# New limited series (function) and using it as an efficient new numerical method for integration, differentiation, calculating curves length and solving equations

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**Abstract:** Any mathematical function or curve or any shape used in any kind of sciences could now be replaced with easy series and this series could be limited if we take the interval that we care and interested in it. This limitation reaches to three simple terms that easily could get involved in any mathematical or complex operations and solved easily. This opens the door for hundreds of applications. In this research I used it to find new numerical integration method, numerical differentiation for any order and also finding any curve length. And here the surprise that operations take 1000 or 10000 or 100000 subintervals in the numerical method become 5 or 3 and 1 in some times and with higher accuracy sometimes!!!!!!!. In the research I use the generated function to solve accurately any variable equations and no matter how hard is it and this is become very easy. A lot of examples and programs code in this research prove and simulate these applications.

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**1-The idea of generated function**

Any equation or function or curve could be understood mathematically with a lot of angles using algebra formulas, graphs or some mathematic characteristics to this equation like its derivatives or integrals or may be its mean and variance if we look from the statistics point of view.

In this research I will give the light on my idea for generating a simple series for any function and speaks by details about its properties.

I call it sameh generated function.

- **Sameh:** because this is my name.
- **Generated function:** because this function will be generated dependently on the function or equation that we want to replace it.

Any f (x) ≅ g (x) where g (x) is sameh generated function.

$$g(x) = \sum_{n=0}^{n=\infty} a_n \mathbf{x}^n$$ (Sameh generated function main formula)

Where $a_n$ are coefficients. This formula is like the one of the power series and really it was but totally different in its properties and the difference are clear because the method of calculating the coefficients. Because as we will know in this research after a while that this series can be limited to several terms only in a specific interval 2 or 3 or 4 … terms as we want we can generate. We generate the function by equate the series with the function and in our case that happen if the series equation specify the f(x) that we want to replace with the generated function and by looking to the main formula of the generated function then we have the below mathematic notes

$a_0x^0 + a_1x^1 + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + a_7x^7 + a_8x^8 + a_9x^9 + a_{10}x^{10}$……………=f(x)

So if we substitute with infinite value of x and infinite value of f(x) then we have infinite numbers of equations and by solving them we can deduce $a_0$, $a_1$, $a_2$, $a_3$……………

Every equation $F(x_o) = a_o + a_1 * x_o + a_2 * x_o^2 + a_3 * x_O^3$…………

So if we want to generalize the equations formula that we deduce from it the coefficients in matrix form we can follow the steps below.

A= [ao a1 a2 a3 ..........................] where it dimensions 1*∞

$$X=\begin{bmatrix} xo^\wedge 0 & x1^\wedge 0 & x2^\wedge 0 \ldots\ldots\ldots\ldots \\ xo^\wedge 1 & x1^\wedge 1 & x2^\wedge 1 \\ xo^\wedge 2 & x1^\wedge 2 & x2^\wedge 2 \\ xo^\wedge 3 & x1^\wedge 3 & x2^\wedge 3 \\ xo^\wedge 4 & x1^\wedge 4 & x2^\wedge 4 \\ . & & \\ . & & \\ . & & \\ . & & \end{bmatrix}$$

where it dimensions ∞*∞

And A.X=F where

$F = [F(xo)\ F(x1)\ F(x2)\ F(x3)\ldots\ldots\ldots]$ where its dimensions 1*∞ from 1*∞.∞*∞ .The above matrix equation represent the charactristics and the meaning of my series and the problem is in the matrix A values so to get them

A.X=F So to get [A] using matrix charactristics

$A=F.\ X^{-1}$ where $X^{-1}$ is inverse matrix of X

And by MATLAB program we just deduce A=X\F

**The properties:**

**(1) limitation & calculation:**

We determine g(x) by taking interval from any function and get number of equatious equal to the number of terms from $\sum a_n x^n$

– For example – Ln (x) = f (x) and we take an interval from x = 4 to x= 7 then we can say.

$F(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$....... Theoretically if we take (n) to ∞ we will get curve and values equal to f(x)=ln(x) from x=0 to x=∞ but practically we will limit the series to 6 or 5 or 4 terms because we want the
series to be useful and applicable not like maclorine and tailor series.

–For example– In case of 3 terms $g(x) = a_0 + a_1 x + a_2 x^2$ only.

Then to determine $a_0$ , $a_1$, and $a_2$ we need three value from f(x) and its related equations.

$\ln(4) = a_0 + 4a_1 + 16\, a_2$

$\ln(5.5) = a_0 + 5.5\, a_1 + 30.25\, a_2$

$\ln(7) = a_0 + 7\, a_1 + 49\, a_2$

We know that Ln (4) =1.3862, Ln (5.5)= 1.70474 and Ln (7) =1.94591

By soling the three equation together $a_0=0.1584$, $a_1=0.37573$ and $a_2=0.0171968$.

Then Ln (x) from 4 to7 $\cong 0.1584 + 0.37573*x - 0.0171968*x^2 = g(X)$

If we compare between the original function Ln (x) and our generated function we will see how related are they in the below table look also figure (1).

| Value of X | Ln(x) values | g(x) values |
|---|---|---|
| 4.00000 | 1.38629 | 1.38617 |
| 4.50000 | 1.50408 | 1.50095 |
| 5.00000 | 1.60944 | 1.60713 |
| 5.50000 | 1.70475 | 1.70471 |
| 6.00000 | 1.79176 | 1.79370 |
| 6.50000 | 1.87180 | 1.87408 |
| 7.00000 | 1.94591 | 1.94587 |

If we look to the above table we can deduce the meaning of limitation beside the method of calculation.

First limitation: that we can't take all the values of the curve from (o) to (∞) because in practice we can't make infinite series to replace any function with our generated function or it will be useless or in its best shot like maclorine and tailor series to the scientists. So we limit our target by taking a specific interval that we interested in it or care about to cover it with countable number in the series.
Second limitation: it's hard to take large series to cover the target so we choice a number of terms in the series like three or four or more.

The best way to cover the curve or the interval is to take number of points in equal distance from the first point in the interval and with equal distant step till we reach to the end point in the interval

For example interval from x=2 to x5 if we want to generate g(x) with four terms then we have to take 4 points on the curve to get four equations and by solving then we can deduce $a_o$, $a_1$, $a_2$ and $a_3$ values

$F(2) = a_o + a_1*2 + a_2*4 + a_3*8$

$F(3) = a_o + a_1*3 + a_2*9 + a_3*27$

$F(4) = a_o + a_1*4 + a_2*16 + a_3*64$

$F(5) = a_o + a_1*5 + a_2*25 + a_3*125$
Generally we can say $F(x_o) = a_o + a_1*x_o + a_2*x_o^2 + a_3*x_O^3$ and to make the curve more accurate we distribute the points to cover the curve like the above example.

**(2)number of terms & the accuracy:**

When we discuss this property we can divide it to three main sectors.

- Functions that has sudden change.
- The effect of the number of terms and its related accuracy in the same interval.
- Number of terms and accuracy how to be specified?

**First, functions that has sudden change in its values:**

Some of the functions have a suddenly change so this functions can't be treated like other functions that have a curve that slowly change or in other sentence (the change in the value of the function is few compared to the change of x in x axis) the value is few compared to distance that the curve exist in x-axis.

We can summery our meaning by the below two functions.
Ln(x) & sin(x)
What will happen if we take an interval from x=2 to x=6 in the above two functions??

Let us see after solving them
For Ln(x) taking points in x=2, 4, 6

$0.693147 = a_0 + a_1*2 + a_2*4$

$1.386294 = a_0 + a_1*4 + a_2*16$

$1.7917594 = a_0 + a_1*6 + a_2*36$

$g_1(x) \cong Ln(x)$ from (2 - 6) $\cong - 0.28768 + 0.5623397*x - 0.0359602*x^2$

For sin (x) taking points x=2, 4, 6
this values angles 114.545° , 229.299° , 343.949°

Then we have the points value sin(114.545°) , sin(229.299°) and
sin(343.949°)

$0.9 = a_0 + a_1*2 + a_2*4$

$- 0.7581 = a_0 + a_1*4 + a_2 + 16$

$- 0.2764 = a_0 + a_1*6 + a_2*36$

$g_2(x) \cong \sin(x)$ from x = 2 to x=6 or x = 0.63 Π to 1.91 Π
$\cong 4.6979 - 2.4339*x+0.267475*x^2$
Now let's compare between the two generated function values VS their initial functions values & also look to figure (2)
The first comparison between Ln(x) VS $g_1(x)$ in the below table

| Value of x | Ln(x) values | $g_1(x)$ values |
|---|---|---|
| 2.00000 | 0.69315 | 0.69316 |
| 2.50000 | 0.91629 | 0.89342 |

| | | |
|---|---|---|
| 3.00000 | 1.09861 | 1.07570 |
| 3.50000 | 1.25276 | 1.24000 |
| 4.00000 | 1.38629 | 1.38632 |
| 4.50000 | 1.50408 | 1.51465 |
| 5.00000 | 1.60944 | 1.62501 |
| 5.50000 | 1.70475 | 1.71739 |
| 6.00000 | 1.79176 | 1.79179 |

And the below table represent the comparison between sin(x) VS $g_2(x)$

| Value of x | Sin(x) values | $g_2(x)$ values |
|---|---|---|
| 2.00000 | 0.90930 | 0.90000 |
| 2.50000 | 0.59847 | 0.28487 |
| 3.00000 | 0.14112 | -0.19653 |
| 3.50000 | -0.35078 | -0.54418 |
| 4.00000 | -0.75680 | -0.75810 |
| 4.50000 | -0.97753 | -0.83828 |
| 5.00000 | -0.95892 | -0.78473 |
| 5.50000 | -0.70554 | -0.59743 |
| 6.00000 | -0.27942 | -0.27640 |

From the previous tables we can deduce that in the functions like $10^x$ , sin(x) that have a high rate of change compared to the change in x axis this functions need high number of terms from the series compared to slow change functions like Ln(x) or we can take low number in the series like 3 terms but we have to take narrower intervals with low terms like the below example.
Sin(x) from x = 2 to x = 4
by taking 3 points (3 different values of sin(x)) at 2, 3 and 4 to get g(x) with three terms

$0.9093 = a_0 + a_1*2 + a_2*4$

$0.14112 = a_0 + a_1*3 + a_2*9$

$- 0.7568 = a_0 + a_1*4 + a2*16$

So sin(x) from (2 – 4) ≅ $g(x) = 2.05644 - 0.44383*x - 0.06487*x^2$
We will make a comparison between sin(x) and g(x) in the below table see also figure (3).

| Value of x | sin(x) values | g(x) values |
|---|---|---|
| 2.00000 | 0.90930 | 0.90930 |
| 2.50000 | 0.59847 | 0.54143 |
| 3.00000 | 0.14112 | 0.14112 |
| 3.50000 | -0.35078 | -0.29162 |

| | | |
|---|---|---|
| 4.00000 | -0.75680 | -0.75680 |

The differences now are clear. And if we take narrower interval the difference will be clear more and also if we take more terms in the wider interval.

**Second, the effect of the number of terms and the related accuracy in the Same interval:**

In the same interval from any function or curves if we increase the number of terms we have more accurate generated function so to collect my meaning is this sector lets see the below example

F(x) = ln(x) from x=10 to X = 16

$$g(x) = \sum_{n=0}^{n=\infty} a_n \mathbf{x}^n$$

Let's see g(x) in case of 2 terms of the series, 3 terms of the series and 4 terms and compare the results to feel the deference & look at figures (4).

*if we take two terms by taking points Ln(15) and Ln (16)

$2.3025 = a_0 + a_1 * 40$

$2.7725 = a_0 + a_1 * 15$

$g_1(x) = 1.519166 + 0.078333*x$

*if we take three terms at valves Ln ( 10 ) , Ln (13 ) and Ln (16 )

$2.3025 = a_0 + a_1 * 10 + a_2 * 100$

$2.5649 = a_0 + a_1 * 13 + a_2 * 169$

$2.7725 = a_0 + a_1 * 16 + a_2 * 256$

$g_2(x) = 1.032055 + 0.1574888\ X$
$- 3.04444 * 10^{-3} * x^2$

* if we take four terms then we need four equations which mean that we need four points at the values Ln (10) , Ln (12) , Ln(14 ) and Ln (16)

$2.3025 = a_0 + a_1*10 + a_2*100 + a_3*1000$

$2.4849 = a_0 + a_1*12 + a_2*144 + a_3*1728$

$2.639 = a_0 + a_1*14 + a_2*196 + a_3*2744$

$2.7725 = a_0 + a_1*16 + a_2*256 + a_3*4096$

$g_3(x) = 0.6965 + 0.23768333*x$
$-0.93125*10^{-2}*x^2 + 0.16041666*10^{-3}*x^3$

Let's make a compare between $g_1(x)$ , $g_2(x)$ and $g_3(x)$ VS Ln(x) in the previous example to feel this properties.

| Value of x | Ln(x) values | $g_1(x)$ values | $g_2(x)$ values | $g_3(x)$ values |
|---|---|---|---|---|
| 10.00000 | 2.30259 | 2.30250 | 2.30250 | 2.30250 |
| 10.50000 | 2.35138 | 2.34166 | 2.35004 | 2.35117 |
| 11.00000 | 2.39790 | 2.38083 | 2.39605 | 2.39772 |
| 11.50000 | 2.44235 | 2.42000 | 2.44055 | 2.44225 |
| 12.00000 | 2.48491 | 2.45916 | 2.48352 | 2.48490 |
| 12.50000 | 2.52573 | 2.49833 | 2.52497 | 2.52578 |
| 13.00000 | 2.56495 | 2.53750 | 2.56490 | 2.56501 |
| 13.50000 | 2.60269 | 2.57666 | 2.60330 | 2.60271 |
| 14.00000 | 2.63906 | 2.61583 | 2.64019 | 2.63900 |
| 14.50000 | 2.67415 | 2.65499 | 2.67555 | 2.67401 |
| 15.00000 | 2.70805 | 2.69416 | 2.70939 | 2.70784 |
| 15.50000 | 2.74084 | 2.73333 | 2.74170 | 2.74064 |

| | | | | |
|---|---|---|---|---|
| 16.00000 | 2.77259 | 2.77249 | 2.77250 | 2.77250 |

As we see and note that $g_3(x)$ more accurate than $g_2(x)$ more than $g_1(x)$ and we can choice which generated function to be used due to our needing and applications

**Third, number of terms & accuracy how to be specified? :**

Really this sector is very important because it needs some sense to make a judgment and this part depends on the application and the function and the accuracy needed.

Generally to have an accepted accuracy we have to predict the behavior of our function (curve) like the below difference.

Ln (x) and exp(x)

Ln 3 = 1.0986

Ln 6 = 1.7917

Ln 10 = 2.3025

Clearly we can deduce that the change in Ln(x) which represents f(x) here is relatively small compared to change in x axis but in exp (x)

Exp (3) = 20.0855

Exp (6) = 403.4287

Exp (15) = 22026.46579

It's clear that the change is very very big in f(x) = exp (x) values compared to x-axis.

