Numerical Analysis of Magnetic Field with the Hemodynamics Of Large Blood Vessel

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Abstract: The function of blood is to deliver necessary substances to the body cells such as nutrients and oxygen through the blood vessels. In earlier works on hemodynamics of blood, the effect of the uniform magnetic field applied to the flow channel was investigated. In this current work, the problem was modelled and the non-dimensionalised equation is then solved numerically involving asymptotic expansion for fixed injection Reynolds Number and Maxwell Visco-elastic analysis. The effect of visco-elastic K and the viscosity variation parameters on the velocity field are advanced.

[Babadipe, E.S. Numerical Analysis of Magnetic Field with the Hemodynamics Of Large Blood Vessel. Researcher. 2010;2(11):82-94]. (ISSN: 1553-9865). http://www.sciencepub.net.

Keywords: Hemodynamics, Flow Channel, Visco Elastic, Asymptotic Expansion, Reynolds Number

1.Introduction

The visco-elastic model of the study of blood flow in blood vessels assuming a constant blood viscosity for non Newtonian blood flow has been variously discussed [1-5] under different flow conditions.

In this work, a study of the interaction of magnetic field with the hemodynamic of large blood vessels [6-7] is extended so as to examine the effect of Hematocrit variation on the velocity profile when magnetic field is included. Furthermore is to examine the effect of increase in magnetic field variation on hemodynamics and the effect of magnetic field on pressure gradients

We consider the viscous, incompressible, homogeneous and laminar flow under the influence of uniform external magnetic field flowing through a large blood vessel. The geometry of the flow under consideration in this present study is given below.

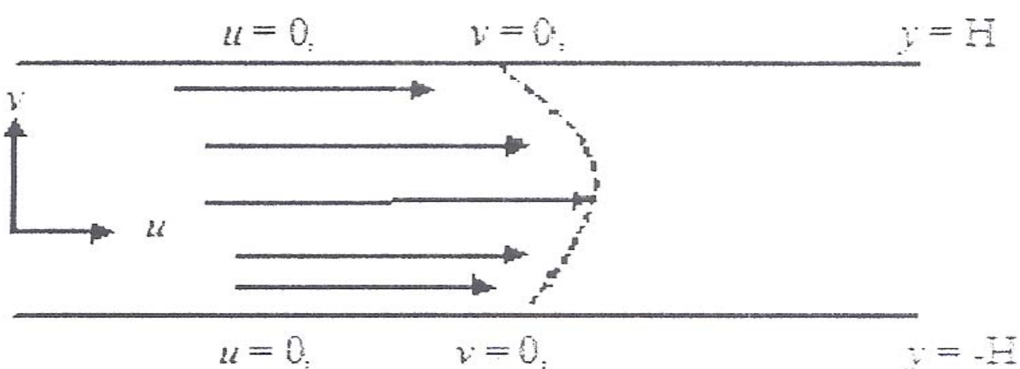

Figure 1: Flow geometry

The continuity equation is

$$\frac{du}{dx} = 0 \qquad \text{.............................. (1.1)}$$

$$\rho\frac{du}{dt} = -\frac{dp}{dx} + \frac{d}{dy}\left(\mu\frac{du}{dy}\right) - \sigma\beta_0^2 u \qquad \text{........................... (1.2)}$$

Where μ represents the dynamic viscosity and σ = electrical conductivity, u = velocity, ρ = density, μ = viscosity, p = pressure, x = co-ordinate in the direction of flow, y = coordinate across the flow.

Introducing the following dimensionless parameters

$$u' = \frac{u}{u_0},\ y' = \frac{y}{h},\ t' = \frac{t}{t_0},\ x' = \frac{x}{h} \qquad \text{........................... (1.3)}$$

We obtain the dimensionless equation after dropping (') for clarity with the appropriate initial and boundary condition.

$$\frac{du}{dt} = G + \frac{d}{dy}\left(\mu\frac{du}{dy}\right) - H^2 u \qquad \text{..................... (1.4)}$$

With

$$\begin{aligned} t \le 0 &: \ u = 0 \ \forall\ y \\ t > 0 &: \ u = 0, \ \ y = -1 \\ & \ \ \ u = 0, \ y = 1 \end{aligned} \qquad (1.5)$$

Where $G = \dfrac{-t_0}{\rho h u_0}\dfrac{dp}{dx}$ and H^2 = represent the dimensionless pressure gradient.

