

Jiang true singular series and Hardy-Littlewood wrong singula series

Jiang, Chun-Xuan

Institute for Basic Research, Palm Harbor, FL34682-1577, USA

And: P. O. Box 3924, Beijing 100854, China

jiangchunxuan@sohu.com, cxjiang@mail.bcf.net.cn, jcxuan@sina.com, Jiangchunxuan@vip.sohu.com,
jcxxxx@163.com

Abstract: Using Jiang function we prove Jiang prime k -tuple theorem with true singular series. Hardy and Littlewood conjectured Hardy-Littlewood prime k -tuple conjecture with wrong singular series.. Jiang prime k -tuple theorem will replace the Hardy-Littlewood prime k -tuple conjecture.

[Jiang, Chun-Xuan. **Jiang true singular series and Hardy-Littlewood wrong singula series**. *Rep Opinion* 2016;8(7):72-76]. ISSN 1553-9873 (print); ISSN 2375-7205 (online). <http://www.sciencepub.net/report>. 12. doi:[10.7537/marsroj080716.12](https://doi.org/10.7537/marsroj080716.12).

Keywords: Jiang function; prime; k -tuple; theorem; singular series; conjecture.

(A) Jiang prime k -tuple theorem with true singular series[1, 2].

We define the prime k -tuple equation

$$p, p + n_i, \quad (1)$$

where $2 \mid n_i, i = 1, \dots, k-1$.

we have Jiang function [1, 2]

$$J_2(\omega) = \prod_P (P-1 - \chi(P)) \quad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{i=1}^{k-1} (q + n_i) \equiv 0 \pmod{P}, \quad q = 1, \dots, P-1 \quad (3)$$

which is true.

If $\chi(P) < P-1$ then $J_2(\omega) \neq 0$. There exist infinitely many primes P such that each of $P + n_i$ is prime. If $\chi(P) = P-1$ then $J_2(\omega) = 0$. There exist finitely many primes P such that each of $P + n_i$ is prime. $J_2(\omega)$ is a subset of Euler function $\phi(\omega)$ [2].

If $J_2(\omega) \neq 0$, then we have the best asymptotic formula of the number of prime P [1, 2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : P + n_i = \text{prime} \right\} \right| \sim \frac{J_2(\omega) \omega^{k-1}}{\phi^k(\omega)} \frac{N}{\log^k N} = C(k) \frac{N}{\log^k N} \quad (4)$$

$$\phi(\omega) = \prod_P (P-1)$$

$$C(k) = \prod_P \left(1 - \frac{1 + \chi(P)}{P} \right) \left(1 - \frac{1}{P} \right)^{-k} \quad (5)$$

is Jiang true singular series.

Example 1. Let $k = 2, P, P+2$, twin primes theorem.

From (3) we have

$$\chi(2) = 0, \quad \chi(P) = 1 \quad \text{if } P > 2, \quad (6)$$

Substituting (6) into (2) we have

$$J_2(\omega) = \prod_{P \geq 3} (P-2) \neq 0 \quad (7)$$

There exist infinitely many primes P such that $P+2$ is prime. Substituting (7) into (4) we have the best asymptotic formula

$$\pi_k(N, 2) = \left| \left\{ P \leq N : P+2 = \text{prime} \right\} \right| \sim 2 \prod_{P \geq 3} \left(1 - \frac{1}{(P-1)^2} \right) \frac{N}{\log^2 N}. \quad (8)$$

Example 2. Let $k = 3, P, P+2, P+4$.

From (3) we have

$$\chi(2) = 0, \quad \chi(3) = 2 \quad (9)$$

From (2) we have

$$J_2(\omega) = 0 \quad (10)$$

It has only a solution $P = 3, P+2 = 5, P+4 = 7$. One of $P, P+2, P+4$ is always divisible by 3.

Example 3. Let $k = 4, P, P+n$, where $n = 2, 6, 8$.

From (3) we have

$$\chi(2) = 0, \chi(3) = 1, \chi(P) = 3 \quad \text{if } P > 3. \quad (11)$$

Substituting (11) into (2) we have

$$J_2(\omega) = \prod_{P \geq 5} (P-4) \neq 0 \quad (12)$$

There exist infinitely many primes P such that each of $P+n$ is prime.

Substituting (12) into (4) we have the best asymptotic formula

$$\pi_4(N, 2) = \left| \left\{ P \leq N : P+n = \text{prime} \right\} \right| \sim \frac{27}{3} \prod_{P \geq 5} \frac{P^3(P-4)}{(P-1)^4} \frac{N}{\log^4 N} \quad (13)$$

Example 4. Let $k = 5, P, P+n$, where $n = 2, 6, 8, 12$.