The question here is there will be a difference treatment between them or we will deal with them by same ways to generate g(x)??!!

The answer is yes of course there will be a difference because if I have the same interval for the two functions Ln(x) and exp(x) we will not take the same number of terms.

for the interval from x = 10 to x = 16 we can take three terms only in the series $\sum a_n x^n$ to get a suitable generated function for Ln(x) from 10-16 but this number will not be suitable for exp(x) because the high rate of change between the points and the points behind it so we can take a larger number of terms or very narrow interval with few terms in some cases can be a unit only in x axis or less and to prove this point of view lets see the next example

Exp(x) from x=10-13

With three terms and five terms and see how much this will differ in the results of their generated functions see also figure (5).

*First with three terms [our points 10, 11.5, 13]

Exp (10) = $a_0 + 10 * a_1 + 100 * a_2$

Exp (11.5) = $a_0 + 11.5 * a_1 + 132.25 * a_2$

Exp (13) = $a_0 + 13 * a_1 + 169 * a_2$

Exp(x) from (10-13) ≅ $g_1(x) = 6334310.279 – 1224580.194*x + 59335.18128*x^2$

*Second with five terms our points [10, 10.75, 11.5, 12.25 and 13]

Exp (10) = $a_0 + a_1*10 + a_2*100 + a_3*1000 + a_4*10000$

Exp (10.75) = $a_0 + a_1*10.75 + a_2*115.2625 + a_4*1242.296875$

Exp (11.5) = $a_0 + a_1*11.5 + \ldots\ldots\ a_4*11.5\text{^}4$

Exp (12.25) = $a_0 + a_1*12.25 + \ldots\ a_4*12.25\text{^}4$

Exp (13) = $a_0 + a_1*13 + \ldots\ldots\ldots\ a_4*13\text{^}4$

Exp(x) from (10-13) with 5 terms from the series ≅ $g_2(x) = 55710095.824076833-21004470.26371250198*x +2980129.2869905486104*x^2-188810.995060626634*x^3 +4515.46996404147657*x^4$

Now let's make our comparison between exp(x) from x=10 to x=13 VS $g_1(x)$ which represent the generated function with three terms VS $g_2(x)$ which represent the generated function with five terms to see the differences

| Value of x | Exp(x) values | $g_1(x)$ values | $g_2(x)$ values |
|---|---|---|---|

| | | | |
|---|---|---|---|
| 10.00000 | 22026.46579 | 22026.46700 | 22026.46579 |
| 10.50000 | 36315.50267 | 17921.97812 | 35903.41854 |
| 11.00000 | 59874.14172 | 43485.07988 | 60127.96693 |
| 11.50000 | 98715.77101 | 98715.77228 | 98715.77101 |
| 12.00000 | 162754.79142 | 183614.05532 | 162455.69577 |
| 12.50000 | 268337.28652 | 298179.92900 | 268909.81114 |
| 13.00000 | 442413.39201 | 442413.39332 | 442413.39201 |

We can see the difference Cleary between $g_1(x)$ values and $g_2(x)$ values VS the original function exp (x) it's clear that if we increase more than five terms or take narrower intervals the accuracy goanna be increased. So it's preferred before taking any decision to study enough or try to predict the change in the values patterns.

*in figures (6) I collect some functions VS their generated functions to show how the generated functions can be useful and effective.

**2-The second part in this research: generated function for solving equations:**

**But our question here before we start in this sector is what kind of equations?**

We will show now our meaning and our target with the below simple example.

**Ln (x) – x $^2$ + x = 0 how to be solved?**

Of course we can solve this equation by graph method or by computer, in my research I have another idea that will solve this equation by another methods using g(x).

From the first we know that sameh generated

function = g (x) = $\sum_{n=0}^{n=\infty} a_n \, \mathbf{x}^n$

So any function can be represented by an infinite series

$a_0 x^0 + a_1 x + a_2 x^2 + a_3 x^3$ ……..

and using the properties we shown that this infinite series can be limited and also we have a trade of between number of terms in the series and the accuracy putting the width of the interval in our consideration so If we say that we want to represent any function by $a_0 x^0 + a_1x^1+a_2x^2$ then we can solve any second order equation then we can solve the equation which we will assume it f(x) and we get g(x) which is the generated function for f(x) and solve g(x) to get the value of (x) that solve the equation.

And to be organized I will take an example and walk step by step to show my idea

Step (1): $\ln(x) + x^2 - 2x = 0 = a_0+a_1x+a_2x^2$

Step (2): taking points and substitute in the f(x) (the equation) to decide our interval to be sure that this interval cross x axis (y=0) to find after that the value of x that make

the equation = 0

Step (3): gets g(x)

Step (4): get the value of (x) by solving g (x) (second order equation).

Let's see with the pervious example what I meant step by step

Step (1): Ln (x) + x^2 – 2x = 0 = f(x) ≅ g(x) = $a_0+ a_1x+a_2x^2$

Step (2): we look to f(x) graph or substitute with some points in the equation to know the interval that this function cross x axis and let us take the hardest way by substituting with some points

at $\longrightarrow$ x =0 f(x) - ∞

at x =10 f (x) + 80.30

at x = 5 f (x) 16.6

at x= 1 f(x) -1

So it's between (1) and (5)

We can take the interval from (1) to (5) or find narrower interval

F(x) = 4.0986

So it's between 1 and F(x) = 5.6931

So it's between 1 and 2

Step (3): so we know our interval

from x=1 to x=2

So $f(x) = \text{Ln}(x)+x^2-2x$ From x =1 to x = 2

We will take three values to get g(x)

At x = 1, 1.5 and 2.

$-1 = a_0 + a_1 + a_2$
$-0.34453 = a_0 + a_1 * 1.5 + a_2 * 2.25$
$0.6931 = a_0 + a_1 * 2 + a_2 *4$

Then after solving the above three equations we can deduce our generated function
$g(x) = - 1.16446 - 5.59986*x + 5.76432*x^2$
Step (4): let g(x) =0. And by solving it
$x \cong 1.6876$ (1)
By computer $x = 1.6895$ (2) (1) ≅ (2)
$x \cong 1.6875$ is an approximate solve for
$\text{Ln}(x) + x^2 – 2x = 0$.
*Now I will make a detailed description and notes about the 4 steps.

**Step (1):** in this step we start to assume our g(x) from (number of terms) we can assume g(x) more than three terms if we can solve it like put the equation which represent $f(x) = a_0 + a_1 x+ a_2 x^2+ a_3 x^3$ but in the normal we put the equation = 0 = $f(x) \cong g(x) = a_0 + a_1 x+ a_2 x^2$ because a second order equation is always solvable by easy methods .

**Step (2):** now we know that f(x) will be replaced with g(x) so we have to use the limitation properties by selection a appropriate interval this interval must cover the part of the function (our equation) that cross x axis to be sure that this interval contain a specific value that make g(x) ≅ o to make this step we have there options.

**Option(1) :** you have a graph or figure of the function ( the equation ) so you can know or predict that for example the f (x) cross x axis between x=4 and x=6 so we can directly obtain our interval.

**Option (2):** you know from the begging the expected interval contain the part that cross x axis because of a certain condition or scientific reason for what this equation represent. So in this case we start directly to get our points and get g (x).

**Option (3):** we didn't have either option (1) or option (2) then we have to substitute with several points to deduce the interval that cover the equation and contain the x value that make it =0.
at $x = x_0$ $f(x) = A$
at $x = x_{00}$ $f(x) = B$
and our interval between positive + and negative – value of f (x) like.
at $x = 4$ $f(x) = +2$
at $x = 5$ $f(x) = -1$
Then f(x) between (4-5) cross x axis and this is our interval.

**Step (3):** after getting the wanted interval we must get g(x) which normally $\cong a_0 + a_1 x + a_2 x^2$ by getting three different values to f(x) ( the equation ) like the below example

$\ln(x) + x^2 – 2x = 0$ from x =1 to x =2

We will take $\ln(x) + x^2 – 2*x$ values at x =1, 1.5 and 2

$\ln(1) + 1^2 – 2*1 = a_0 + a_1*1 + a_2*1$

$\ln(1.5) + (1.5)^2 – 2*1.5 = a_0 + a_1* 1.5 + a_2*(1.5)^2$

$\ln(2) + (2)^2 – 2*2 = a_0 + a_1*2 + a_2*(2)^2$

Then we get g(x)

**Step (4):** by solving the second order equation by its general rule ($- b \pm \sqrt{b^2-4\cancel{ac)/(2a}}$) or by calculator we figure out x vale that make
g(x) = 0 and here f(x) ≅0
*Now we will see sets of examples covering this application

**<u>Ex (1.A)</u>**

$x^7+ x^3 + \exp(x) – 3.73286516*10^3=0$

The expected value of x between 31 & 32

Sol.

$g(x)= a_0 + a_1 x+ a_2 x^2 \cong f(x)$ = above equation taking values of f(x) at 31,31.5 and 32 then we have the next three equations

$-8.252199291*10^{12} = a_0+31*a_1+31*31*a_2$

$1.059557805*10^{13} =a_0+31.5*a_1+31.5*31.5*a_2$

$4.166875479*10^{13} = a_0+32*a_1+32*32*a_2$

Then after solving the above three equations

$g(x)=2.44507988*10^{13} * x^2$
$– 1.49047937*10^{15}*x$
$+2.269939063*10^{16} = 0$

Then $x \cong 31.25939$ ⟶ (1)

By computer using mat lab x = 31.25 (2)…… (1) ≅ (2)

**<u>Ex (2.A)</u>**
Looking at the figure (7) solve the equation.

$x^7+x^3+\exp(x)-40427713.3=0$

Sol.
$g(x) =a_0+a_1x+a_2* x^2 \cong$ the equation above & by looking at the graph the wanted interval is locating between

x=12 & x=13 by taking three different values for the equation in this interval.

$-4431422.509=a_0+12a_1+144a_2$

$7526292.932=a_0+12.5a_1+12.5*12.5a_2$

$22765414.09=a_0+13a_1+13*13a_2$

Then $g(x) = 6562811.434*x^2-136873449.3*x+693005122=0$

Then $x\cong12.2018$….(1) & by computer x=12.2…(2) so (1) ≅ (2)

**Ex (3A)**

The next equation can be solved by x value that locate between x =100,110

$120*x^3+\ln(x)-x^5+1.311436913*10^{10}=0$

At x= 100 $\longrightarrow$ F(x) =3234369135

At x=105 $\longrightarrow$ F(x) =490468509

At x =107 $\longrightarrow$ F(x) =764143012.

So it's between x = 101,107

At x =106 $\longrightarrow$ F(x) = - 124964721

It's location between x = 106, x = 105

$490468479.7=a_0+105*a_1+105*105*a_2$

$18567835.6=a_0+105.5*a_1+105.5*105.5*a_2$

$-124964721.3=a_0+106*a_1+106*106*a_2$

$g(x)=0= 6.56362344*x^2-1.391867787*10^3*x+7.374585247*10^4$
$x\cong105.53$ by computer = 105.8

**Ex(4.A)**

Solve the below equation

$x^3-x^2+x-60=0$

Sol.

$g(x) = a_0+a_1x+a_2x^2$ & $f(x) = x^3-x^2+x-60=0$

At x = 0 $\longrightarrow$ f(x) = -6

At x = 100 $\longrightarrow$ f(x) =990040 >>

At x =10 $\longrightarrow$ f(x)=850>>

At x =3 $\longrightarrow$ f (x)=-39

At x =4 $\longrightarrow$ f (x)=-8

At x =5 $\longrightarrow$ f (x)=45

Then our interval between x = 4.5

Taking F(4) . F(4.5) and F(5)

$-8 = a_0+4a_1+16a_1$

$15.375 = a_0+4.5a_1+4.5*4.5\ a2.$

$45 = a_0+5a_1+25C_2$

Then $g(x) = 12.5x^{2-}-59.5x+30=0$

After solving $x\cong4.186 \longrightarrow 1$

by computer x=4.185 $\longrightarrow$ 2 ………(1)≅(2)

**Ex (5.A)**

Solve $5x^6+x^{3-}20x^2-10=0$

Sol.
$g(x)=0= a_0+a_1x+a_2x^2$ & $F(x)= 5x^6+x^{3-}20x^2-10$

At x=0 $\longrightarrow$ F(x) = -10

At x=100 $\longrightarrow$ $F(x) = 5*10^{12}$ >>>

At x=10 $\longrightarrow$ F(x) = 4998990 >>

At x=3 $\longrightarrow$ F(x) = 3482

At x=2 $\longrightarrow$ F(x) = 238 √√

At x=1 $\longrightarrow$ F(x) = -24

Our interval between x=1, x =2

By taking the values of F(1).F(1.5 )and F(2) to get g(x)
$-24 = a_o + a_x + a_2$

$341/64= a_o + 1.5a_x + 1.5*1.5a_2$

$238 = a_o + 2a_1 + 4a_2$

Then $g(x) = 0 = 406.687x^2- 958.0525x + 527.375$

Then $x\cong1.4789 \longrightarrow$ (1)

by computer x=1.457 $\longrightarrow$ (2)
…………… (1)≅(2)

**Ex (6.A)**

Solve $x^8 - x^5 - x^2 - 100 = 0$

Sol .
g(x) $\longrightarrow$ second order = 0

At x=0 $\longrightarrow$ F(x) = -100

At x=100 $\longrightarrow$ F(x) 9.9…*$10^{15}$ >>>

At x=10 $\longrightarrow$ F(x) = 99899800 >>>>

At x=3 $\longrightarrow$ F(x) = 6209 >>

At x=2 $\longrightarrow$ F(x) = 120 √√

At x=1 $\longrightarrow$ F(x) = -101√

The wanted interval that cross x axis

Between x=1 and x=2

F(1) , F(1.5) and F(2) values to get g(x)

$-101 = a_o + a_1 + a_2$

$-(21559/256) = a_o + 1.5a_x + 1.5*1.5a_2$

$120 = a_o + 2a_1 + 4a_2$

Then after solving the above three equations

$g(x) = 0 = (2399x/64)\, x^2 - (27829/64)\, x + (13667/32)$

After solving g (x) $\Longrightarrow$ x ≅ 1.7648 (1) By computer x=1.8260 $\Longrightarrow$ (2) $\longrightarrow$ ………… (1)≅(2)

*In all of the next examples we assume $g(x) = a_o + a_1x + a_2x^2$ and F(X) = the equation that needed to be solved

**EX (7.A)**

$X^5 - 50x^3 + x - 50 = 0$

Sol.