At steady state we have,

$$0 = G + \frac{d}{dy}\left(\mu\frac{du}{dy}\right) - H^2 u \qquad (1.6)$$

With boundary conditions

$$u(-1) = 0 = u(1) \qquad (1.7)$$

The relationship between the red blood cell and viscocity is given by (Makinde 2008) to be

$$\mu = e^{\beta\left(1-y^2\right)} \qquad \text{…………………………….(1.8)}$$

Where β represents the viscosity variation parameter

Substituting (8) into equation (6) we obtain

$$0 = G + \frac{d}{dy}\left(\exp \beta\left(1-y^2\right)\frac{du}{dy}\right) - H^2 u \qquad \text{………... (1.9)}$$

$$u(-1) = 0 = u(1)$$

Definition 1.1:

(Corddington and Leviton, 1955) for a system of equation

$$x_i' = \sum_{j=1}^{n} a_{ij}(t)x_j \qquad (i = 1,\ldots,n) \qquad \text{…………(1.10)}$$

Where a_{ij} are continuous functions on some closed bounded t interval [a, b]. If f is the vector with components f_i defined by

$$f_i(t,x) = \sum_{j=1}^{n} a_{ij}(t)x_j \qquad (i = 1,\ldots,n) \qquad \text{……………………….. (1.11)}$$

then f satisfies the Lipschitz condition on the (n+1) –dimensional region $D: a \le t \le b \quad |x| < \infty$ infact

$$|f(t,x_1) - f(t,x_2)| \le k|x_1 - x_2| \qquad (1.12)$$

Where

$$k = \sum_{i=1}^{n} |a_{ij}(t)| \qquad (t\varepsilon[a,b];\ j = 1,\ldots,n) \qquad \text{…………………….(1.13)}$$

Theorem 1.1: (Existence and Uniqueness Theorem) for the linear system (L), where the functions a_{ij} ε C on [a, b], there exists one and only one solution φ of (L) on [a, b] passing through any point $(\tau, \xi) \varepsilon D$ that is $\varphi(\tau) = \xi$

We shall use the theorem of (Corddington and Leviton, 1955) to prove our result

Proof:- we have to show that the vector f satisfies the Lipschiitz condition on D, then the existence and uniqueness is guaranteed at the initial point x_1 (0) = -1. It suffices to show that the solution can be continued to a unique solution in the entire interval [a, b]

Now

Let $x_1 = y$, $x_2 = u$, $x_3 = u'$ ……………………….. (1.14)

Then, (2.11) becomes

$$\begin{pmatrix} x_1' \\ x_2' \\ \\ x_3' \end{pmatrix} = \begin{pmatrix} 1 \\ x_3 \\ \\ \dfrac{H\ {}^2u - G + 2\beta x_1 x_3 \exp \beta\left(1 - x_1^2\right)}{\exp \beta\left(1 - x_1^2\right)} \end{pmatrix} = \begin{pmatrix} f_1(x_1, x_2, x_3) \\ f_2(x_1, x_2, x_3) \\ \\ f_3(x_1, x_2, x_3) \end{pmatrix}$$

……………………… (1.15)

subject to the initial conditions

x_1 (0) = 0

x_2 (0) = -1

x_3 (0) = D …………………… (1.16)

Clearly $\frac{df_i}{dx_j} = K$ therefore (1.15) is Lipschitz continuous which implies that it is bounded below and above with D as a guess value set to meet the other boundary condition

Now that we have proved that the solution exists and it's unique, we can proceed to solve (1.15) which represents a non-linear equation

Then,

$$\begin{pmatrix} x_1' \\ x_2' \\ \\ x_3' \end{pmatrix} = \begin{pmatrix} 1 \\ x_3 \\ \\ \dfrac{H\ {}^2u - G + 2\beta x_1 x_3 \exp \beta\left(1 - x_1^2\right)}{\exp \beta\left(1 - x_1^2\right)} \end{pmatrix}$$

………………. (1.17)

Subject to the initial conditions

$$x_1(0) = 0$$
$$x_2(0) = -1$$
$$x_3(0) = D \quad \text{………………… (1.18)}$$

then (1.17) is evaluated with a Pascal code to give the result in Table 1-3 for various values of the different parameters

2. FURTHER MATHEMATICAL ANALYSIS

We consider the visco-elastic fluid model given (Singh, 1983) whose constitutive equation is characterized by

$$1 + (\lambda \frac{\delta}{\delta t})\tau^n = 2\mu e^{ik} \quad \text{……………….. (2.1)}$$

Where

$$e_{ik} = \frac{1}{2}(v_{ij} + v_{kj}) \quad \text{………..……..….. (2.2)}$$

τ^n - is the stress tensor, λ – is the relaxation, μ - is the dynamic viscosity, e^{ik} - the rate of strain tensor.

