# The Greatest Prime Discovery That Was Ever Made On The Singular Series In The Jiang Prime K-Tuple Theorem

Jiang Chunxuan

Institute for Basic Research, Palm Harbor, FL34682-1577, USA
And: P. O. Box 3924, Beijing 100854, P. R. China
jiangchunxuan@sohu.com, cxjiang@mail.bcf.net.cn, jcxuan@sina.com, Jiangchunxuan@vip.sohu.com, jcxxxx@163.com

**Abstract:** Using Jiang function we prove Jiang prime $k$-tuple theorem.We find true singular series. Using the examples we prove the Hardy-Littlewood prime $k$-tuple conjecture with wrong singular series.. Jiang prime $k$-tuple theorem will replace the Hardy-Littlewood prime $k$-tuple conjecture.
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**(A) Jiang prime $k$-tuple theorem with true singular series[1, 2].**

We define the prime $k$-tuple equation

$$p, p+n_i, \qquad (1)$$

where $2|n_i, i=1,\cdots k-1$.

we have Jiang function [1, 2]

$$J_2(\omega)=\prod_P(P-1-\chi(P)), \qquad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{i=1}^{k-1}(q+n_i)\equiv 0 \pmod P, \quad q=1,\cdots,p-1. \qquad (3)$$

which is true.

If $\chi(P)<P-1$ then $J_2(\omega)\neq 0$. There exist infinitely many primes $P$ such that each of $P+n_i$ is prime. If $\chi(P)=P-1$ then $J_2(\omega)=0$. There exist finitely many primes $P$ such that each of $P+n_i$ is prime. $J_2(\omega)$ is a subset of Euler function $\phi(\omega)$ [2].

If $J_2(\omega)\neq 0$, then we have the best asymptotic formula of the number of prime $P$ [1, 2]

$$\pi_k(N,2)=\left|\{P\le N: P+n_i=prime\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)}\frac{N}{\log^k N}=C(k)\frac{N}{\log^k N} \qquad (4)$$

$$\phi(\omega)=\prod_P(P-1)$$

$$C(k)=\prod_P\left(1-\frac{1+\chi(P)}{P}\right)\left(1-\frac{1}{P}\right)^{-k} \qquad (5)$$

is Jiang true singular series.

It is The greatest prime discovery that was ever made

**Example 1**. Let $k=2, P, P+2$, twin primes theorem.

From (3) we have

$$\chi(2)=0, \quad \chi(P)=1 \text{ if } P>2, \tag{6}$$

Substituting (6) into (2) we have

$$J_2(\omega)=\prod_{P\geq 3}(P-2)\neq 0 \tag{7}$$

There exist infinitely many primes $P$ such that $P+2$ is prime. Substituting (7) into (4) we have the best asymptotic formula

$$\pi_k(N,2)=\left|\{P\leq N: P+2=prime\}\right|\sim 2\prod_{P\geq 3}\left(1-\frac{1}{(P-1)^2}\right)\frac{N}{\log^2 N}. \tag{8}$$

**Example 2**. Let $k=3, P, P+2, P+4$.

From (3) we have

$$\chi(2)=0, \quad \chi(3)=2 \tag{9}$$

From (2) we have

$$J_2(\omega)=0. \tag{10}$$

It has only a solution $P=3$, $P+2=5$, $P+4=7$. One of $P, P+2, P+4$ is always divisible by 3.

**Example 3**. Let $k=4, P, P+n$, where $n=2,6,8$.

From (3) we have

$$\chi(2)=0, \chi(3)=1, \chi(P)=3 \text{ if } P>3. \tag{11}$$

Substituting (11) into (2) we have

$$J_2(\omega)=\prod_{P\geq 5}(P-4)\neq 0, \tag{12}$$

There exist infinitely many primes $P$ such that each of $P+n$ is prime.