At x = 0 $\longrightarrow$ f (x) = -50

At x = 100 $\longrightarrow$ f (x) = 9950000050 >>

At x = 20 $\longrightarrow$ f (x) = 2799970 >>

At x = 10 $\longrightarrow$ f (x) = 49960 >

At x = 5 $\longrightarrow$ f (x) = -3170

At x = 6 $\longrightarrow$ f (x) = -3068

At x = 7 $\longrightarrow$ f (x) = -386 √

At x = 8 $\longrightarrow$ f (x) = 7126 √

Our interval between x= 7, x = 8 & our points f (7) , f (7.5) & f (8)

$-386 = a_0 + 7a_1 + 49a_2$

$(83015 / 32) = a_0 + 7.5a_1 + 7.5*7.5a_2$

$7126 = a_0 + 8a_1 + 64a_2$

$g(x) = 0 = -39034.875x + 120805 + 3103.125x^2$

x =7.0827 by computer x = 7.07961

**EX (8.A)**

$x^5 - x^4 - x^3 - x^2 - x - 10000 = 0$

Sol.

at x = 0 $\longrightarrow$ f (x) = -10000

at x = 100 $\longrightarrow$ f (x) = 989898000 >>>>

at x = 50 $\longrightarrow$ f (x) = 306112450 >>>>

at x = 10 $\longrightarrow$ f (x) = 78890 >

at x = 5 $\longrightarrow$ f (x) = -7655 <

at x = 6 $\longrightarrow$ f (x) = -3778 √

at x = 7 $\longrightarrow$ f (x) = 4007 √

The needed interval between x = 6 , x = 7 and the three values f (6), f (6.5) and f (7)

$-3778 = a_0 + 6a_1 + 36a_2$

$-505.53125 = a_0 + 6.5a_1 + 6.5*6.5a_2$

$4007 = a_0 + 7a_1 + 49a_2$

Then $g(x) = 0 = 2480.125x^2 - 24456.625x + 53677.25$

x = 6.56364 by computer x = 6.56442

**EX (9.A)**

$x^5 - 100x^4 - 100x^3 - x^2 - 10000 = 0$

Sol.

At x = 0 $\longrightarrow$ f (x) = -10000

At x = 100 $\longrightarrow$ f (x) = -100020000

At x = 200 $\longrightarrow$ f (x) = 1.591905*$10^{11}$ >>>

At x = 150 $\longrightarrow$ f (x) = 2.497…*$10^{10}$ >>>

At x = 110 $\longrightarrow$ f (x) = 1.58255*$10^{10}$ >>>

At $x = 103$ ⟶ $f(x) = 228359334$ >

At $x = 101$ ⟶ $f(x) = 1010100$

Our interval cover $f(x)$ from $x = 100$ to $x = 101$ and our vales $f(100)$, $f(100.5)$ and $f(101)$
$-100020000 = a_0 + 100a_1 + 10000a_2$
$-50520087.72 = a_0 + 100.5a_1 + 100.5*100.5a_2$
$1010100 = a_0 + 101a_1 + 101*101a_2$
$g(x) = 0 = 4.06055088*x^2 - 715.1406269*x + 3.080853389$
$x = 100.99038$ by computer $x = 100.990357$

**EX (10.A)**

$x^6 - x^5 - x^4 - x^3 - 50000 = 0$

At $x = 0$ ⟶ $f(x) = -50000$

At $x = 100$ ⟶ $f(x) = 9.8989895*10^{11}$ >>>

At $x = 20$ ⟶ $f(x) = 60582000$ >>

At $x = 10$ ⟶ $f(x) = 839000$ >

At $x = 5$ ⟶ $f(x) = -38250$

At $x = 7$ ⟶ $f(x) = 48098$ √

At $x = 6$ ⟶ $f(x) = -12632$ √

Our interval between $x = 6$ and $x = 7$ values of $f(x)$ are $f(6.5)$ and $f(7)$
$-12632 = a_0 + 6a_1 + 36a_2$
$11756.29688 = a_0 + 6.5a_1 + 6.5*6.5a_2$
$48098 = a_0 + 7a_1 + 49a_2$
$g(x) = 0 = 23906.81248*x^2 - 250058.5622*x + 627074.1242$
$x = 6.2888$ and by computer $x = 6.2845$

**EX (11.A)**

In the example (10.A) solve this equation by finding –ve value of x that satisfy the equation there

At $x = 0$ ⟶ $f(x) = -50000$

At $x = -100$ ⟶ $f(x) = 1\ldots x10^{12}$ >>>>

At $x = -20$ ⟶ $f(x) = 66998000$ >>>>

At $x = -10$ ⟶ $f(x) = 1041000$ >>

At $x = -5$ ⟶ $f(x)$ -31750 √

At $x = -6$ ⟶ $f(x) = 3352$ √

The interval needed between x= -5.x=-6

And the points needed f (5).f (-5.5 ) and f(-6)

$-31750 = a_0+(-5)\,a_1+ (-5*-5)a_2$

$-18035.20313 = a_0+(-5.5)\,a_1+ (-5.5*-5.5)a_2$

$3352 = a_0+(-6)\,a_1+ (-6*-6)a_2$

$g(x)= 15344.81252x^2 + 133690.9377x + 253084.3756$
$x \cong -5.93215361$ and by computer $x = -5.9342$.

**E x (12.A )**

$x^5 - 1000x^4 - 1000x^3 - 1000x^2 - 300000 = 0$

At x= 0 f(x) = -300000

At x= 100 f(x) =- 9.00*10^10

At x= 200 f(x) = -1.200*10^12

At x= 1000 f(x) = -10000* 10^17

At x= 10000 f(x) = -8.99900*10^19 >>>

At x= 5000 f(x) = 2000x10^18 >>>

At x= 2000 f(x) = 1.599000*10^16 >>>

At x= 1500 f(x) = 2.5200x10^15 >

At x= 1100 f(x) = 1.45000*10^14 >

At x= 1010 f(x) = 9.374000*10^12

At x= 1001 f(x) = 702001
Our interval between x= 1000 and x= 1001 our three points values f(1000), f(1000.5) and f(1001)
$-1.0010003 * 10^{12} = a_0 + 1000a_1+ 1000*1000a_2$
$-5.01513001* 10^{11} = a_0+ 1000.5a1+ 1000.5*1000.5a_2$
$702001= a_0 + 1001a_1+1001*1001a_2$
$g(x)=0= 4.006004400x^2 - 7.015013808*10^3\,x +3.008008404*10^6$

$x = 1001.00002$ and by computer $x=1000.99999$

**E x (13.A)**

$x^4 + \ln(3x) - \log_{10}(x) - 66000=0$

Sol.

At x = 0 f(x)= -∞

At x =100 f(x)=99934003.7 >>>

At x = 30 f(x)= 6184003.312>>

At x = 20 f(x)= 94Ø02.79331>

At x = 10 f(x)= -55997.5988

At x = 15 f(x)= -15372.36943

At x = 17 f(x)= 17523.70138 √

At x = 16 f(x)= -461.332919 √
Our interval between x=16, x= 17
Our points value are F(16), F(16.5) and F(17)
$- 461.332919 = = a_0+16a_1+ 16*16a_2$
$8122.746989 = a_0+16.5a_1+ 16.5*16.5a_2$
$17523.70138 = a_0+17a_1+ 17*17a_2$
Then $g(x)= 0 =16633.748966*x^2$
$- 35928.68158*x+156157.837$
After solving g(x)
x ≅ 16.028 by computer x= 16.028

**Ex(14.A)**

$x^3+1000*\sin(x) - 59500=0$

Sol.
note : x inside the sin = x*180/ 3.14

At x = 0 f(x) = -59500

At x =100 f(x)=939959.3592

At x = 50 f(x)= 65218.26744

At x = 20 f(x)=-50590.368

At x = 30 f(x)= -33489.82144

At x = 40 f(x)= 5255.749574

At x = 39 f(x)= 778.4929736 √

At x = 38 f(x)= -4346.267443 √

Then our interval between x= 38 and x= 39 and our values to get g(x) & F(38). F(38.5) and f(39)
$-4346.267443= a_0+ 38a_1 + 38*38a_2$
$- 1726.268219= a_0+ 38.5a_1+ 38.5*38.5a_2$
$778.4929736= a_0+ 39a_1 + 39*39a_2$
Then g(x) =0=
$-230.4760628*x^2+22871.41725*x-540652.6883$
x ≅38.8421 and by computer x= 38.83989

I think that it is enough examples to show this application of g (x) but before we leave this sector and move on to the next one I have an important note.
In the last example EX(14,A) we have sin (x) in one of the equation terms. The problem can be appear Cleary if we speak about equation this type of function control it and control the shape of the curve because in this case the function cross x axis infinite number of times so its preferred to have a graph for this types at equations to be sure that its not controlled by these types of function.

**3 -The third sector in this research, Sameh generated function as anew numerical method for integration process in calculus:**

In this sector we will discuss the second application I found for sameh generated function. By converting any f (x) to series of $a_nx^n$ then we can use the result of conversion to make a numerical integration we know that

$$\int_a^b a_nx^n = (a_n/(n+1))*( x^{n+1} )_a^b$$

Which mean that g (x) always easily to be integrated so we will deal with this numerical integration from two points of views. One of them about the length of series $g (x) = a_0 + a_1x$ or $g (x) = a_0 + a_1x + a_2x^2$

Or $g (x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$ and the second point of view is about the step or the subintervals (width of the interval).
Now we will see the main idea of this application in the next example f (x) = ln (x) and we want to integrate it from (3) to (4) then we can do this integration normally but we concentrate in numerical methods so we will speak about our numerical method step by step.

**Step (1)** the choice of the width of interval to know the number of intervals inside the integration limits.

**Step (2)** start solving the intervals and gets the generated functions that cover f (x) in the integration limits.

**Step (3)** Make the integration process of $g_1(x)$ , $g_2(x)$….

And make a summation to them.

Now I take some examples and after that I will make a general comparison between my method and other numerical methods in numerical integration.

**EX (1.B)**

$\int$ ln (x) dx from x = 4 to x = 7

**Step (1)**: Ln (x) is a slowly changeable function so I will take the step with (1) unit in x-axis which mean that we will have three subintervals and also 3 generated function.

**Step (2):** to get $g_1(x)$ from x = 4 to x = 5
$Ln(4) = 1.38629 = a_0 + 4a_1 + 16a_2$
$Ln(4.5) = 1.50408 = a_0 + 4.5a_1 + 4.5*4.5a_2$
$Ln(5) = 1.60944 = a_0 + 5a_1 + 25a_2$
After solving the three equations above.
$g_1(x) = -3.51/1000 + 0.44689*x - 0.02486*x^2$
To get $g_2$ (x) from x = 5 to x = 6
$Ln(5) = 1.60944 = a_0 + 5a_1 + 25a_2$
$Ln(5.5) = 1.70475 = a_0 + 5.5a_1 + 5.5*5.5a_2$
$Ln(6) = 1.79176 = a_0 + 6a_1 + 36a_2$
$g_2$ (x) after solving the above three equations = $0.19984 + 0.36492x - 0.0166x^2$
to get $g_3$ (x) from x = 6 to x = 7
$\ln(6) = a_0 + 6a_1 + 36a_2$
$\ln(6.5) = a_0 + 6.5a_1 + 6.5*6.5* a_2$
$\ln(7) = a_0 + 7a_1 + 49a_2$
$g_3(x) = 0.36874 + 0.30833*x - 0.01186*x^2$

**Step (3):** $\int \ln(x)\,dx$ from x = 4 to x = 7

$= \int g_1(x)$ from x = 4 to x = 5

$+ \int g_2(x)$ from x= 5 to x = 6

$+ \int g_3(x)$ from x = 6 to x = 7

Using the rule $\int a_n x^n = (an/n+1) * x^{n+1}$

$\int \ln(x)$ from x = 4 to x = 7 = 1.502 + 1.703 + 1.870

= 5.075 the below table show a compare between our result with three subintervals and other numerical methods with 100 & 10000 sub intervals!!

| Result of integration process | Method of numerical integration |
|---|---|
| 5.075 | Generated function with three subint. |
| 5.06779 | Left endpoint with 100 subint. |
| 5.07620 | Midpoint with 100 subint. |
| 5.08458 | Right endpoint with 100 subint. |
| 5.07619 | Trapezoid with 100 subint. |
| 5.07619 | Parabola with 100 subint. |
| 5.07611 | Left endpoint with 10000 subint. |
| 5.07619 | Midpoint with 10000 subint. |
| 5.07628 | Right endpoint with 10000 subint. |
| 5.07619 | Trapezoid with 10000 subint. |
| 5.07619 | Parabola with 10000 subint. |

By three sub intervals only we reach to such accuracy of course if we take taller series $a_0 + a_1x + a_2x^2 + a_3x^3$…….. or more sub intervals we get more accuracy that with other methods need thousands of subintervals. We can also see how 100 subintervals with the other methods in a ln(x) function that change slowly didn't reach the accuracy of my method with three terms only with three subintervals!!!.

**EX (2.B)**

$\int Exp(x)/x\,dx$ from x=10 to x=12

We will take two subinterval to get first g(x) we will put in exp (x)/x the next values 10,10.5

and 11
$2202.64658 = a_0 + 10a_1 + 100a_2$
$3458.61930 = a_0 + 10.5a_1 + 10.5*10.5a_2$
$5443.10379 = a_0 + 11a_1 + 12a_2$

Then $g_1(x) = 130070.6639 – 27357.03713*x + 1457.02354*x^2$
To get $g_2(x)$ we will do the same but with the points 11,11.5 and 12
$5443.10379 = a_0 + 11a_1 + 121a_2$
$8583.98009 = a_0 + 11.5a_1 + 11.5*11.5a_2$
$13562.89928 = a_0 + 12a_1 + 144a_2$
Then $g_2(x) = 401368.6764 – 76430.17745*x + 3676.08578*x^2$The result of integration

$$= \int_{10}^{11} g_1(x) + \int_{11}^{12} g_2(x) = 12470.35856$$

By computer midpoint method (100000 subintervals) = 12467.30369
In the Ex (2.B) we take two sub interval in a function that increased strongly and the error only 0.00024% .What happen if we increased the number of intervals ( sub intervals ) that's what we will see in the next example.

**EX (3.B)**

$\int$ Exp(x) d x from 15 to 16

We will solve it by

(a) 1 sub interval
(b) 2 sub interval
(c) 4 sub interval
(d) 8 sub interval

And we will compare at last all the results with each other and VS the true solution result.

Sol.

(a) $\int$ Exp (X) dx from 15-16 in one interval Exp(15)

$=3269017.372 = a_0 +15a_1+225a_2$

$Exp(15.5)=5389698.476=a_0+15.5a_1 +15.5*15.5a_2$

$Exp(16) = 8886110.521 = a_0+16a_1+16256a_2$

Then $g(x) = 579363471.8 – 79678225.19*x +2751461.882*x^2$

$\int$ g(x) from (15-16) = 5618775.02

(b) $\int$ Exp(x) from 15-16 in two subintervals

To get $g_1(x)$ (intervals 15-15.5).

