

## Chaotic behavior in Flexible Assembly Line of Manufacturing System

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**Abstract:** Chaos theory has been applied in manufacturing systems in the recent years. An investigation has been done on chaotic behavior of assembling system. The assembly line is flexible which can accommodate variety of product types. An algorithm is proposed and a model developed is implemented on real life data in which analysis is performed to obtain time persistent data. The behavior of the system is observed for work in process as this is sensitive in the process. It has been found that the flexible assembly line exhibits chaotic behavior by showing that the computed average Lyapunov exponent is positive.

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**Keywords:** Flexible assembly line; Chaos; Lyapunov exponent

### 1. Introduction:

Due to rapid development in assembling technologies and advancement in methods, there is a need to understand an assembly system's environment. There has been an interest to describe the system's dynamic behavior. The identification and prediction of a system is also a fundamental task, not limited to the field of engineering but in all kinds of other tasks as well. During the last few decades, researchers have been carried out to analyze chaotic behavior in manufacturing. The significant research work in this direction has been done by [1-6]. In the work of Alfaro and Sepulveda[6], the dynamic behavior of a reactive system has been studied; a system where there is no determined schedule and the tasks of operations to machines are assigned according to the state of the system. This research work was performed using discrete event simulation to represent the system and its analysis was done by using nonlinear dynamic systems theory.

Nonlinearity of a system is a necessary condition for chaos. Chaos is the phenomena of occurrence of bounded non-periodic evolution in completely deterministic nonlinear dynamical systems with high sensitive dependence on initial conditions. Sometimes, chaos is called deterministic randomness and is associated with the impediments of forecasting the future. The chaotic behavior generates a kind of randomness and a loss of information about initial conditions, which might explain somewhat complex behavior in real systems. Moreover, a system that is chaotic has long-term behavior that can be hard to describe, hard to predict, and hard even to simulate. The theoretical tool used for quantifying chaotic behavior is the notion of a time-series of data for the real system. The Lyapunov exponent is one of very

important tool to measure chaos. It is known [7-9] that a system's behavior is chaotic if average Lyapunov exponent is a positive number. The Lyapunov exponent, for a one-dimensional time-series data, is computed by:

$$\lambda = \frac{1}{n} \ln \frac{d_n}{d_0}$$

Where  $d_n = |x_{j+n} - x_{i+n}|$  and  $x_i, x_j$  are two close values on the two different time series. In order to detect chaotic behavior in flexible assembly system, there is a need to describe the assembling process. In this research work, Work In Process (WIP) of assembling system has been chosen to describe system behavior, as it imposes dynamism if not properly controlled [10]. A real flexible assembly line (with various performance variables) has been analyzed by means of process modeling software and the Lyapunov exponent (also called Lyapunov's fractal dimension) is computed.

### 2. Research Methodology:

As mentioned in the previous section, several researchers have done research on chaos in manufacturing, but the fundamental work is credited to Taken [11] who proposed a theorem. The methodology adopted in our case is analyzing of system, obtaining a time series of assembly lines, and computing chaotic value. The proposed algorithm is presented after reviewing the selection of the tool/technique for analysis.

In flexible assembling systems, parts are assembled in a respective station and there exists an interaction of parts with the machines. Apart from interaction, dominating relationship also exists in

which change in any parameter in the respective station affects the work in process in the subsequent stations. The system chosen for analysis consists of stations where different parts arrive and are assembled in a series of steps.

Many approaches can be applied, e. g., analytic modeling, queuing theory and simulation. The analytical models demand too many assumptions while in queuing theory the estimates of expected value of the inter-arrival time distribution  $\mu_A$  and service-time distribution  $\mu_S$  (average waiting time in queue is  $\mu_S^2 / \mu_A - \mu_S$ ) are not exact. The queuing theory formula does not provide any information on the natural variability (dynamism) in the system [12]. On the other hand, simulation involves modeling of a process or system in such a way that the model mimics the response of the actual system to events that take place over time[13]. In this way, simulation provides a concrete way of directly dealing with the model and it is the most suitable methodology to model system characteristics. A simulation run gives a number of variables in the output, including total production, average waiting time in queue, maximum waiting time in the queue, flow times, work in process, etc. Time persistent data of work in process is selected for the system. The mathematical form of work in process is given by [10] and calculated as:

$$WIP = \frac{\left( \frac{MTBF - MTTR}{MTBF} \right) (Q) [n_o (T_{su} + Q_b T_c + T_{no})]}{\left( \frac{n S_w H_s R_p}{n_o} \right)} \tag{1}$$

Where MTBF = Mean Time Between Failure; MTTR = Mean Time To Repair; Q = Quantity actually produced;  $n_o$  = Number of operations in the routing;  $T_{su}$ = Setup time;  $T_c$ = Cycle time per part;  $T_{no}$  = Non Operation time;  $Q_b$ = Batch Quantity;  $n$ = number of machines;  $S_w$  = shifts per week;  $H_s$ = Hours in a shift;  $R_p$  = production rate.