For any contra-variant tensor b^{tk}

$$\frac{\delta b^{ik}}{\delta t} = \frac{\partial b^{ik}}{\partial t} + v^m \frac{\partial b^{ik}}{\partial x_m} - \frac{\partial v^k}{\partial x_m} b^m - \frac{\partial v^i}{\partial x_m} b^{mb} \quad \text{………………… (2.3)}$$

The continuity equation for the incompressible unsteady flow of fluid of density ρ is

$$(\rho v^i)_{,i} = 0 \quad \text{………………. (2.4)}$$

Let us assume that v is every where negative that is v = $-v_o$ (constant) and u = u(y, t) and then the momentum equation gives

$$\rho(\frac{\partial u}{\partial t} - v_0 \frac{\partial u}{\partial y}) = -\frac{\partial p}{\partial x} + \frac{\partial \tau^{xy}}{\partial y} \quad \text{………………… (2.5)}$$

Equation (1) gives,

$$\tau^{xy} = \frac{\mu \frac{\partial u}{\partial y}}{(1 + \lambda \frac{\partial}{\partial t} - v_0 \lambda \frac{\partial}{\partial y})} \quad \text{………………….. (2.6)}$$

We assume a variable viscosity (Makinde, 2008)

$$\mu(y) = e^{\beta(1-y^2)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2.7)$$

Where

μ is the dynamic velocity, u= velocity, ρ = density, p = pressure, x = co-ordinate in the direction of flow, y = coordinate across the flow, v_o is the constant vertical substituting (2.6) in (2.5)

$$\rho\left(\frac{\partial u}{\partial t} - v_0\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \lambda\frac{\partial}{\partial t}\left(-\frac{\partial p}{\partial x}\right) + 2\rho\lambda v_0\frac{\partial^2 u}{\partial t\partial y} - \rho\lambda\frac{\partial^2 u}{\partial t^2} + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) - \rho\lambda v_0^2\frac{\partial^2 u}{\partial y^2} \ldots\ldots (2.8)$$

Now introducing the following dimensionless parameters

$$u' = \frac{u}{v_0}, x' = \frac{x}{h}, y' = \frac{y}{h}, t' = \frac{v_0 t}{h}, p' = \frac{p}{\rho v_0^2}, Re = \frac{v_0 h}{v}, v = \frac{\mu_0}{\rho}, K = \frac{v_0^2\lambda}{v} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2.9)$$

We obtain

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = -\frac{dP}{dx} + K\frac{\partial}{\partial t}\left(-\frac{dP}{dx}\right) + 2K\frac{\partial^2 u}{\partial t\partial y} - K\frac{\partial^2 u}{\partial t^2} + \frac{\partial}{\partial y}\left(e^{\beta(1-y^2)}\frac{\partial u}{\partial y}\right) - K\frac{\partial^2 u}{\partial y^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.10)$$

Subject to initial and boundary conditions

$$u(0, y) = Sin(\pi y), \qquad \frac{\partial u}{\partial t}(0, y) = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.11)$$

$$u(t, -1) = 0 = u(t, 1),$$

3. STEADY STATE SOLUTION

We assume that blood flow steadily therefore (2.10) reduces to

$$0 = \lambda + \frac{\partial u}{\partial y} + \frac{\partial}{\partial y}\left(e^{\beta(1-y^2)}\frac{du}{dy}\right) - K\frac{d^2 u}{dy^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.1)$$

Subject to the boundary conditions

$$u(-1) = 0 = u(1) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.2)$$

Let $0 < \beta << 1$ therefore by asymptotic expansion

We take $u = u_o + \beta u_1 + \beta^2 u_2$ neglecting other terms ……..………… (3.3)

Taking the Taylor's expansion of $e^{\beta(1-y^2)}$ about β

$$e^{\beta(1-y^2)} = 1 + \beta(1 - y^2) + 0.5\beta^2(1 - y^2)^2 \qquad \text{…………… (3.4)}$$