$Exp(15) = 3269017.372 = a_0+ 15a_1+15*15a_2$

$Exp(15.25)=4197501.394=a_0+15.25a_1 +15.25*15.25a_2$

$Exp(15.5)=5389698.476=a_0+15.5a_1 +15.5*15.5a_2$

Then $g_1(x) = 430154875.9 – 60104624.43*x + 2109704.48*x^2$

$Exp(15.5)=5389698.476=a_0+15.5a_1 +15.5*15.5a_2$

$Exp(15.75)=6920509.832=a_0+15.75a_1 +15.75*15.75a_2$

$Exp(16) = 8886110.521 = a_0+ 16a_1+256a_2$

Then $g_2(x) = 759622961.8 – 102574087.8*x + 3478314.664*x^2$

The result $\int_{15}^{15.5} g_1(x) + \int_{15.5}^{16} g_2(x)$

=2120726.8245+3496487.58866=5617214.413

(c) $\int$ Exp (x) from 15 to 16 (4 subintervals) To get

$g_1(x)$ its interval from x=15 to x=15.25

$Exp(15) = 3269017.372 = a_0+ 15a_1+15*15a_2$

$Exp(15.125)=3704281.979=a_0+15.125a_1 +15.125*15.125a_2$

$Exp(15.25)=4197501.394=a_0+15.25a_1 +15.25*15.25a_2$
Then $g_1(x) = 371789170.6 – 52386318.06*x +1854553.856*x^2$
To get $g_2(x)$ its interval from (15.25-15.5)

$Exp(15.25)=4197501.794=a_0+15.25a_1 +15.25*15.25a_2$

$Exp(15.375)=4756392.211=a_0+15.375a_1 +15.375*15.375a_2$

$Exp(15.5)=5389698.476=a_0+15.5a_1+15.5*15.5a_2$

Then $g_2(x) = 494351928.1 - 68456012.5*x + 2381294.336*x^2$

To get $g_3(x)$ which cover the interval from (15.5-15.75)

$Exp(15.5)=a_0+15.5a_1+15.5*15.5a_2 =5389698.476$

$Exp(15.625)=a_0+15.625a_1+15.625*15.625a_2 = 6107328.491$

$Exp(15.75)=a_0+15.75a_1+15.75*15.75a_2 =6920509.832$

Then $g_3(x) = 656926353.1 – 89428080.58*x +3057642.432*x^2$

To get $g_3(x)$ which cover the interval from (15.75-16)

$Exp(15.75)=a_0+15.75a_1+15.75*15.75a_2 = 6920509.832$

$Exp(15.875)=a_0+15.875a_1+15.875*15.875a_2 = 7841965.01$

$Exp(16) = a_0+ 16 a_1+16*16a_2 = 8886110.571$

Then $g_4(x) = 872462511.7-116790975.6*x + 3926090.656*x^2$

The result of integration

$$\int_{15}^{15.25} g_1(x) + \int_{15.25}^{15.5} g_2(x) + \int_{15.5}^{15.75} g_3(x) + \int_{15.75}^{16} g_4(x)$$

$=928485.25029+1192198.72079 +1535813.40837+1965603.22933 = 567100.609$

(d)We will make (8) g(x) to cover exp(x) from x=15tox=16

To get $g_1(x)$ x=15-15.125

$Exp(15) =3269017.37247 = a_0+15a_1+15*15a_2$

$Exp(15.0625)=3479850.87910=a_0+15.0625a_1 +15.0625*15.06125a_2$

$Exp(15.125)=3704281.97867=a_0+15.125a_1 +15.125*15.125a_2$

Then $g_1(x) = 345911363.6 – 48950201.53*x +1740491.896*x^2$

To get $g_2(x)$ x=15.125-15.25

$Exp(15.125)=3269017.37247=a_0+15.125a_1 +15.125*15.125a_2$

$Exp(15.1875)=3479850.87910=a_0+15.1875a_1 +15.1875*15.1875a_2$

$Exp(15.25)=3704281.97867=a_0+15.25a_1 +15.25*15.25a_2$

Then $g_2(x) = 398933223.2 – 55960904.04*x +1972235.699*x^2$

To get $g_3(x)$ x= (15.25 – 15.375)

$Exp(15.25) = a_0+ 15.25a_1+15.25*15.25a_2$

$Exp(15.3125)=a_0+15.3125a_1 +15.3125*15.3125a_2$

$Exp(15.375)=a_0+15.375a_1+15.375*15.375a_2$

Then $g_3(x) = 460011955.4 – 63970720.79*x +2234835.831*x^2$

To get $g_4$ x = (15.375-15.5)

$Exp(15.375) = a_0+ 15.375a_1+15.375*15.375a_2$

$Exp(15.4375)=a_0+15.4375a_1 +15.4375*15.4375a_2$

$Exp(15.5) = a_0+15.5a_1+15.5*15.5a_2$

Then $g_4(x) = 530362478.4 – 73121423.44*x + 2532400.763*x^2$

To get $g_5(x)$ x= 15.5-15.625

$Exp (15.5) = a_0+15.5a_1+15.5*15.5a_2$

$Exp(15.5625)=a_0+15.5625a_1 +15.5625*15.5625a_2$

$Exp (15.625) = a_0+15.625a_1+15.625*15.625a_2$

Then $g_5(x) = 611381438.4 – 83574824.43*x + 2869586.009*x^2$

To get $g_6(x)$ x= 15.625 – 15.75

$Exp (15.625) = a_0+15.625a_1+15.625*15.625a_2$

$Exp(15.6875)=a_0+15.6875a_1 +15.6875*15.6875a_2$

$Exp (15.75) = a_0+15.75a_1+15.75*15.75a_2$

Then $g_6(x) = 704674573.8 – 95575599.76*x +3251666.948*x^2$

To get $g_6(x)$ x= 15.75 – 15.875

$Exp (15.75) = a_0+15.75a_1+15.75*15.75a_2$

$Exp(15.8125)=a_0+15.8125a_1 +15.8125*15.8125a_2$

$Exp\ (15.875) = a_0+15.875a_1+15.875*15.875a_2$

Then $g_7(x)= 3684621.371*x^2 – 109154509.4*x + 812087644.4$

To get $g_8$ (x) x= 15.875 -16

$Exp\ (15.875) = a_0+15.875a_1+15.875*15.875a_2$

$Exp(15.9375)=a_0+15.9375a_1 +15.9375*15.9375a_2$

$Exp\ (16) = a_0+16a_1+16*16a_2$

Then $g_8(x) = 935742128.9 - 12432069.3*x + 4175223.007*x^2$

The total result

$$= \int g_1(x)+ \int g_2(x)+ \int g_3(x) +\int g_4(x)$$

$$+\int g_5(x) + \int g_6(x)+ \int g_7(x)+ \int g_8(x)$$

By making this integration for each g(x) in its own intervals the result = 435264.62767+493219.45011+558890.85286+633356.30809+717630.0541+813181.41434 +921455.30208+1044145.52030=5617093.541

* We will make a comparison between the result from (a) .(b) .(c) and (d) from the above example VS other numerical methods using (1000) subintervals result VS exact result by normal method VS exact

$\int$ exp(x) = exp (x) .

| The result value | limit | Function to be integrated | Method |
|---|---|---|---|
| 561877.502 | 15 to 16 | Exp(x) | (a) in EX(3.b) |
| 5617214.413 | 15 to 16 | Exp(x) | (b) in EX(3.b) |
| 5617100.609 | 15 to 16 | Exp(x) | (c) in EX(3.b) |
| 5617093.541 | 15 to 16 | Exp(x) | (d) in EX(3.b) |
| 5614285.06955 | 15 to 16 | Exp(x) | Left endpoint |
| 5617092.91399 | 15 to 16 | Exp(x) | Midpoint |
| 5619902.16270 | 15 to 16 | Exp(x) | Right endpoint |
| 5617093.61613 | 15 to 16 | Exp(x) | Trapezoid. |
| 5617093.14804 | 15 to 16 | Exp(x) | Parabola |
| 5617093.148 | 15 to 16 | Exp(x) | Normal (exact) |

By looking at the previous table we can deduce that in (d) in ex(3.b) we reach to highest accuracy with 8 subintervals that the other numerical methods didn't achieve with 1000 subintervals!!!!! .Also we can deduce that in a function with high values change its very safe to take a constant step in any function needed to be integrated and I think it will be better if we take (1/8) as a step like ex(3.B) the part (d).I make the MATLAB code in the folder (2) and activate it in the matlab program so I will make a large table with different examples and compare with other numerical method .

The (matlab) code divide any integration limit to steps each is (1/8) over x axis.

**E.x(4.B)**

Multiple functions with different integration limits compared with other numerical methods result.

First to make the table organized I will give the functions that I will make the integration process on it a numbers in the next table then I will make the table which include the result and the comparison see also figures (8)

| Given number | Function to be integrated |
|---|---|
| 1 | Ln(x) |
| 2 | x*sin(x) |
| 3 | ln(x)*sin(x) |
| 4 | (ln(x^2))^3/(x^2) |
| 5 | (ln(x^2))^3/sin(x) |
| 6 | 1/sin(x) |
| 7 | x^ln(x) |
| 8 | x^sin(x^2) |
| 9 | sin(x)^ln(x) |
| 10 | x*sin(x)*ln(x) |
| 11 | (2^x)/(x^6) |
| 12 | (x*sin(x))/cos(1/x^2) |
| 13 | sin(x)/1.1^x |
| 14 | sin(x)/1.1^x |
| 15 | ln(1000+100*x^2+x^4) |

In the next table:

- (N.) means the number
- The first column N. include the functions to be integrated according to their number which we gave in the past table
- The second column include the limits of the integration process by the order first limit, second limit
- The last column (Subint. N.) include the number of subintervals that the other numerical methods take to get the accurate results in the 3,4,5,6 and 7$^{th}$ columns
- The font is small because I mean to put all this result together but I put this table by clear font in the figures

| Subint. N. | parabola | trapezoid | Right endpoint | midpoint | Left endpoint | Generated function step = 1/8 | limits | N. |
|---|---|---|---|---|---|---|---|---|
| 1000 | 9.97866 | 9.97866 | 9.98039 | 9.97866 | 9.97693 | 9.97866 | 5,10 | 1 |
| 1000 | 0.29559 | 0.29559 | 0.31747 | 0.29558 | 0.27371 | 0.29558 | 4,8 | 2 |
| 1000 | -0.75935 | -0.75935 | -0.75314 | -0.75935 | -0.76556 | -0.75935 | 12,19 | 3 |
| 1000 | 8.42588 | 8.42588 | 8.42518 | 8.42589 | 8.42659 | 8.42588 | 3,10 | 4 |
| 1000 | 183.43666 | 183.43667 | 183.44828 | 183.43666 | 183.42505 | 183.43809 | 7,9 | 5 |
| 1000 | 2.51607 | 2.51607 | 2.51616 | 2.51607 | 2.51598 | 2.51609 | 7,9 | 6 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1000 | 35.57699 | 35.57701 | 35.62335 | 35.57698 | 35.53066 | 35.57698 | 2,6 | 7 |
| 1000 | 5.13046 | 5.13048 | 5.13010 | 5.13046 | 5.13086 | 5.13069 | 1,5 | 8 |
| 1000 | 1.57696 | 1.57696 | 1.57607 | 1.57696 | 1.57784 | 1.57696 | 1,3 | 9 |
| 10000 | -22.24584 | -22.24583 | -22.19386 | -22.24584 | -22.29779 | -22.24584 | 1,20 | 10 |
| 100000 | 2242667.19 | 2242667.2 | 2242810.37 | 2242667.19 | 2242524.02 | 2242667.22 | 33,55 | 11 |
| 100000 | 54.06650 | 54.06649 | 53.98985 | 54.06650 | 54.14304 | 54.06650 | 20,130 | 12 |
| 1000 | 2.28204 | 2.28202 | 2.26766 | 2.28204 | 2.29637 | 2.28204 | -12,-1 | 13 |
| 10000 | 2.02224 | 2.02224 | 2.02917 | 2.02224 | 2.01530 | 2.02224 | -20,5 | 14 |
| 10000 | 2962.41755 | 2962.417551 | 2962.41755 | 2962.41754 | 2962.41755 | 2962.41754 | -100,100 | 15 |

like we see in the previous table Ex(4.B) and figures (8) that using Sameh generated function in the numerical integration is very useful that replace more than 10000 operations some times to several of tens operations by computer. But there is another problem in numerical integration generally & our method specially and this problem because of some functions like (sin) or (cos) which has a specific frequency. If this frequency more than the sampling frequency it will be a problem and let's see the below example to know exactly the difference in .Ex (5.B)

By taking 1000 subintervals in the next functions

| parabola | trapezoid | midpoint | Right endpoint | Left endpoint | limits | function |
|---|---|---|---|---|---|---|
| 0.71634 | 0.71634 | 0.71634 | 0.71634 | 0.7183 | 0,5 | sin(x) |
| 0.01884 | 0.01844 | 0.01904 | 0.01727 | 0.01961 | 0,5 | sin(100x) |
| -0.01917 | -0.01906 | -0.01923 | -0.02156 | -0.01656 | 0,5 | sin(10000x) |
| 0.0332 | -0.042 | 0.07109 | -0.0519 | -0.0328 | 1,7 | 10*sin(1000000x) |

By looking to the result we can deduce that at sin(x) (low relative frequency lower than the sampling one) the result was accurate. At sin(100x) the accuracy decreased and at sin(10000x) it become low and wrong result at 10*sin(1000000x) so if we do my method it will be wrong faster because the sampling frequency in my method if we take the step =1/8 equal 8 per unit over x-axis. So we have two solutions to this problem

*First: take high sampling frequency that can cover the given function.

*Second: transfer the variables by make the frequency in the range.

Ex: $\int$ sin(1000x) dx from x=5 to x=7 by assuming

1000x=y

$$dx = \left(\frac{1}{1000}\right)dy$$

At x=2 y = 2000

At x=7

y = 7000

Then $\int_{2}^{7}$ sin(1000x)dx

$$= \int_{2000}^{7000} \left(\frac{1}{1000}\right) \sin (y)\, dy$$

And we can program the computer programs with this operation by checking the Input function and if the computer find any function that have a frequency make the transformation directly and start the solving procedure by the normal way like the previous example and the below one .

$$\int_{3}^{8} \ln(x)*\cos(10000x)\, dx$$

10000x=y

& dx = $\left(\frac{1}{10000}\right)$ dy & x = $\left(\frac{y}{10000}\right)$

At x= 3 y = 30000

At x= 8 y = 80000

$$\therefore \int_{3}^{8} \ln(x)*\cos(10000x)dx=$$

$$\int_{30000}^{80000} \ln\left(\frac{y}{10000}\right) * \cos (y) * \left(\frac{1}{10000}\right) dy$$

*The previous examples and MATLAB GUI we deal with a constant step = 1/8 using Sameh generated function to get equations of three terms to integrate it but we also have another method. Instead of make the steps narrower and $g(x) = a_0 + a_1 x + a_2x^2$ we will take the subinterval by unit over x-axis but in the other hand we will not conceder $g(x) = a_0 + a_1 x + a_2x^2$ we will conceder it $g(x) = a_0 + a_1 x + a_2x^2 + a_3x^3 + a_4x^4$ or $= a_0 + a_1 x \ldots\ldots\ldots a_6x^6$ or $+ a_0 + a_1 x \ldots. a_{10}x^{10}$ . Which mean that we will take larger interval width but with larger terms in the series. Then after making integration by the rule ( $\int a_n x^n = \frac{a_n}{n+1} x^{n+1}$ ) for the g(x) that approximately cover our function We will get the result . Now let's see some examples that will show us the previous meaning.