From (3) we have

$$\chi(2) = 0, \chi(3) = 1, \chi(5) = 3, \chi(P) = 4 \quad \text{if } P > 5 \quad (14)$$

Substituting (14) into (2) we have

$$J_2(\omega) = \prod_{P \geq 7} (P-5) \neq 0 \quad (15)$$

There exist infinitely many primes P such that each of $P+n$ is prime. Substituting (15) into (4) we have the best asymptotic formula

$$\pi_5(N, 2) = \left| \left\{ P \leq N : P+n = \text{prime} \right\} \right| \sim \frac{15^4}{2^{11}} \prod_{P \geq 7} \frac{(P-5)P^4}{(P-1)^5} \frac{N}{\log^5 N} \quad (16)$$

Example 5. Let $k = 6, P, P+n$, where $n = 2, 6, 8, 12, 14$.

From (3) and (2) we have

$$\chi(2) = 0, \chi(3) = 1, \chi(5) = 4, \quad J_2(5) = 0 \quad (17)$$

It has only a solution $P = 5, P+2 = 7, P+6 = 11, P+8 = 13, P+12 = 17, P+14 = 19$. One of $P+n$ is always divisible by 5.

(B) The Hardy-Littlewood prime k -tuple conjecture with wrong singular series[3-14].

We define the prime k -tuple equation

$$P, P+n_i \quad (18)$$

where $2|n_i, i = 1, \dots, k-1$.

In 1923 Hardy and Littlewood conjectured the asymptotic formula

$$\pi_k(N, 2) = \left| \{P \leq N : P + n_i = \text{prime}\} \right| \sim H(k) \frac{N}{\log^k N}, \tag{19}$$

where

$$H(k) = \prod_P \left(1 - \frac{\nu(P)}{P} \right) \left(1 - \frac{1}{P} \right)^{-k} \tag{20}$$

Is Hardy-Littlewood wrong singula series,

$\nu(P)$ is the number of solutions of congruence

$$\prod_{i=1}^{k-1} (q + n_i) \equiv 0 \pmod{P}, \quad q = 1, \dots, P. \tag{21}$$

which is wrong.

From (21) we have $\nu(P) < P$ and $H(k) \neq 0$. For any prime k -tuple equation there exist infinitely many primes P such that each of $P + n_i$ is prime, which is false.

Conjecture 1. Let $k = 2, P, P + 2$, twin primes theorem

From (21) we have

$$\nu(P) = 1 \tag{22}$$

Substituting (22) into (20) we have

$$H(2) = \prod_P \frac{P}{P-1} \tag{23}$$

Substituting (23) into (19) we have the asymptotic formula

$$\pi_2(N, 2) = \left| \{P \leq N : P + 2 = \text{prime}\} \right| \sim \prod_P \frac{P}{P-1} \frac{N}{\log^2 N} \tag{24}$$

which is false see example 1.

Conjecture 2. Let $k = 3, P, P + 2, P + 4$.

From (21) we have

$$\nu(2) = 1, \nu(P) = 2 \text{ if } P > 2 \tag{25}$$

Substituting (25) into (20) we have

$$H(3) = 4 \prod_{P \geq 3} \frac{P^2(P-2)}{(P-1)^3} \tag{26}$$

Substituting (26) into (19) we have asymptotic formula

$$\pi_3(N, 2) = \left| \{P \leq N : P + 2 = \text{prime}, P + 4 = \text{prim}\} \right| \sim 4 \prod_{P \geq 3} \frac{P^2(P-2)}{(P-1)^3} \frac{N}{\log^3 N} \tag{27}$$

which is false see example 2.

Conjecture 3. Let $k = 4, P, P + n$, where $n = 2, 6, 8$.

From (21) we have

$$\nu(2) = 1, \nu(3) = 2, \nu(P) = 3 \text{ if } P > 3 \tag{28}$$

Substituting (28) into (20) we have

$$H(4) = \frac{27}{2} \prod_{P > 3} \frac{P^3(P-3)}{(P-1)^4} \tag{29}$$

Substituting (29) into (19) we have asymptotic formula

$$\pi_4(N, 2) = \left| \{P \leq N : P + n = \text{prime}\} \right| \sim \frac{27}{2} \prod_{P>3} \frac{P^3(P-3)}{(P-1)^4} \frac{N}{\log^4 N} \quad (30)$$

Which is false see example 3.