Equating coefficients

$$0 = 1 + \frac{du_0}{dy} + (1 - K)\frac{d^2u_0}{dy^2} \qquad \text{………………………… (3.5)}$$

$$u_o\,(-1) = 0 = u_o\,(1) \qquad \text{…………………………….. (3.6)}$$

$$0 = \frac{du_1}{dy} + \frac{d}{dy}\left((1 - y^2)\frac{du_0}{dy}\right) + (1 - K)\frac{d^2u_1}{dy^2} \qquad \text{………………….. (3.7)}$$

$$u_1\,(-1) = 0 = u_1\,(1) \qquad \text{…………………… (3.8)}$$

$$0 = \frac{du_2}{dy} + \frac{d}{dy}\left((1 - y^2)\frac{du_1}{dy}\right) + \frac{d}{dy}\left((1 - y^2)\frac{du_1}{dy}\right) + (1 - K)\frac{d^2u_2}{dy^2} \qquad \text{…………… (3.9)}$$

$$u_2\,(-1) = 0 = u_2\,(1) \qquad \text{……………. (3.10)}$$

Using mathematical version (3.6) the solution of (3.3) is given in the graphical results of figure 2 and 3.

4. NUMERICAL RESULTS

The numerical result are herein presented in both tabular and graphical forms subject to (1.17).

Table 01

y	u, β = 0.1	u, β = 0.3	u, β = 0.5	u, β = 0.7
-1	0	0	0	0
-0.9	0.0691	0.0653	0.0618	0.0585
-0.8	0.1286	0.1205	0.1129	0.1058
-0.7	0.1794	0.1667	0.1545	0.1439
-0.6	0.2221	0.205	0.1891	0.1744
-0.5	0.2574	0.2362	0.2167	0.1986
-0.4	0.2854	0.261	0.2382	0.2173
-0.3	0.3072	0.2797	0.2544	0.2313
-0.2	0.3224	0.2929	0.2657	0.2409
-0.1	0.3315	0.3006	0.2724	0.2466
0	0.3345	0.3032	0.2746	0.2485
0.1	0.3315	0.3007	0.2724	0.2466
0.2	0.3224	0.2929	0.2657	0.241
0.3	0.3072	0.2798	0.2545	0.2313
0.4	0.2857	0.261	0.2383	0.2174
0.5	0.2574	0.2363	0.2167	0.1987
0.6	0.2222	0.2051	0.1892	0.1745
0.7	0.1795	0.1668	0.155	0.144
0.8	0.1287	0.1205	0.1129	0.1059
0.9	0.0691	0.0653	0.0618	0.0586
1	0	0	0	0

Table 02

y	u, $\beta = 0.1$	u, $\beta = 0.3$	u, $\beta = 0.5$	u, $\beta = 0.7$
-1	0	0	0	0
-0.9	0.0539	0.1617	0.2696	0.3774
-0.8	0.0959	0.2874	0.4795	0.6714
-0.7	0.1287	0.3861	0.6436	0.901
-0.6	0.1543	0.4629	0.7715	1.0802
-0.5	0.1741	0.5223	0.8706	1.2188
-0.4	0.1892	0.5675	0.9459	1.3242
-0.3	0.2002	0.6007	1.0011	1.4016
-0.2	0.2078	0.6234	1.0389	1.4545
-0.1	0.2122	0.6366	1.061	1.4854
0	0.2137	0.641	1.0683	1.4956
0.1	0.2122	0.6367	1.0612	1.4856
0.2	0.2079	0.6236	1.0393	1.455
0.3	0.2003	0.601	1.0016	1.4023
0.4	0.1873	0.5679	0.9465	1.3251
0.5	0.1743	0.5228	0.8714	1
0.6	0.1545	0.4638	0.7724	1.0814
0.7	0.1289	0.3867	0.6445	0.9023
0.8	0.0961	0.2883	0.4804	0.6726
0.9	0.0541	0.1621	0.2702	0.3783
1	0	0	0	0

Table 03

y	u, H = 1	u, H = 3	u, H = 5	u, H = 7
-1	0	0	0	0
-0.9	0.0539	0.0404	0.0326	0.0275
-0.8	0.0959	0.0713	0.0571	0.0479
-0.7	0.1287	0.095	0.0757	0.0631
-0.6	0.1543	0.1133	0.0898	0.0745
-0.5	0.1741	0.1273	0.1004	0.083
-0.4	0.1892	0.1378	0.1083	0.0893
-0.3	0.2002	0.1454	0.114	0.0938
-0.2	0.2078	0.1506	0.1179	0.0968
-0.1	0.2122	0.1536	0.1201	0.0985
0	0.2137	0.1546	0.1209	0.0991
0.1	0.2122	0.1536	0.1202	0.0985
0.2	0.2079	0.1506	0.118	0.0968
0.3	0.2003	0.1455	0.1142	0.0939
0.4	0.1873	0.1379	0.1085	0.0894
0.5	0.1743	0.1274	0.1006	0.0832
0.6	0.1545	0.1135	0.09	0.0747
0.7	0.1289	0.0952	0.0759	0.0634
0.8	0.0961	0.0715	0.0573	0.0481
0.9	0.0541	0.0405	0.0327	0.0277
1	0	0	0	0