Substituting (12) into (4) we have the best asymptotic formula

$$\pi_4(N,2)=\left|\{P\leq N: P+n=prime\}\right|\sim\frac{27}{3}\prod_{P\geq 5}\frac{P^3(P-4)}{(P-1)^4}\frac{N}{\log^4 N} \tag{13}$$

**Example 4.** Let $k=5$, $P$, $P+n$, where $n=2,6,8,12$.

From (3) we have

$$\chi(2)=0, \chi(3)=1, \chi(5)=3, \chi(P)=4 \text{ if } P>5 \tag{14}$$

Substituting (14) into (2) we have

$$J_2(\omega)=\prod_{P\geq 7}(P-5)\neq 0 \tag{15}$$

There exist infinitely many primes $P$ such that each of $P+n$ is prime. Substituting (15) into (4) we have the best asymptotic formula

$$\pi_5(N,2)=\left|\{P\leq N: P+n=prime\}\right|\sim\frac{15^4}{2^{11}}\prod_{P\geq 7}\frac{(P-5)P^4}{(P-1)^5}\frac{N}{\log^5 N} \tag{16}$$

**Example 5**. Let $k=6$, $P$, $P+n$, where $n=2,6,8,12,14$.

From (3) and (2) we have

$$\chi(2)=0, \ \chi(3)=1, \ \chi(5)=4, \quad J_2(5)=0 \tag{17}$$

It has only $a$ solution $P=5$, $P+2=7$, $P+6=11$, $P+8=13$, $P+12=17$, $P+14=19$. One of $P+n$ is always divisible by 5.

**(B) The Hardy-Littlewood prime $k$-tuple conjecture with wrong singular series[3-14].**

We define the prime $k$-tuple equation

$$P, P+n_i \tag{18}$$

where $2|n_i, i=1,\cdots,k-1$.

In 1923 Hardy and Littlewood conjectured the asymptotic formula

$$\pi_k(N,2)=\left|\{P\le N: P+n_i=prime\}\right|\sim H(k)\frac{N}{\log^k N}, \tag{19}$$

where

$$H(k)=\prod_P\left(1-\frac{\nu(P)}{P}\right)\left(1-\frac{1}{P}\right)^{-k} \tag{20}$$

is Hardy-Littlewood wrong singula series,

It is the greatest prime mistake that was ever made.

$\nu(P)$ is the number of solutions of congruence

$$\prod_{i=1}^{k-1}(q+n_i)\equiv 0 \pmod P, \quad q=1,\cdots,P. \tag{21}$$

which is wrong.

From (21) we have $\nu(P)<P$ and $H(k)\ne 0$. For any prime $k$-tuple equation there exist infinitely many primes $P$ such that each of $P+n_i$ is prime, which is false.

**Conjecture 1**. Let $k=2, P, P+2$, twin primes theorem

From (21) we have

$$\nu(P)=1 \tag{22}$$

Substituting (22) into (20) we have

$$H(2)=\prod_P\frac{P}{P-1} \tag{23}$$

Substituting (23) into (19) we have the asymptotic formula

$$\pi_2(N,2)=\left|\{P\le N: P+2=prime\}\right|\sim\prod_P\frac{P}{P-1}\frac{N}{\log^2 N} \tag{24}$$

which is wrong see example 1.

**Conjecture 2**. Let $k=3, P, P+2, P+4$.

From (21) we have

$$\nu(2)=1,\ \nu(P)=2 \text{ if } P>2 \tag{25}$$

Substituting (25) into (20) we have

$$H(3)=4\prod_{P\ge 3}\frac{P^2(P-2)}{(P-1)^3} \tag{26}$$

Substituting (26) into (19) we have asymptotic formula

$$\pi_3(N,2)=\left|\{P\le N: P+2=prime, P+4=prim\}\right|\sim 4\prod_{P\ge 3}\frac{P^2(P-2)}{(P-1)^3}\frac{N}{\log^3 N} \tag{27}$$

which is wrong see example 2.

**Conjecture 3**. Let $k=4$, $P, P+n$, where $n=2,6,8$.

From (21) we have

$$\nu(2)=1,\ \nu(3)=2,\ \nu(P)=3 \text{ if } P>3 \tag{28}$$

Substituting (28) into (20) we have

$$H(4)=\frac{27}{2}\prod_{P>3}\frac{P^3(P-3)}{(P-1)^4} \quad (29)$$

Substituting (29) into (19) we have asymptotic formula

$$\pi_4(N,2)=\left|\{P\le N: P+n=prime\}\right|\sim\frac{27}{2}\prod_{P>3}\frac{P^3(P-3)}{(P-1)^4}\frac{N}{\log^4 N} \quad (30)$$

Which is wrong see example 3.