The following algorithm is proposed for identification whether system is chaotic or not:

**Algorithm:**

(i) Finding a time series of the variable to be analyzed and the response obtained from the simulation model.

(ii) Taking the absolute difference of the series in question and obtain logarithm of the difference values.

(iii) Reconstructing the variables for describing behavior using separation distance plots and measuring the difference for each data values.

(iv) Estimating the sensitivity by the Lyapunov exponent; if it is greater than zero then the system has a chaotic behavior.

The algorithm is implemented on an assembly system that consists of a series of steps where parts are passed through decision process as described in subsequent section.

**3. Case of Flexible Line (Process description):**

The arriving parts are cast metal bodies that have already been machined to accept electronic parts. There are two parts; A and B which are produced in adjacent departments (out of bound for this model). Part A arrives in the system with mean time of 5 minutes. Part A is prepared where mating faces of the cases are machined, deburred and cleaned with combined process time of 6.5 minutes. After preparation, work part is transferred to the sealer. The work part B arrives in batches of four units and time between arrivals of successive batches with a mean of 30 minutes. The batch is separated into four individual units (processed individually) arrived at work part B preparation area. The combined processing time for work unit B preparation is 8.5 minutes after which it is sent to the sealer. At sealer (decision) electronic components are inserted, case is assembled, sealed and then tested. The process time at sealer depends on part type i.e. 3.5 minutes for part A and 5.3 minutes for part B respectively. Approximately 90% parts passed the quality test and leave system as ‘Yield Good’, while 10% parts are sent for further reworking station. At the rework stage 80% parts passed the quality (decision) and leave system as ‘Yield after reworking’ while remaining parts are ‘scrapped’. This is shown in Figure 1.

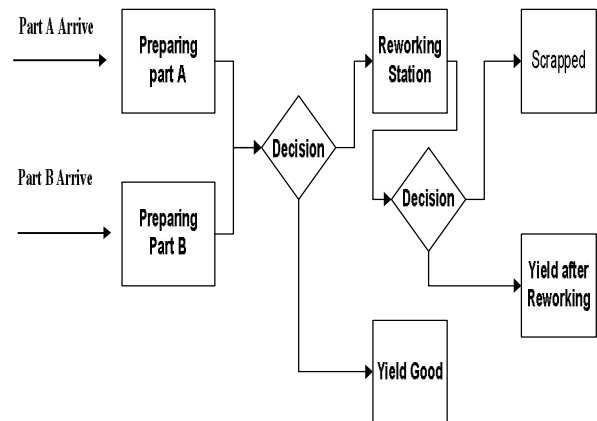


Figure 1: The Process flow of assembly line

**Analysis and Graphical Display:**

The averaged value data has been used for the analysis. The simulation is run for 480 hours (for warm up) and replicated with different scenarios for ensuring the adequacy of the data. The experimental data is validated and verified and production runs(in simulation) are set for 12000 hours with replications. It can be argued that with finite amounts of data, it is difficult to find specified line segment in the reconstructed phase space, therefore long range data for the research problem is selected. The parameters of interest which are affecting performance of assembly system are Work In Process (WIP), Number in Queue (NQ), Average Time in Queue (ATQ), and Utilization (U). It is pertinent to note that WIP is the most sensitive in assembling process as it directly affects the system and is calculated using Equation (1). The time persistent plots for the parts A are shown in Figure 2. The most prominent characteristic of chaos is the unpredictability of the future regardless of deterministic time evolution. There may be an average error when forecasting the response of a future measurement which increases very rapidly with time.

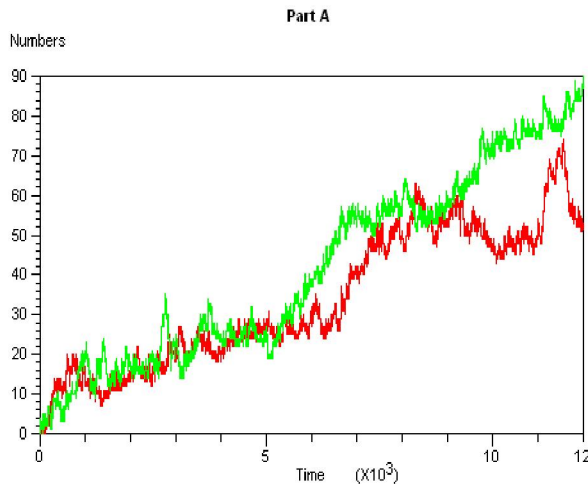


Figure 2: Work in Process for Part A

This unpredictability is an outcome of the inbuilt instability of the solutions, reflected by what is called sensitive dependence on initial conditions.