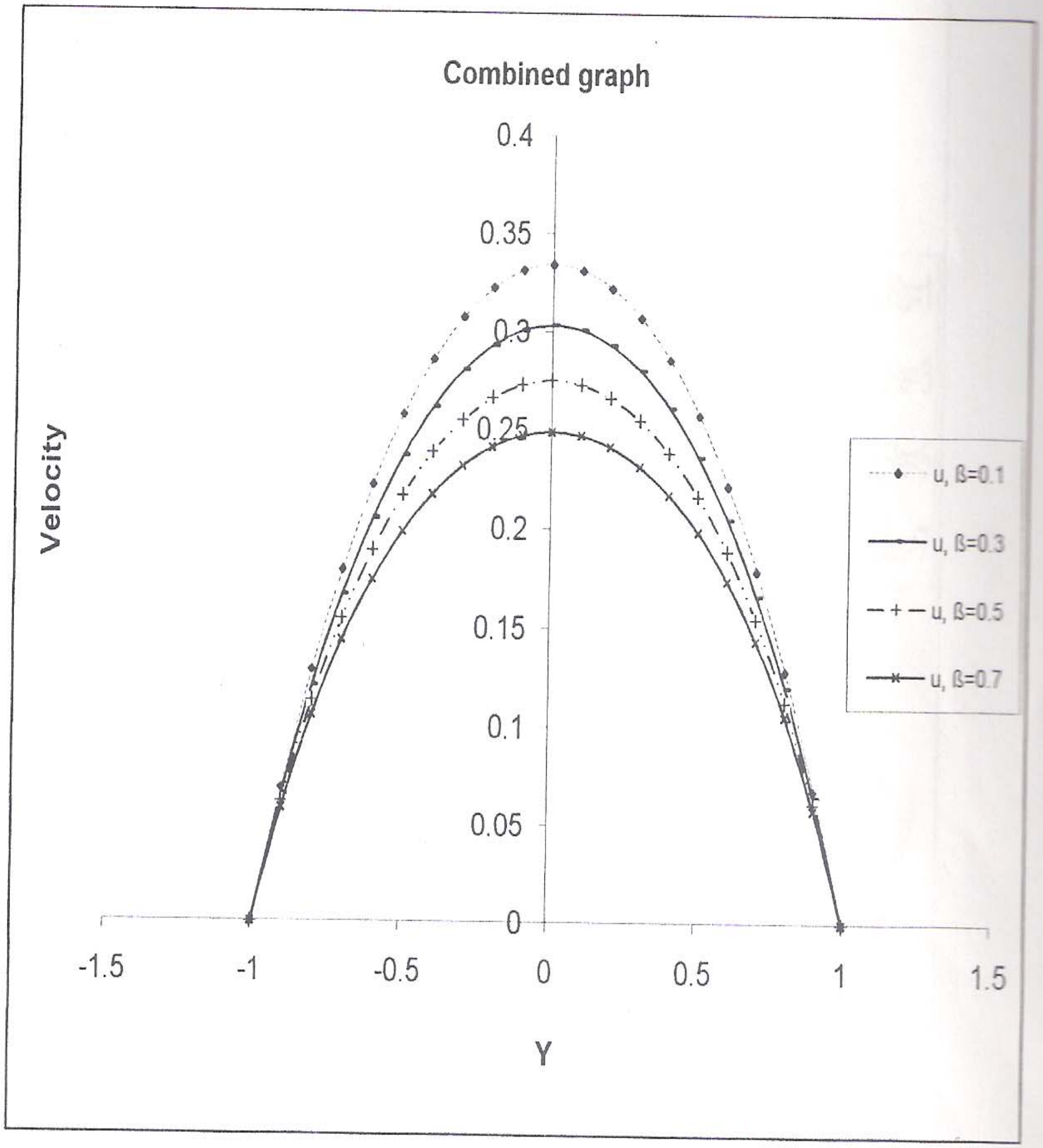

Figure 2: Graph of Velocity against distance between plates, Y involving table 1 and 2. It is evident that U(Y) is a well behaved function, since the boundary conditions are satisfied and variations of parameters due to cause and effect are seen.