**Conjecture 4**. Let $k=5,\ P, P+n$, where $n=2,6,8,12$

From (21) we have

$$\nu(2)=1,\ \nu(3)=2,\ \nu(5)=3,\ \nu(P)=4 \text{ if } P>5 \quad (31)$$

Substituting (31) into (20) we have

$$H(5)=\frac{15^4}{4^5}\prod_{P>5}\frac{P^4(P-4)}{(P-1)^5} \quad (32)$$

Substituting (32) into (19) we have asymptotic formula

$$\pi_5(N,2)=\left|\{P\le N: P+n=prime\}\right|\sim\frac{15^4}{4^5}\prod_{P>5}\frac{P^4(P-4)}{(P-1)^5}\frac{N}{\log^5 N} \quad (33)$$

Which is wrong see example 4.

**Conjecture 5**. Let $k=6$, $P$, $P+n$, where $n=2,6,8,12,14$.

From (21) we have

$$\nu(2)=1,\ \nu(3)=2,\ \nu(5)=4,\ \nu(P)=5 \text{ if } P>5 \quad (34)$$

Substituting (34) into (20) we have

$$H(6)=\frac{15^5}{2^{13}}\prod_{P>5}\frac{(P-5)P^5}{(P-1)^6} \quad (35)$$

Substituting (35) into (19) we have asymptotic formula

$$\pi_6(N,2)=\left|\{P\le N: P+n=prime\}\right|\sim\frac{15^5}{2^{13}}\prod_{P>5}\frac{(P-5)P^5}{(P-1)^6}\frac{N}{\log^6 N} \quad (36)$$

which is wrong see example 5.

**Conclusion**. The Jiang prime k-tuple theorem has true singular series. The Hardy-Littlewood prime $k$-tuple conjecture has wrong singular series.. The tool of additive prime number theory is basically the Hardy-Littlewood wrong prime k-tuple conjecture which are wrong [3-14]. Using Jiang true singula series we prove almost all prime theorems. Jiang prime $k$-tuple theorem will replace Hardy-Littlewood prime $k$-tuple Conjecture. There cannot be really modern prime theory without Jiang function.

**References**


1. Chun-Xuan Jiang, Foundations of Santilli's isonumber theory with applications to new cryptograms, Fermat's theorem and Goldbach's conjecture. Inter. Acad. Press, 2002, MR2004c:11001, (http://www.i-b-r.org/docs/jiang.pdf) (http://www.wbabin.net/math/xuan13. pdf).
2. Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution. (http:// www. wbabin. net/math/ xuan2. pdf) (http://vixra.org/pdf/0812.0004v2.pdf)
3. G. H. Hardy and J. E. Littlewood, Some problems of 'Partition Numerorum', III: On the expression of a number as a sum of primes, Acta Math, 44(1923), 1-70.
4. B. Green and T. Tao, The primes contain arbitrarily long arithmetic progressions, Ann. Math., 167(2008), 481-547.
5. D. A. Goldston, S. W. Graham, J. Pintz and C. Y. Yildirim, Small gaps between products of two primes, Proc.

London Math. Soc., (3) 98 (2009) 741-774.
6. D. A. Goldston, S. W. Graham, J. Pintz and C. Y. Yildirim, Small gaps between primes or almost primes, Trans. Amer. Math. Soc., 361(2009) 5285-5330.
7. D. A. Goldston, J. Pintz and C. Y. Yildirim, Primes in tulpes I, Ann. Math., 170(2009) 819-862.
8. P. Ribenboim, The new book of prime number records, 3rd edition, Springer-Verlag, New York, NY, 1995. PP409-411.
9. H. Halberstam and H.-E. Richert, Sieve methods, Academic Press, 1974.
10. A. Schinzel and W.Sierpinski, Sur certaines hypotheses concernant les nombres premiers, Acta Arith.,4 (1958)185-208.
11. P.T. Bateman and R.A. Horn, A heuristic asymptotic formula concerning the distribution of prime numbers, Math. Comp., 16(1962)363-367.
12. W. Narkiewicz, The development of prime number theory,From Euclid to Hardy and Littlewood, Springer-Verlag, New York,NY,2000,333-53.
13. B.Green and T. Tao, Linear equations in primes, Ann. Math. 171(2010)1753-1850.
14. T. Tao, Recent progress in additive prime number theory, http://terrytao.files.wordpress.com/2009/08/prime-number-theory1.pdf.

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