The time series plot for the situation as given in Figure 2 is redrawn from 250 to 350 for part A and shown in Figure 3. It is evident that two trajectories are separated which shows the divergence of nearby trajectories and it is apparent that Lyapunov exponents measure the rate of divergence of initially close trajectories.

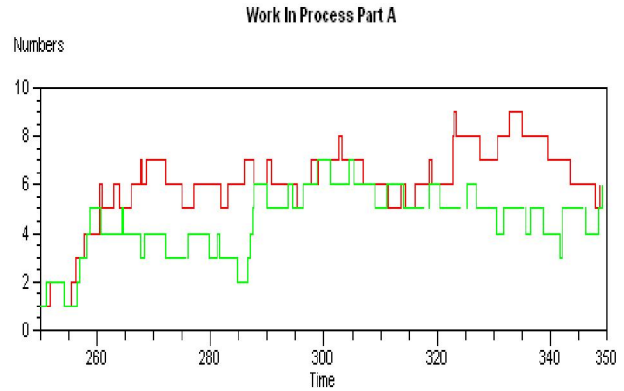


Figure 3: Trajectories of time series data part A

Two data series may be considered to define the separation distance plots. The separation distance (magnitude of difference) plots of two trajectories are given in Figure 4 for time series data obtained in Figure 2. It is observed that the separation distance between these two trajectories varies irregularly; it indicates the sensitive dependence on initial conditions. The nature of divergence of the nearby trajectories and sensitive dependence on initial condition is quantified and characterized using the Lyapunov exponent which is computed by using the algorithm described above.

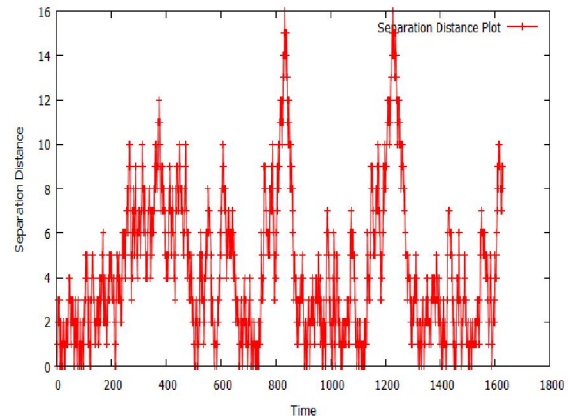


Figure 4: Separation Distance Plot for WIP Part A

Lyapunov exponents measure the rate of divergence of initially close trajectories and the sign of the Lyapunov exponent provides a qualitative picture of a system's dynamics. For a deterministic process the Lyapunov exponent have to be a positive finite number, for a linear process it should be zero and for a stochastic process it should be infinite. The average Lyapunov exponents for data is computed from slopes of trend line fit and these are found to be 0.00066 as given in Figure 5.

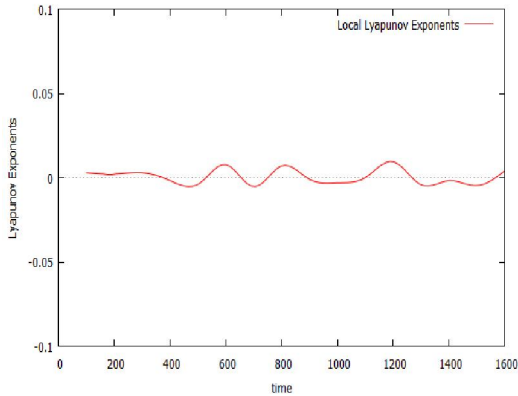


Figure 5: Local Lyapunov Exponents for WIP Part A

The result obtained from the two time series using Lyapunov exponent is affirmative but the slope value is not very sensitive but showing beginning of the chaos. From the above analysis of the time series data, it has been proved that flexible assembly system exhibit chaotic behavior.

#### 4. Conclusion:

In this research work chaos in assembling system has been investigated. It is concluded that flexible assembly line may have chaotic behavior as very small changes may lead to deviations in the performance indicators such as work in process. The main interest is focused on the Lyapunov exponents since it can be calculated relatively easily and it yields confirmation of the presence of chaos in the observed data. This has been computed with an aid of process-based simulation software and responses of time series obtained. The average Lyapunov exponent computed is positive and shows that chaotic behavior occurs in the system. It is recommended that the shop floor management must be careful when carrying out assembling with different scenarios as a chaotic behavior may result from system. More research work is required to investigate other similar processes, issues related to control and scheduling.

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