac{\varphi_0}{\varphi_N} ,$$

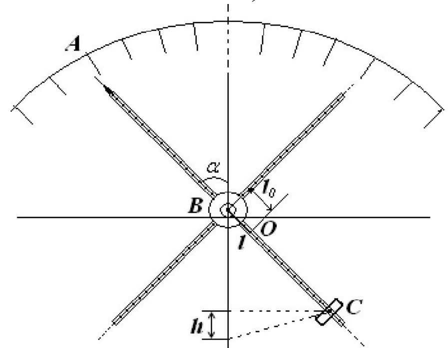

Fig. 1

It should be noted that, knowing T, we can find the inertia moment of the Oberbeck pendulum.

Defined with the help of the above-stated method, the values of the inertia moment of the Oberbeck pendulum have the equal order. The values found by the methods where the friction forces are neglected and those where the friction forces are taken into account differ from each other by 10-12%.

A method of defining the rolling friction coefficient from damped rotary motion of the Oberbeck pendulum is also stated in this work. The main point of the theory and methods of defining the rolling friction coefficient is as follows. While rotation the pendulum does the work against the rolling friction forces therefore the amplitude value of the angle of deviation from the equilibrium position decreases. The friction mainly takes place between the body – spindle and rotation axis. This friction is the rolling one. In this case, the motion equation is complicated; therefore, it is convenient to use the equation of energy conservation. We assume that the coefficient of rolling friction is independent of the velocity of pendulum motion.

The initial potential energy of the pendulum can be as follows

$$W_0 = Mgh = Mgl_0\left(1-\cos\alpha_0\right)$$

Here α_0 is the initial angle of pendulum deviation, h is the displacement of the gravity centre of pendulum, M is the pendulum mass, l_0 is the distance between the gravity centre of pendulum and its axis (see Fig. 1).

After one period of pendulum oscillations the potential energy has the form

$$W_1 = Mgl_0\left(1-\cos\alpha_1\right),$$

where α_1 is the angle of pendulum deviation after one full period of oscillations.

A decrease in the potential energy of pendulum for one full period is as follows

$$\Delta W = Mgl_0\left(\cos\alpha_0 - \cos\alpha_1\right) = 2Mgl_0\left(\sin\frac{\alpha_0^2}{2} - \sin\frac{\alpha_1^2}{2}\right)$$

If we take into account that the angle α is small ($\alpha \leq 0.1$ radn) then the decrease in the potential energy of pendulum can be written as follows

$$\Delta W = 0{,}5Mgl_0\left(\alpha_0^2 - \alpha_1^2\right).$$

If the force of air resistance is not taken into account then the decrease in the potential energy is equal to the work against the rolling friction forces. For one full period of pendulum oscillations the work is equal to

$$A = \mu P\alpha = \mu Mg\left(\alpha_0 + \alpha_{\frac{1}{2}} + \alpha_{\frac{1}{2}} + \alpha_1\right)$$

Here μ is the rolling friction coefficient, $\alpha_{\frac{1}{2}}$ is the angle of pendulum deviation after one semi-period, $\mu P = \mu Mg$ is the moment of the rolling friction force.

If the decrease in the deviation angle for one semi-period is $\Delta\alpha$ then $\alpha_{\frac{1}{2}} = \alpha_0 - \Delta\alpha$, $\alpha_1 = \alpha_{\frac{1}{2}} - \Delta\alpha$. In this case, the work expression has the form $A = 2\mu Mg\left(\alpha_0 + \alpha_1\right)$.

Equating this formula with that for potential energy decrease, we obtain

$$\mu = \frac{1}{4} l_0\left(\alpha_0 - \alpha_1\right)$$

If the pendulum makes N oscillations then that expression has the form

$$\mu = \frac{1}{4}\frac{l_0}{N}\left(\alpha_0 - \alpha_N\right)$$

where α_N is the deviation angle after N oscillations. Finding the distance between the gravity center and the rotation axis l_0, the initial and final angles of deviation for damped oscillations from experiments, we can obtain the rolling friction coefficient μ.

Thus, with the above-stated methods used for the Oberbeck pendulum it is possible to test the main law of dynamics for rotary motion and the Steiner theorem, define the inertia moment of the Oberbeck pendulum, study the regularities of rotary motion and damped rotary one and define the rolling friction coefficient.

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