**EX(6.B)** $\int$ Exp(x) from x= 15 to 16

(a) Taking $g(x) = a_0 + a_1 x + a_2x^2 + a_3x^3 + a_4x^4$

(b) Taking $g(x) = a_0 + a_1 x \ldots\ldots.. a_5x^5$

(c) Taking $g(x) = a_0 + a_1 x \ldots\ldots\ldots a_{10}x^{10}$

(d) Compare the results VS the exact valve VS the result from other numerical methods.

Sol.

(a) $\int$ exp(x) dx from x= 15 to x= 16 in one interval

using (5) points to get (5) terms in g(x)

$\text{Exp}(15) = a_0 + a_1 15 + a_2 15^2 + a_3 15^3 + a_4 15^4$

$\text{Exp}(15.25) = a_0 + a_1 15.25 + a_2 15.25^2 + a_3 15.25^3 + a_4 15.25^4$

$\text{Exp}(15.5) = a_0 + a_1 15.5 + a_2 15.5^2 + a_3 15.5^3 + a_4 15.5^4$

$\text{Exp}(15.75) = a_0 + a_1 15.75 + a_2 15.75^2 + a_3 15.75^3 + a_4 15.75^4$

$\text{Exp}(16) = a_0 + a_1 16 + a_2 16^2 + a_3 16^3 + a_4 16^4$

After solving these five equations with five variables using the matlab see also figure (9.a) which represent the error

g(x)=226921.04469333333*x^4 -13156697.986560*x^3+287374502.0692*x^2 - 2800632920.111760*x+10269477670.50387

$$\int_{15}^{16} g(x) = 5617092.92762.$$

(b) $\int$ Exp(x) dx form x =15 to x = 16

Using one interval by unit in x axis but g(x) with (6) terms

So we need (6) points

Exp (15) = $a_0 + a_1 *15 + a_2*15^2$ ……………… $+a_5*15^5$

Exp(15.2)=$a_0+a_1*15.2+a_2*15.2^2$………………$+a_5 *15.2^2$

Exp(15.4)=$a_0+a_1*15.4+a_2*15.4^2$………………$+a_5 *15.4^5$

Exp(15.6)=$a_0+a_1*15.6+a_2*15.6^2$………………$+a_5*15.6^5$

Exp(15.8)=$a_0+a_1*15.8+a_2*15.8^2$………………$+a_5*15.8^5$

Exp(16)=$a_0+a_1 16+a_2 16^2$ ………………$+a_5*16^5$

After solving the previous (6) equations with (6) variables we can get $a_0,a_1,a_2,a_3,a_4$ and $a_5$ (see figure (9.b) which represent the error)

g(x)=45289.87604166667*$x^5$ -3282761.681250*$x^4$+95620552.29833333*$x^3$-1398098435.623750*$x^2$ +10255632378.3786650*x-30180622155.69813

$$\int_{15}^{16} g(x) = 5617094.73195.$$

(c) $\int$ Exp(x) dx from x= 15 to x=16

g(x) = $a_0+a_1x$ ……………………….. $a_{10}x^{10}$ so we need (11) point to get $a_0$ , $a_1$ ,$a_2$……….. $a_{10}$ from (11) equations.

Exp (15) = $a_0 + 15a_1 + 15^2 a_2 + 15^3 a_3$………… $15^{10} a_{10}$

Exp(15.1)= $a_0 + 15.1a_1 + 15.1^2a_2 + 15.1^3a_3$………… $15.1^{10}a_{10}$

Exp(15.2)= $a_0 + 15.2a_1 + 15.2^2a_2 + 15.2^3a_3$………… $15.2^{10}a_{10}$

Exp(15.3)= $a_0 + 15.3a_1 + 15.3^2a_2 + 15.3^3a_3$………… $15.3^{10}a_{10}$

Exp(15.4)= $a_0 + 15.4a_1 + 15.4^2a_2 + 15.4^3a_3$………… $15.4^{10}a_{10}$

Exp(15.5)= $a_0 + 15.5a_1 + 15.5^2a_2 + 15.5^3a_3$………… $15.5^{10}a_{10}$

Exp(15.6)= $a_0 + 15.6a_1 + 15.6^2a_2 + 15.6^3a_3$………… $15.6^{10}a_{10}$

Exp(15.7)= $a_0 + 15.7a_1 + 15.7^2a_2 + 15.7^3a_3$………… $15.7^{10}a_{10}$

Exp(15.8)= $a_0 + 15.8a_1 + 15.8^2a_2 + 15.8^3a_3$………… $15.8^4a_{10}$

Exp(15.9)= $a_0 + 15.9a_1 + 15.9^2a_2 + 15.9^3a_3$………… $15.9^{10}a_{10}$

Exp(16)=$a_0+16a_1+ 16^2a_2+16^3a_3$……… $16^{10} a_{10}$

After solving the above 11 equations we get $a_0$ . $a_1$………. $a_{10}$

g(x)=0.88183421516754*$x^{10}$-121.6380070546737*$x^9$+7568.783068783*$x^8$-279473.50859788359*$x^7$ +6775294.76018518*$x^6$-112588518.14155*$x^5$+1297690018.78413*$x^4$-10235055255.6465485*$x^3$ +52816751927.0576138*$x^2$-160857686477.55929563492*x +219287168126.18347

$$\int_{15}^{16} g(x) = 5617093.1482.$$

(d)from the fact $\int$ Exp(x) dx = Exp(x) which we assume it the normal method we will make a comparison between the result of integration of Exp(x) by normal method VS the results from my methods in Ex(6.b) VS the other numerical methods in the below table

| The result value | The method | the limits | Function to be integrated |
|---|---|---|---|
| <u>5617093.148</u> | Normal method | 15 to 16 | Exp(x) |
| 5617095.95762 | Ex (6.b) (a) | 15 to 16 | Exp(x) |
| 5617094.73195 | Ex (6.b) (b) | 15 to 16 | Exp(x) |
| <u>5617093.1482</u> | Ex (6.b) (c) | 15 to 16 | Exp(x) |

| | | | |
|---|---|---|---|
| 5614285.06955 | left endpoint 1000 subint. | 15 to 16 | Exp(x) |
| 5617092.91399 | midpoint 1000 subint. | 15 to 16 | Exp(x) |
| 5619902.16270 | Right endpoint 1000 subint. | 15 to 16 | Exp(x) |
| 5617093.61613 | Trapezoid 1000 subint. | 15 to 16 | Exp(x) |
| 5617093.14804 | Parabola 1000 subint. | 15 to 16 | Exp(x) |

Form the above table we see how the length of the series of g(x) affect the result which appear in Ex (6.B) (a) .(b) and (c) where (a) is accurate .but (b) is more and (c) is the most accurate equal the exact sol . Which no method achieve except parabola but with (1000) subinterval and we achieve with one interval. the MATLAB code and GUI in folder (3) calculate the numerical integration with $g(x) = a_0 + a_1 x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + a_7x^7 + a_8x^8 + a_9x^9 + a_{10}x^{10}$ across an interval equal unit over x-axis like E(6.B) (d) .

Now we will use this GUI to get final answer to solve functions with our method compared to the other numerical integration methods.

EX(7.b)

Multiple functions with different integration limits compared with other numerical methods result.

First to make the table organized I will give the functions that I will make the integration process on it a numbers in the next table then I will make the big table which include the result and the comparisons see also the figures (10).

| Given number | Function to be integrated |
|---|---|
| 1 | x*sin(x) |
| 2 | ln(x)*sin(x) |
| 3 | ln(x)*sin(x)*x^5 |
| 4 | x*(2^sin(x)) |
| 5 | x*(2^sin(x)) |
| 6 | (x*ln(x))^sin(x) |
| 7 | (x^3)*sin(x)/ln(x) |

In the next table:

- (N.) means the number
- The first column N. include the functions to be integrated according to their number which we gave in the past table
- The second column include the limits of the integration process by the order first limit, second limit
- The last column (Subint. N.) include the number of subintervals that the other numerical methods take to get the accurate results in the 3,4,5,6 and 7th columns
- The font is small because I mean to put all this result together but I put this table by clear font in the figures

| Subint. N. | parabola | trapezoid | Right endpoint | midpoint | Left endpoint | Generated function step = 1 | limits | N. |
|---|---|---|---|---|---|---|---|---|
| 10000 | -4.75447 | -4.75447 | -4.75447 | -4.75447 | -4.75417 | -4.75447 | -5,5 | 1 |
| 100000 | -2.66347 | -2.66347 | -2.66369 | -2.66347 | -2.66324 | -2.66346 | 2,18 | 2 |
| 100000 | -4519886.92 | -4520215.05 | -4520543.17 | -4520215.05 | -451988.692 | -4520215.06 | 2,18 | 3 |
| 1000 | -3.53591 | -3.53590 | -3.47445 | -3.53592 | -3.59736 | -3.53604 | -5,5 | 4 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 100000 | -3.53591 | -3.53591 | -3.53530 | -3.53591 | -3.53653 | -3.53604 | -5,5 | 5 |
| 10000 | 43.29991 | 43.29991 | 43.31049 | 43.29994 | 43.28939 | 43.29982 | 7,14 | 6 |
| 10000 | 43.57325 | 43.57329 | 44.18499 | 43.57323 | 42.69159 | 43.573177 | 2,14 | 7 |

As we see in the previous example.(Ex 7.B) taking 11 terms in generated function and make to them an integration this will give very accurate answer by taking an interval = 1 and I in some functions in this example I take 10000 or (100000) because (1000) subintervals is not enough some times!!! to get accurate results in the other numerical methods unlike my method which give accuracy with little number of intervals .And we can see the function
x*(2^sin(x)) in ex7.b we need (100000) subinterval to get the same result that I get with 10 intervals only!!!!!

*Note: from the previous pages we can deduce that using Sameh generated Function as a method for doing numerical integration depend on (2) things .

*the number of term $a_0 + a_1 x + a_2x^2 + a_3x^3$ ………………&
*the step which specify the number of subintervals. And we have the freedom to trade of between them. If we take more terms we can take large step and via verse .If we take few terms we take small step. In our examples I give the (two) things by taking step = 1/8 with $g(x) = a_0 + a_1 x + a_2x^2$ and step = 1 with $g(x) = a_0 + a_1 x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6 x^6 + a_7 x^7 + a_8 x^8 + a_9x^9 + a_{10} x^{10}$

**4-Using Sameh generated function as a method to get any curve length numerically.**

We have two methods:

**First method :**

The general rule for finding the curve length is

$$\int_{x1}^{x2} \sqrt{dx^2 + dy^2}$$

$$= \int_{x1}^{x2} \sqrt{(1 + (dy / dx)^{\wedge} 2)}\, dx$$

Any curve like the figure (12) we can assume it as infinity hypotenuses and like the figure (12) each one can be calculated. By $\sqrt{(\Delta x)^2 + (\Delta y)^2}$ and because we speak about curve or variable function we make dx & dy and make a summation by $\int$ .

The other numerical methods depend on $\sqrt{(\Delta x)^2 + (\Delta y)^2}$ and take small $\Delta$ x and small $\Delta$ y .then by making 1000 or 100000 subintervals we can calculate the curve length. But our method we go directly to the rule $\int_{x1}^{x2} \sqrt{(1 + (dy / dx)^{\wedge} 2)}$ dx and y = f(x) = the curve needed to be calculated. We will get g(x) ≅ F(x) and get dy/dx to it and by get general rule for integration by few numbers of intervals we can get the result.

**proof (1):**

By assuming $y = f(x) \cong g(x) = a_0 + a_1 x + a_2x^2$ then $dy/ dx = a_1 + 2a_2x$

then $\int_{x_1}^{x_2} \sqrt{1 + (a_1 + 2a_2 x)^{\wedge} 2}$ dx will calculate the curve length for its interval

Let $a_1 + 2a_2x = z$ & $dx = dz /2a_2$

$z_2 = a_1 + 2a_2x_2$
Then the form of the integration will be

$$\frac{1}{2a_2} \int_{z_1}^{z_2} \sqrt{1 + z^2}\, dz$$

let z= tan t , $t= \tan^{-1} z$ & $dz = (\sec(t))^2 dt$

At $z_1$ $t_1 = \tan^{-1} z_1$ & at $z_2$
$t_2 = \tan^{-1} z_2$

By using the rule $1+\tan^2 (t)= \sec^2(t)$

$$\frac{1}{2a_2}\int_{t_1}^{t_2}\sqrt{1+(\tan(t))^2}\ *(\sec(t))^2\ dt$$

$$=\frac{1}{2a_2}\int_{t_1}^{t_2}(\sec(t))^3\ dt\ .$$

By using integration by parts

$$\frac{1}{2a_2}\int_{t_1}^{t_2}(\sec(t))^3 dt$$

$$=\frac{1}{2a_2}\left[\int_{t_1}^{t_2}\sec(t)\ d\tan(t)\right]$$

$$=\frac{1}{2a_2}\left[\sec(t)\tan(t)-\int\tan(t)\ d\sec(t)\right]$$

$$=\frac{1}{2a_2}\left[\sec(t)\tan(t)-\int\sec(t)\,(\tan(t))^2\ dt\right]$$

$$=\frac{1}{2a_2}\left[\sec(t)\tan(t)-\int(\sec(t))^3\ dt\right.$$
$$\left.+\int\sec(t)\ dt\right]$$

Let $\int(\sec(t))^3\ dt = I$

$$\frac{1}{2a_2}I=\frac{1}{2a_2}\left[\sec(t)\tan(t)-I+\mathrm{Ln}(\sec(t)+\tan(t))\right]$$

Then $I=\frac{1}{2}\left[\sec(t)\tan(t)+\mathrm{Ln}(\sec(t)+\tan(t))\right]$

And $\frac{1}{2a_2}I=\frac{1}{4a_2}\left[\sec(t)\tan(t)\right.$
$\left.+\mathrm{Ln}(\sec(t)+\tan(t))\right]$

*I call the previous equation (General rule for curve length using sets of Sameh generated function with 3 terms). Equation ( 1.c)

By using this rule in Equation (1.c) we just take our step in intervals and get g(x) for the interval and substitute in the rule and below I clear the steps

**First step :** get the g(x) for a specific interval In the curve and solve it .

**Second step :** substitute with it in the rule then we get length of $g_1$ (x) ≅ length of the curve in first interval.

**Third step :** make a summation for curve length at $g_1(x),g_2(x)$ …………

In the next example we will see a comparison between some curves lengths VS the curve length calculated by other numerical method using 1000 subintervals see also figures (11).