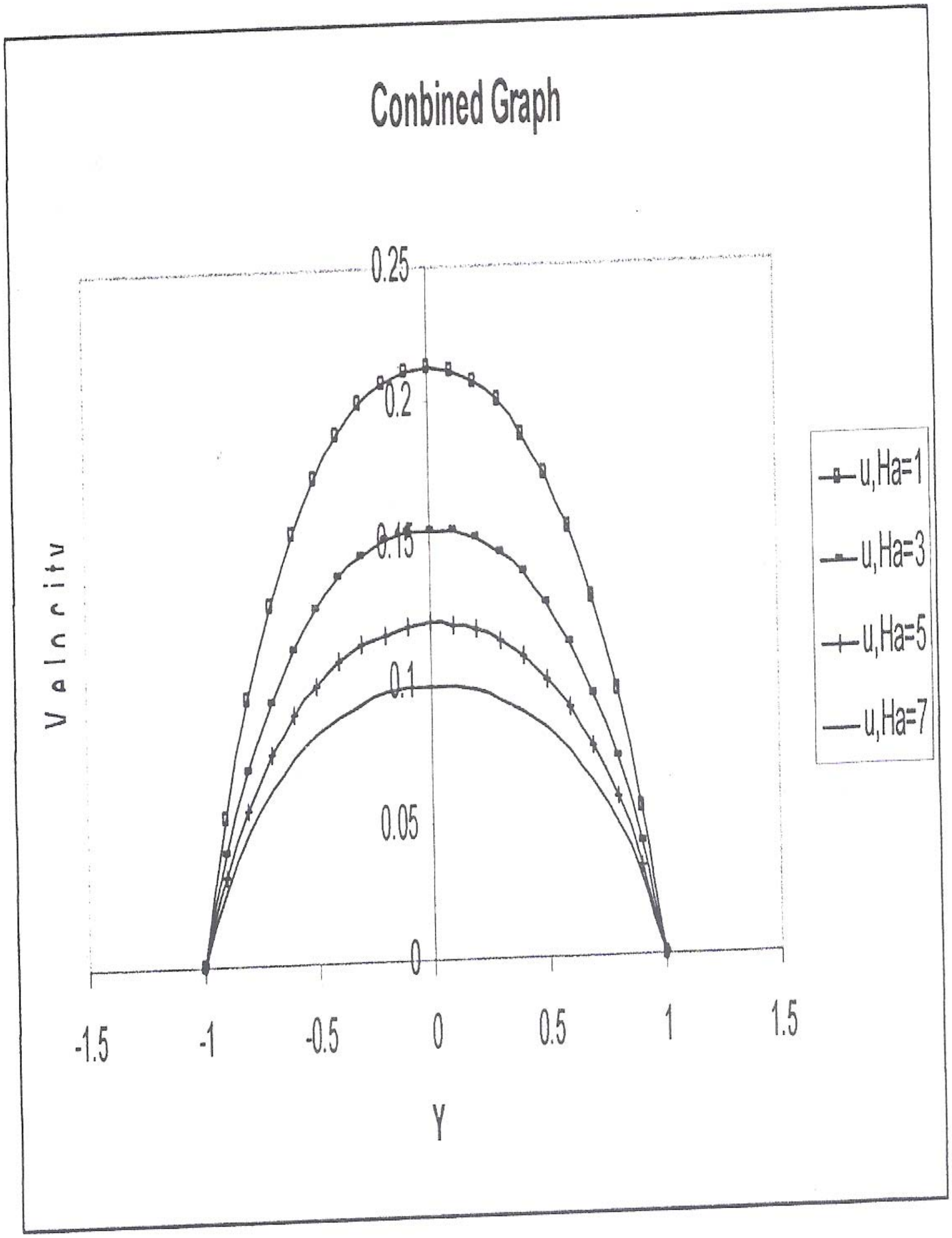

Figure 3i: Graph of Velocity against distance between plates, Y involving pressure gradients.

CONCLUSION

The magnetic field interaction with herodynamics in steady state increases the viscosity parameter as the flow velocity. This is a pre-condition associated with hypertension. Furtherance to our study of the steady flow of Maxwell fluid at steady state, we noted that as relaxation time reduces, the fluid shows Newtonian behaviour while velocity reduces with increase in viscosity. The application is in the treatment and diagnosis of cardiovascular diseases and stenosis.

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10/18/2010