Conjecture 4. Let $k = 5$, $P, P + n$, where $n = 2, 6, 8, 12$

From (21) we have

$$\nu(2) = 1, \nu(3) = 2, \nu(5) = 3, \nu(P) = 4 \quad \text{if } P > 5 \quad (31)$$

Substituting (31) into (20) we have

$$H(5) = \frac{15^4}{4^5} \prod_{P>5} \frac{P^4(P-4)}{(P-1)^5} \quad (32)$$

Substituting (32) into (19) we have asymptotic formula

$$\pi_5(N, 2) = \left| \{P \leq N : P + n = \text{prime}\} \right| \sim \frac{15^4}{4^5} \prod_{P>5} \frac{P^4(P-4)}{(P-1)^5} \frac{N}{\log^5 N} \quad (33)$$

Which is false see example 4.

Conjecture 5. Let $k = 6$, $P, P + n$, where $n = 2, 6, 8, 12, 14$

From (21) we have

$$\nu(2) = 1, \nu(3) = 2, \nu(5) = 4, \nu(P) = 5 \quad \text{if } P > 5 \quad (34)$$

Substituting (34) into (20) we have

$$H(6) = \frac{15^5}{2^{13}} \prod_{P>5} \frac{(P-5)P^5}{(P-1)^6} \quad (35)$$

Substituting (35) into (19) we have asymptotic formula

$$\pi_6(N, 2) = \left| \{P \leq N : P + n = \text{prime}\} \right| \sim \frac{15^5}{2^{13}} \prod_{P>5} \frac{(P-5)P^5}{(P-1)^6} \frac{N}{\log^6 N} \quad (36)$$

which is false see example 5.

Conclusion

The Jiang prime k -tuple theorem has true singular series. The Hardy-Littlewood prime k -tuple conjecture has wrong singular series. The tool of additive prime number theory is basically the Hardy-Littlewood prime wrong k -tuple conjecture which are wrong[3-14]. Using Jiang true singular series we prove almost all prime theorems. Jiang prime k -tuple theorem will replace Hardy-Littlewood prime k -tuple Conjecture. There cannot be really modern prime theory without Jiang function.

References

1. Chun-Xuan Jiang, Foundations of Santilli's isonumber theory with applications to new cryptograms, Fermat's theorem and Goldbach's conjecture. Inter. Acad. Press, 2002, MR2004c:11001, (<http://www.i-b-r.org/docs/jiang.pdf>)

(<http://www.wbabin.net/math/xuan13.pdf>).

2. Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution. (<http://www.wbabin.net/math/xuan2.pdf>) (<http://vixra.org/pdf/0812.0004v2.pdf>).
3. G. H. Hardy and J. E. Littlewood, Some problems of 'Partition Numerorum', III: On the expression of a number as a sum of primes, Acta Math, 44(1923), 1-70.
4. B. Green and T. Tao, The primes contain arbitrarily long arithmetic progressions, Ann. Math., 167(2008), 481-547.
5. D. A. Goldston, S. W. Graham, J. Pintz and C. Y. Yildirim, Small gaps between products of two primes, Proc. London Math. Soc., (3) 98 (2009) 741-774.
6. D. A. Goldston, S. W. Graham, J. Pintz and C. Y. Yildirim, Small gaps between primes or almost primes, Trans. Amer. Math. Soc., 361(2009)

- 5285-5330.
7. D. A. Goldston, J. Pintz and C. Y. Yildirim, Primes in tulpes I, *Ann. Math.*, 170(2009) 819-862.
 8. P. Ribenboim, *The new book of prime number records*, 3rd edition, Springer-Verlag, New York, NY, 1995. PP409-411.
 9. H. Halberstam and H.-E. Richert, *Sieve methods*, Academic Press, 1974.
 10. A. Schinzel and W. Sierpinski, Sur certaines hypotheses concernant les nombres premiers, *Acta Arith.*, 4(1958)185-208.
 11. P.T. Bateman and R.A. Horn, A heuristic asymptotic formula concerning the distribution of prime numbers, *Math. Comp.*, 16(1962)363-367.
 12. W. Narkiewicz, *The development of prime number theory*, From Euclid to Hardy and Littlewood, Springer-Verlag, New York, NY, 2000, 333-53.
 13. B. Green and T. Tao, Linear equations in primes, To appear, *Ann. Math.*
 14. T. Tao, Recent progress in additive prime number theory, <http://terrytao.files.wordpress.com/2009/08/prime-number-theory1.pdf>.

7/25/2016