Ex(1.c)

I use the MATLAB GUI in folder (4) to get the below results in the below table.

| value of the numerical method | Value of my method | limits | Curve length to be calculated |
|---|---|---|---|
| 3.08203 | 3.08203 | 3 to 6 | Ln(x) |
| 22.81332 | 22.8133117 | -2 to 7 | x*sin(x) |
| 117.96157 | 117.9615606 | 2 to 8 | (2^x)/ln(x) |
| 73.01325 | 73.0131914 | 50 to 110 | cos(x)*sin(x) |
| 2.31414 | 2.31413433 | 1 to 3 | sin(x)^ln(x) |
| 10.43949 | 10.43949418 | 1 to 9 | (x^3)/(2^x) |
| 21.74923 | 21.749227 | -2 to 3 | Exp(x) |

| | | | |
|---|---|---|---|
| 8.02921 | 8.02921281 | 6 to 13 | x*ln(x)/(1.1^x) |
| 5.1937 | 5.19371903 | -6 to -1 | (2^x)/(x^4) |
| 1065.36893 | 1065.368941 | -4 to 10 | x^3 |
| 2985984.69334 | 2985984.69319508 | 0 to 12 | x^6 |
| 2.364 | 2.37578 | 0 to 2 | sin(x)^cos(x) |

We use in the previous example (Ex l.c) generated function with three terms every step = 1/8 to make this accuracy but the question here is g(x) with width = 1/8 is a standard??

The answer is no of course. we choice this step to be sure about the accuracy but some curves that didn't change with high rate like ln(x) we can represent g(x) that cover larger width then we can find the curve like the below example.

Ex(2.c)
Compare between the curve length of ln(x) Form x=6 to x = 10 using

(a) g (x) with one step = 4
(b) g (x) with step = 1/8 like Ex (1.c)
(c) g (x) with other numerical method
(d) compare (a,b,c)

Sol.

(a) g (x) for in (x) form 6 to 10 in one step taking the points 6,8 & 10

$Ln(6)=a_0+a_1*6+a_2*36$
$Ln(8)=a_0+a_1*8+a_2*64$
$Ln(10) = a_0 + a_1 * 10 + a_2 * 100$
$g(x)=0.5414821+0.2567834*x-8.0673151*10^{-3}*x^2$
Then the carve length by substituting in the equation (1. c) = 4.0332

(b) Using the mat lab code used in Ex (1.c)
g (x) every step = 1/8

The result = 4.03318

(c) The other numerical method with (1000) subinterval

The result = 4.03318

(d)

| Part (c) result | Part (b) result | Part (a) result |
|---|---|---|
| 4.03318 | 4.03318 | 4.0332 |

We can deduce that we didn't have to take step = 1/8 as standard because some functions like ln(x) its generated function could cover large width efficiently unlike exp(x) which g(x) with three terms must cover small width to be efficient.

We have another way to calculate the curve length numerically depending on the generated function but not directly.

**<u>the second method to figure out the curve length numerically:</u>**

We know already form our previous pages that the generated function could be $g(x) = a_0 + a_1 x + a_2 x^2$ …………..

If we take a look to $g(x) = a_{0+} a_1 x + a_0 x + a_0 x^2$

We will find that in some how related to the equation that defines the circle.

**<u>proof (2):</u>**

Circle equation $y^2 + x^2 = R^2$ then $y^2 = R^2 - x^2$

If $y^2 = f(x) \cong g(x)$ then $R^2 = a_0$ , $a_1 = 0$ and $a_2 = -1$

In case of general equation for a circle

$( y – k )^2 + ( x – h )^2 = R^2$ then $( y – k )^2 = R^2 - ( x – h )^2$

$( y – k )^2 = R^2 - x^2 + 2hx - h^2$

if we conceder $f(x) = ( y – k )^2$

Then $g(x) = R^2 - h^2 + 2hx - x^2$ & $a_0 = R^2 - h^2$, $a_0 =2h$ and $a_2 = -1$

Like figure (12) we can take three points to get R,h and K by putting three different close values of (y) and put the (x) values related to them and by solving these three equations in three variables we can get the length of the curve by getting the circumference of the first interval and to the second……… and so on .

Length of the curve limited by $\theta$ like the figure (13) = Θ*R (Θ in radian)

We have three points $(x_1, y_1)$, $(x_2, y_2)$ & $(x_3, y_3)$

$$y_1 = \sqrt{(x_1-h)^2+R^2} + k$$

$$y_2 = \sqrt{(x_3-h)^2+R^2} + k$$

$$y_3 = \sqrt{(x_3-h)^2+R^2} + k$$

By solving them as a three equations with three variables we know h , R and k in the triangle $(x_1 - y_1)$ , $(x_3 - y_3)$ & ( h , k ) like the figure (13)

$$R.R \sin\theta = R.T \sin(90 - \frac{\theta}{2}) \ \& \ T = \sqrt{(y_3-y_1)^2+(x_3-x_1)}^2$$

$$R*\sin\theta = T*\cos\frac{\theta}{2}$$

$$R * 2*\sin\frac{\theta}{2}\cos\frac{\theta}{2} = T*\cos\frac{\theta}{2}$$

Then $\theta = 2*\sin^{-1}\frac{T}{2R}$

$L_1$ (the length at the first curve) = $\theta * R$

$$L_{total} = \sum_{n=1}^{\infty} l_n$$

Now the MATLAB code in the folder (4) named length 2 calculate the curve length with interval step = 1/8 get the result

Now let's take some Examples and compare it with the other numerical methods

EX (3.C)

| Value of other numerical method | Value of my method | limit | Length to be calculated |
|---|---|---|---|
| 3.08203 | 3.08203 | 3 to 6 | Ln(x) |
| 117.96157 | 117.96157 | 2 to 8 | (2^x)/ln(x) |
| 2.31414 | 2.314136 | 1 to 3 | sin(x)^ln(x) |
| 21.74923 | 21.749227 | -2 to 3 | exp(x) |
| 1065.36893 | 1065.36893 | -4 to 10 | x^3 |
| 2985984.69334 | 2985984.69334 | -4 to 10 | x^6 |
| 2.364 | 2.37578 | 0 to 2 | sin(x)^cos(x) |

As we see in the previous example that this method for detecting the curve length is very accurate but!!!!

This method includes a problem and this problem could be figured out from our first assumption that we take a lot of circles to deduce the curve length. We know that (R) of the circle become larger and larger if we have three points their values are too close. This problem is in its top if we speak about square wave or constant function then theoretically to have a circle that pass through three points in the same line (same value) its radius must be ∞. But practically we didn't so this method must be modified. We have two modifications that must be added to the program code that process this operation.

**The first:** is a simple check if the three points in the same line (same f(x) values) then L=point3-point1 in the x axis.

**The second:** is that some functions that didn't have high rate of change like ln(x) we have to take the numbers values in long format for ex: if we have 1.64032468 we take it 1.64032468 not 1.6403 because if we make a rounds we increase the chance for errors to accurse in other meaning we increase the chance for having two points contain same values for f(x) which according to it will cause an error in the equation solving.

## 5- Sameh generated function as a method for numerical differentiation.

We can cover any specific interval from any function or curve $g(x) = a_0 + a_1 x + a_2 x^2$ ………….. if we want to get dy/dx for any interval all what we have to is to make the differentiation to the g(x) that cover this interval or also if we want to get dy/dx at any instant point in this interval then we can get g(x) that its rate of change after and before this instant nearly equal the same characteristics of the original curve (function). For example if we want to get dy/dx to ln(x) at x=5 then we will get g(x) pass with ln(5),ln(5+Δ) &
ln(5-Δ) and now I will show the steps followed by examples.

Step(1): get g(x) cover x-Δx , x & x+Δx

Step(2): get the differentiation by differentiate g(x) according to the below rule.

If $y= a_n x^n$ then $dy/dx = n* a_n x^{n-1}$

Step(3): substitute the result from step(2) by the instant x value.

The below example will clear our meant

Ex(1.d)

Get the first derivative of ln(x) at x=5

Sol.

Step(1): g(x) from x-Δx to x+Δx

Our instant x=5 & let Δx = 0.1

Then the interval will cover from x=4.9 to 5.1 with three points

$\ln(4.9) = a_0 + 4.9a_1 + 4.9^2a_2$

$\ln(5) = a_0 + 5a_1 + 5^2a_2$

$\ln(5.1) = a_0 + 5.1a_1 + 5.1^2a_2$

Then $g(x) = 0.109204+0.400066*x-0.020004*x^2$

Step(2): dy/dx = -0.040008*x + 0.400066 using the below rule

$y= a_n x^n$ then $dy/dx = n* a_n x^{n-1}$

Step (3): by putting x=5 in the equation -0.040008*x + 0.400066

Then the result = 0.200026

We know that dy/dx for ln(x) = 1/x then at x=5 1/x=0.2 then we can see that we have very very very small error 0.000026.

This error and we take Δx = 0.1 but if we take it = 0.0001 g(x) will be more accurate and according to it the derivative.

Now the next example which I made by the MATLAB GUI and code in the folder (5) will show us a multiple functions & a lot of derivatives using $g(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$ ……… $a_{10}x^{10}$ then we can also get first , second , third …… derivatives at any instant.

Ex(2.d)

| Value of the numerical method | Value of our method | N. of the derivatives | Instant value of x | Function to be differentiated |
|---|---|---|---|---|
| 0.33333 | 0.33333 | 1 | 3 | Ln(x) |
| -0.0625 | -0.062499 | 2 | 4 | Ln(x) |
| 0.016 | 0.016 | 3 | 5 | Ln(x) |
| 50.70416 | 50.70416 | 2 | 8 | (2^x)/ Ln(x) |
| 27 | 27 | 1 | 3 | x^3 |
| 0.653643 | 0.653643 | 3 | 4 | sin(x) |

| | | | | |
|---|---|---|---|---|
| 0.27942 | 0.279415 | 2 | 6 | sin(x) |
| 0.6688 | 0.6688 | 1 | 9 | (cos(sin(x))^2 |
| 15.41853 | 15.418526 | 1 | 10 | Exp(x)/(x^3) |
| -0.15315 | -0.15314526 | 3 | 2 | 1/( $x^3$+2x+1) |
| 0.07944 | 0.0794398 | 2 | 4 | 1/ln(x) |
| 0.73576 | 0.73575888 | 2 | 1 | Exp($-x^2$) |
| -10.74503 | -10.74503 | 1 | 15 | xsin($x^2$) |

**Brief summary of the research:**

**In this research I introduced and explained the below points**

- Sameh generated function which we shown that it is an infinite series theoretically but we can limit it as one of its properties for any scientific reason or just to study the part of the function or to make a particular operations on this interval that we take. This show that we can generate this function to achieve our needing in another sentence we can design a function with our needing that qualify and replace an interval from any complex or simple curve of function.
- Using this idea I replace any complex equation with one variable to second order equation using the generated function by replacing the interval that cross x-axis which make the equation equal zero to series of three terms using my series (second order) then it can be easily solved.
- Using this idea of generated function we can generate function with three terms to replace each interval wide = 1/8 over x-axis and integrate it or differentiate it with any order of derivatives or get its curve length and also we can take the interval step = 1 but with generated function with 11 terms and make the same applications and this show that there is a trade of between number of terms and interval step (number of subintervals).

**All the previous applications and explanations are documented with a lot of figures and examples and there is a part in this research called MATLAB that contain simulations. I clear and explain the figures very well also.**

**Why my research is useful??**

**Of course I didn't have a complete answer about this question but I have some predictions beside the fact that the time that this research main idea jump to my head (the generated function) I just found the application followed by another and another and this what make me believe that this idea is very useful.**

**I will show the advantages and the benefits in the below points**

**First:** the generated function itself as an idea in my opinion is a huge thing in math and any related science that depend on the mathematic explanations. With this function we can take any interval that we study or interested in from any function or experimental curve or even random financial data and get to it one or several equations that easily to be involved in any of the mathematics operations where the form of it is easy to be integrated or differentiated or to be used for any mathematic applications.

**Second:** I used the generated function in this research to deduce numerical applications like numerical integration or differentiation or deducing the curves length numerically and I use the idea of generated function to deduce a solution for any complex variable equation.

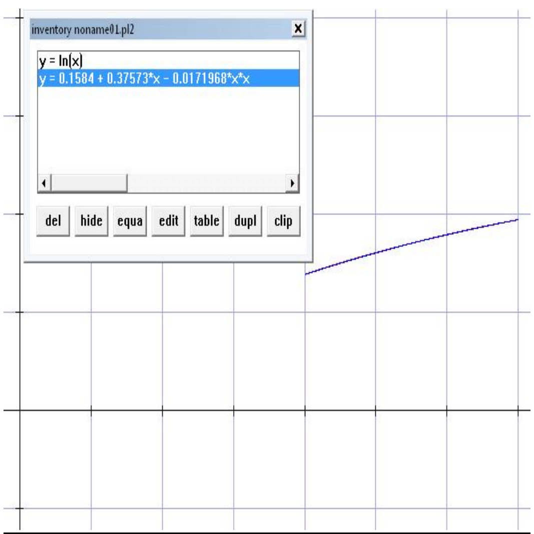
**Third:** it's clear from what I show in the second point and from the idea of generated function itself that this will help a lot in physics, chemistry, communications and signals branches….. These sciences related directly with kinds of functions and interested in study and analyze them by their mathematics characteristics and this will be developed using my ideas.

**Forth:** its clear that this research show the difference between using the old methods in numerical process and mine and the comparisons were clear in the examples I made the simulations with the MATLAB program and this can develop directly in the industry that depend on math like calculators and computer programs and the last versions from a lot of calculators take time to deduce numerical integrations as an example but using my methods it can be easy to make a lot of additional things and using my method there will be a cut in the processing time and number of operations like I shown in my examples 1000 or 10000 subintervals become 3 or 4 subintervals with accurate results sometimes higher the old methods.

**At the end:** these applications in this research is not the only one that I think about…I am working now on other researches that depend on the generated function but I made this research for the numerical applications to keep the specialty.

## The figures and its explanation:

**Figure (1)**




In this figure we can clearly see how ln(x) and its replacement generated function are nearly equal each other

In this figure we show the difference between the functions that change slowly like ln(x) & the functions that change suddenly like sin(x) and by taking an interval like in the figure from x=2 to x=6 for both functions we can see that the generated function with low terms could cover wider intervals in case of slow changeable functions like ln(x) but it will contain a lot of errors in case of high or sudden changeable functions like sin(x) and that could be observed clearly in this figure.

**Figure (2)**

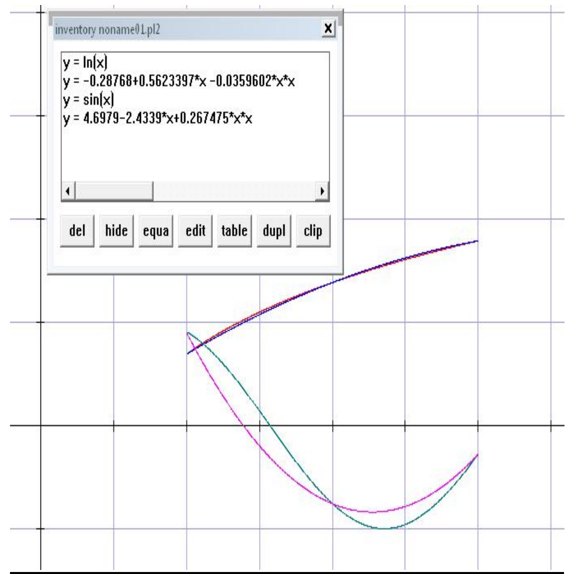
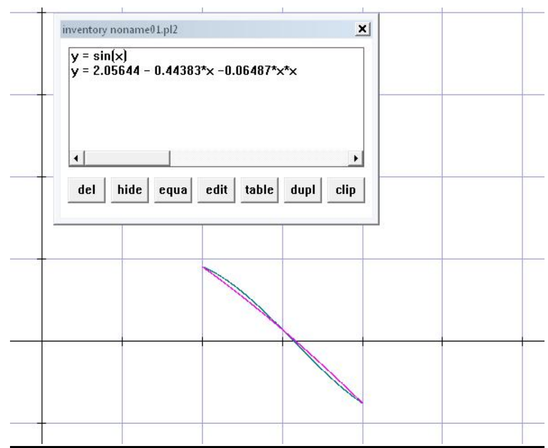


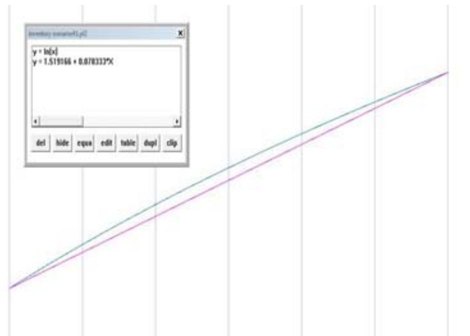

**Figure (3)**




In the previous figure we see the generated function for high changeable function like sin(x) from x=2 to x=6 and the g(x) contain high errors and in our research we assume that to solve this we take narrower intervals with low terms like this figure or we take wider interval with high number of terms. In this figure we take an interval from x=2 to x=4 covering sin(x) with generated function with three terms $g(x) = a_0 + a_1 x + a_2 x^2$

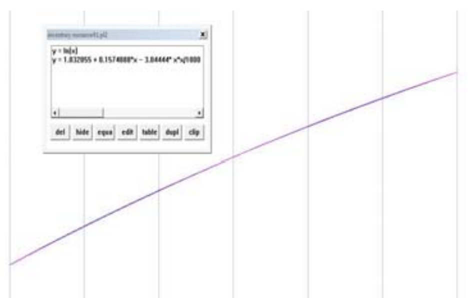
**Figures (4)**

**4.a**

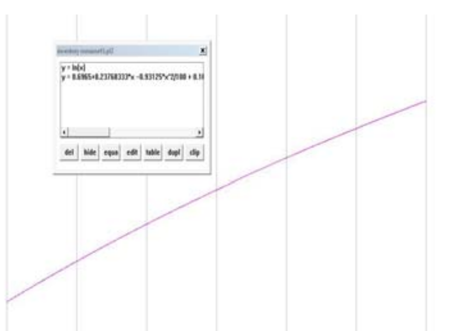


In this figure we see ln(x) from x=10 to x=16 and compare it with g(x) with two terms only g (x) = $a_0 + a_1 x$ and its clear that it has low accuracy

**4.b**



In this figure we take the same interval from ln(x) that we take in the past figure (4.A) but the replacement g(x) with three terms g (x) = $a_0 + a_1 x + a_2 x^2$ And its clear that g(x) with three terms more accurate than g(x) with two terms in the same interval.



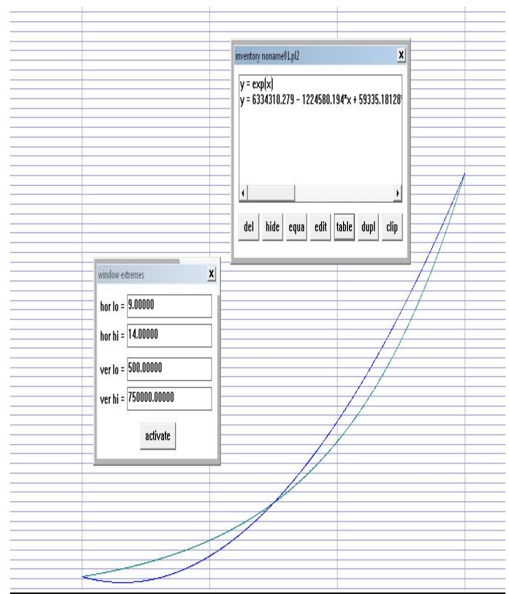
**4.c**

In this figure we continue our proving that in the same interval the accuracy increased with the increasing of the number of the terms in g(x) and in this figure we take four terms g (x) = $a_0 + a_1 x + a_2 x^2 + a_3 x^3$

And it's clear that it is very very accurate more than the past two figures.

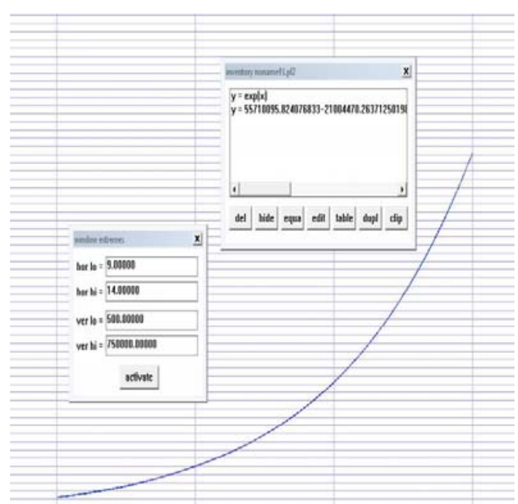
**Figures (5)**

**5.a**




In this figure we see exp(x) from x=10 to x=13 and its generated function g(x) with three terms g (x) = $a_0 + a_1 x + a_2 x^2$ and its clear that it has high rate of error because exp(x) is a function with high rate of change so we must take narrow interval with low terms or more terms in case of wider intervals.

**5.b**

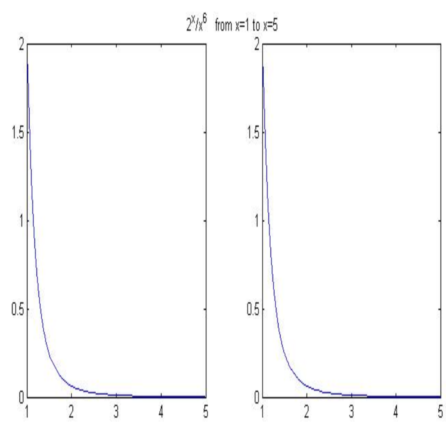



In this figure we compare exp(x) from x=10 to x=13 but unlike figure (5.B) we take g(x) with five terms to increase the accuracy and prove the point of that we increase the terms in case of function with high rate of change if we want to take wider interval and the accuracy are acceptable as we saw in the figure.

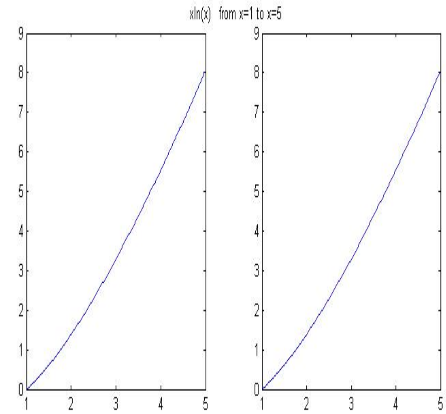
**Figures (6)**

- From figure (6-1) to figure (6-10) each figure contents a function that we plot and its replacement generated functions each generated function with three terms $g(x) = a_0 + a_1 x + a_2 x^2$ cover a step =1/8 from the function over x-axis ex: g1(x) from x=2 to x=2.125 & g2(x) from x=2.125 to x=2.25 and so on…….
- From figure (6-11) to figure (6-20) each figure contents a function that we plot and its replacement generated functions each generated function with eleven terms $g(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \ldots\ldots\ldots + a_{10} x^{10}$ cover a step =complete unit from the function in x-axis ex: g1(x) from x=2 to x=3 & g2(x) from x=3 to x=4 and so on…….
- Each figure from the twenty figures includes the function that we plot it in a specific interval in the right and its generated functions in the left.
- The target from these figures is to show how close my generated function is and how accurate is it compared to the original function on the graph.

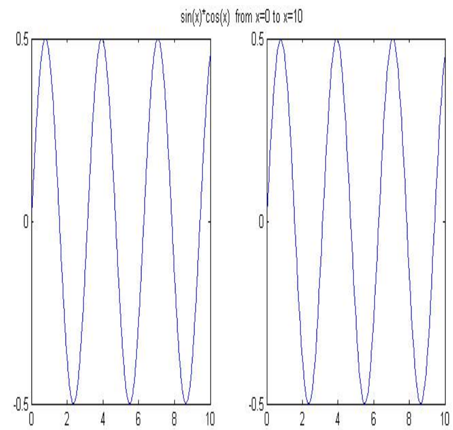
**6-1**



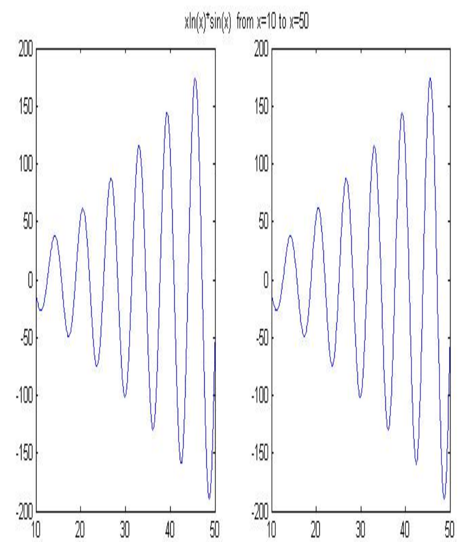

**6-2**



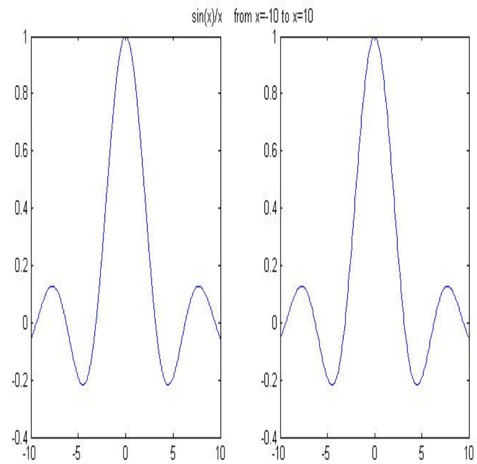

**6-3**



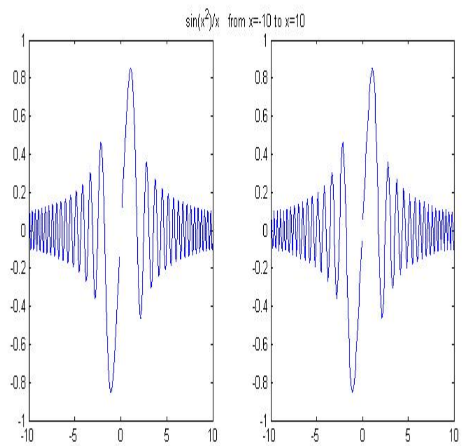

**6-4**



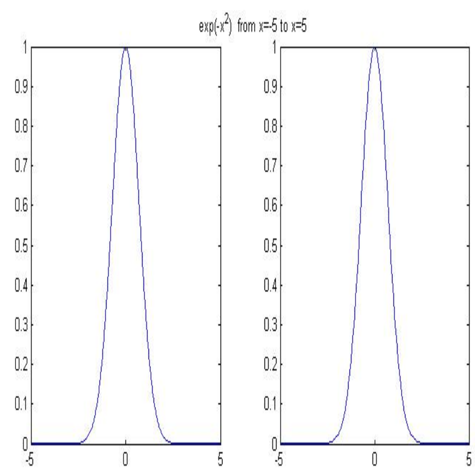
**6-5**



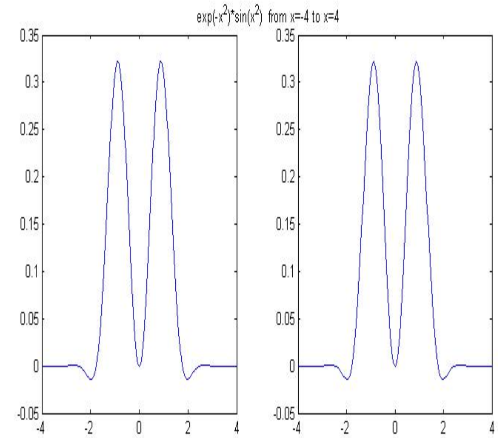

**6-6**



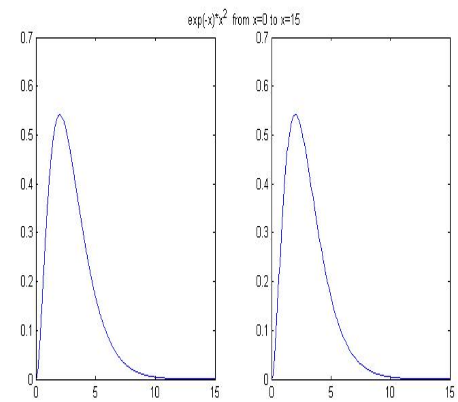

**6-7**



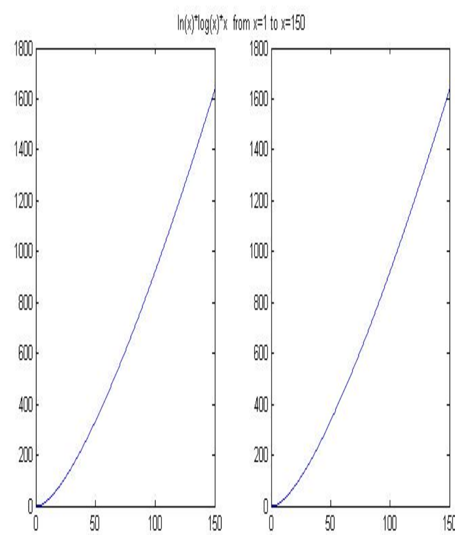

**6-8**



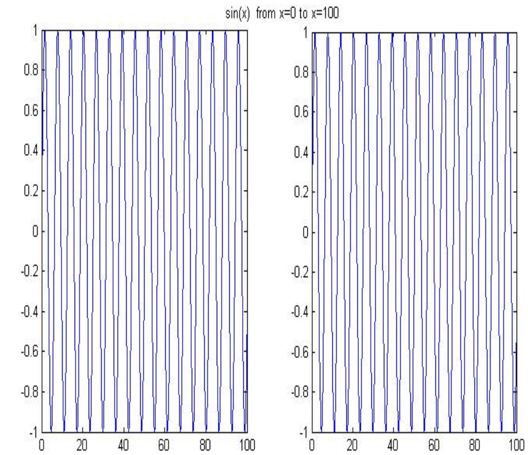

**6-9**



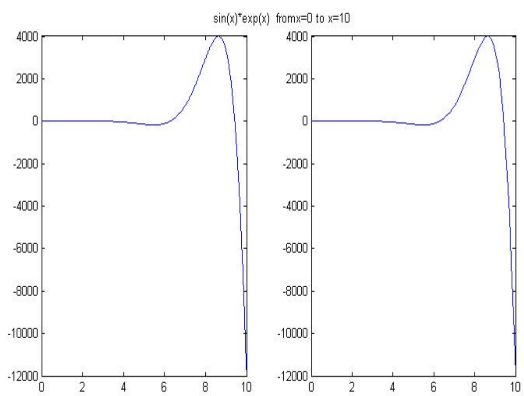

**6-10**



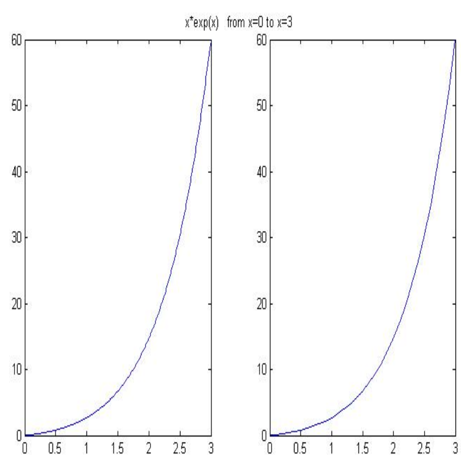
**6-11**



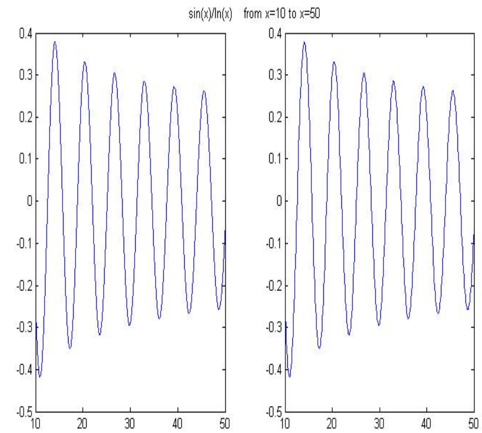

**6-12**



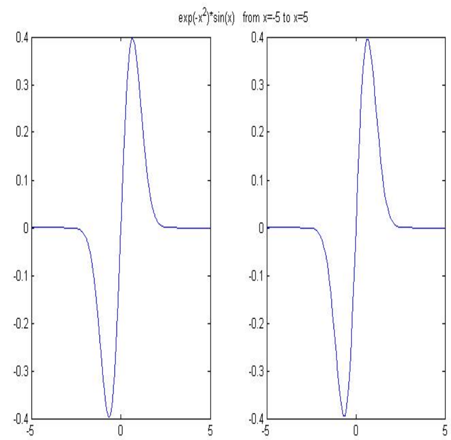

**6-13**



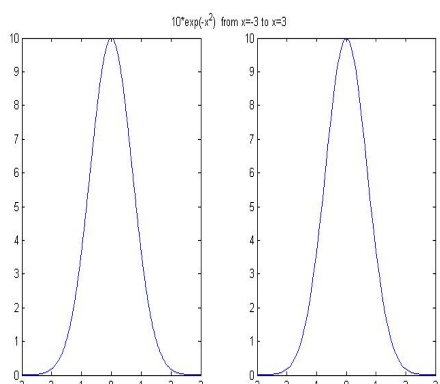

**6-14**



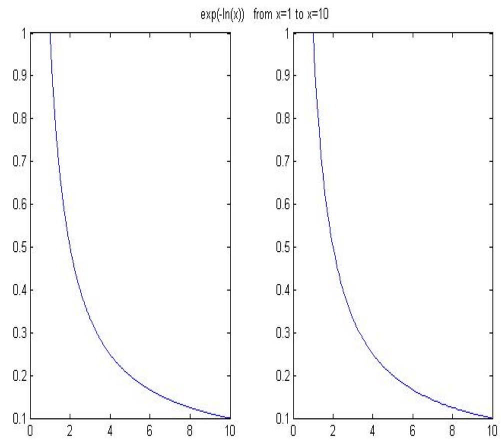

**6-15**



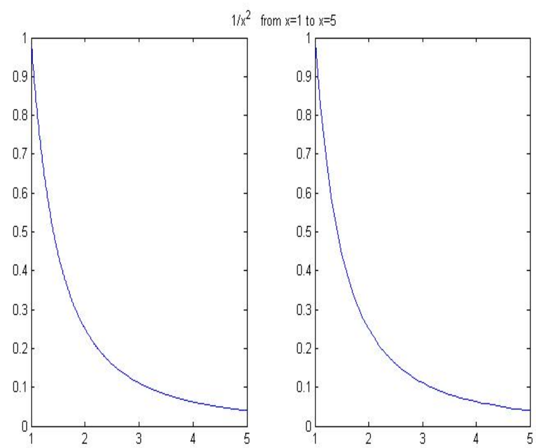

**6-16**



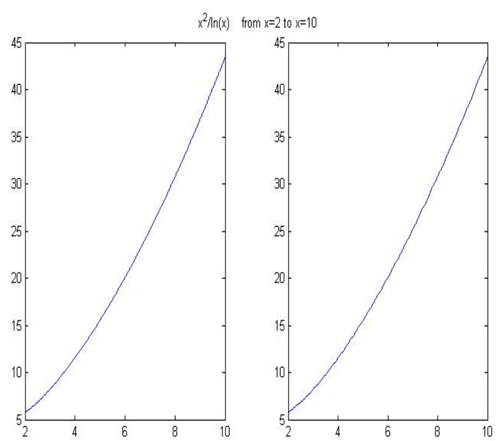
**6-17**




**6-18**




**6-19**




**6-20**

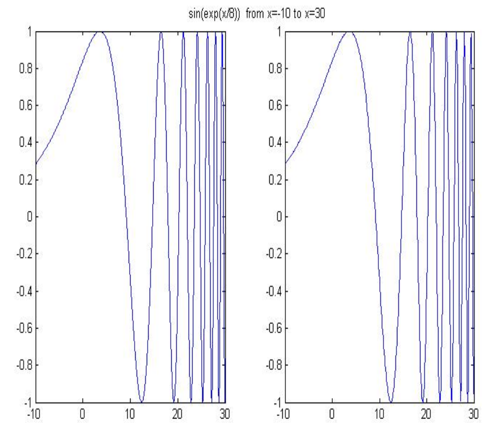
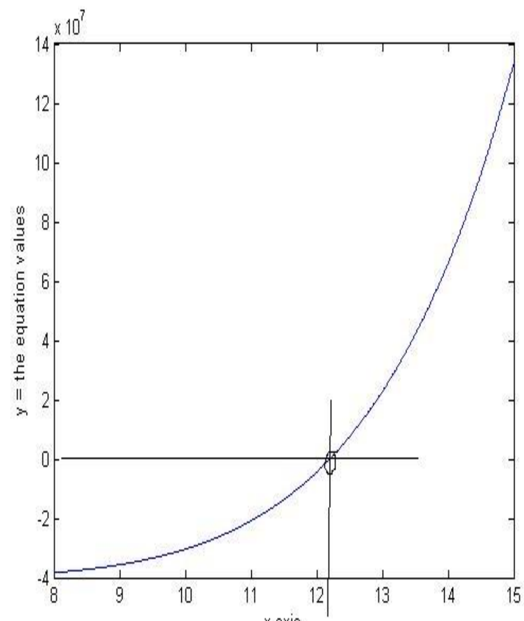



**Figure (7)**




In this figure we can see the values of the equation needed to be solved and it's clear that the curve cross x-axis (y=0) between x=12 and x=13 which means that the x value that make y=equation=0 between these values so we take the generated function to cover the equation curve in this interval between x=12 and x=13 that solve the equation.

**Figures (8)**

The system of these figures figure (8-1), figure (8-2), figure (8-3)………… Include a function that we made to it integration process numerically with our method using Sameh generated function with three terms each step = 1/8 in x-axis and show the result. In other words this figures are exactly the (GUI) of the (MATLAB) code that we make to simulate our method process and each figure include the below elements

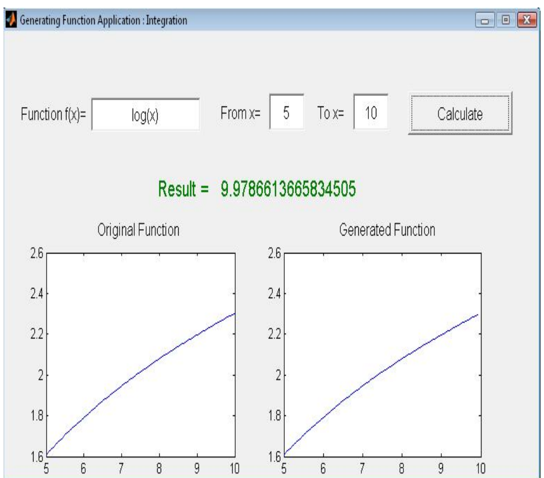
1- the function that we integrate using g(x) with three terms each step =1/8

2- the limit of the integration

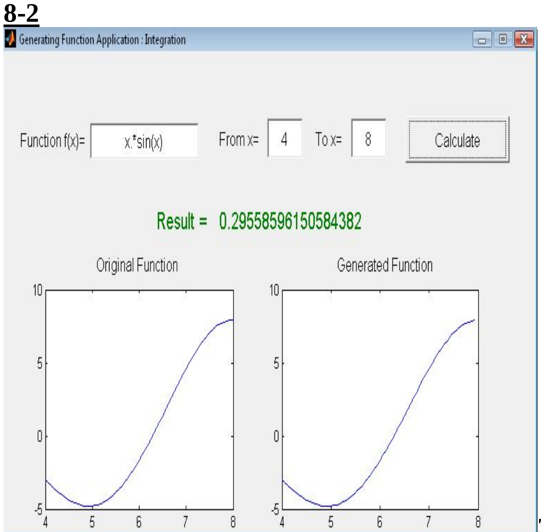
3- the result of the integration process

4- the graphs of the function which we integrate and also the generated functions that cover and replace our function

**8-1**

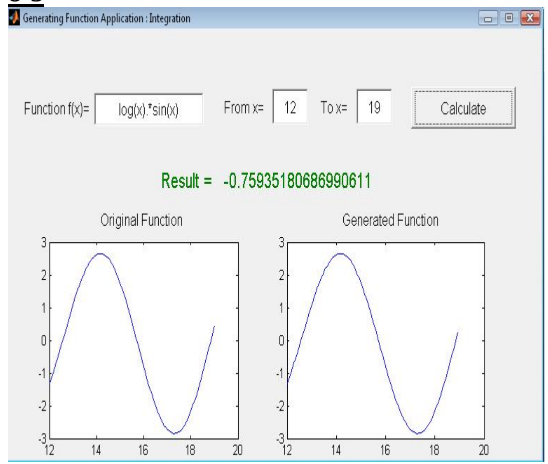



**8-2**

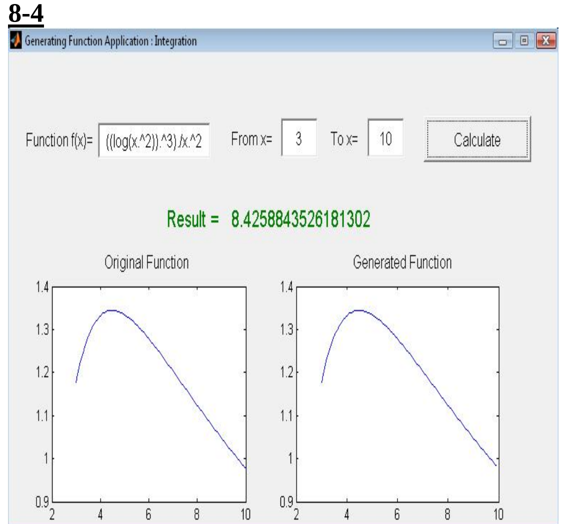



3

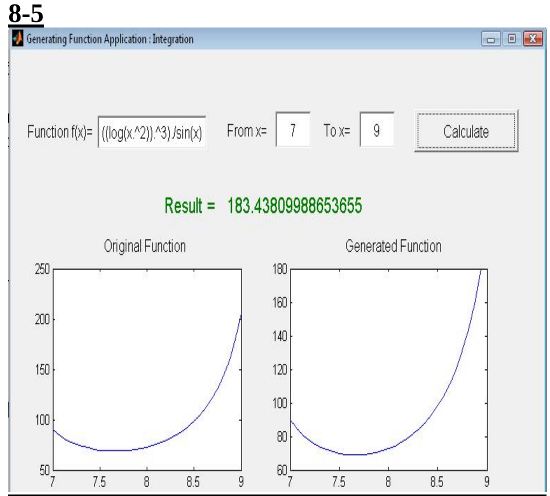
**8-3**



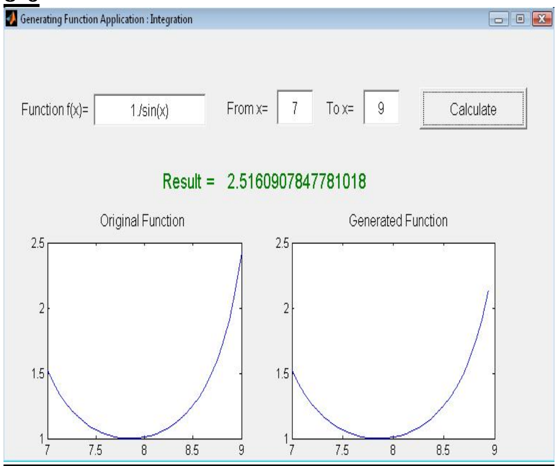

**8-4**



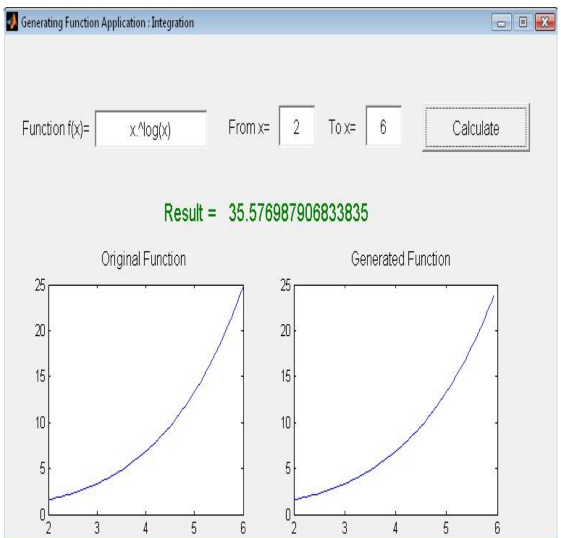

**8-5**



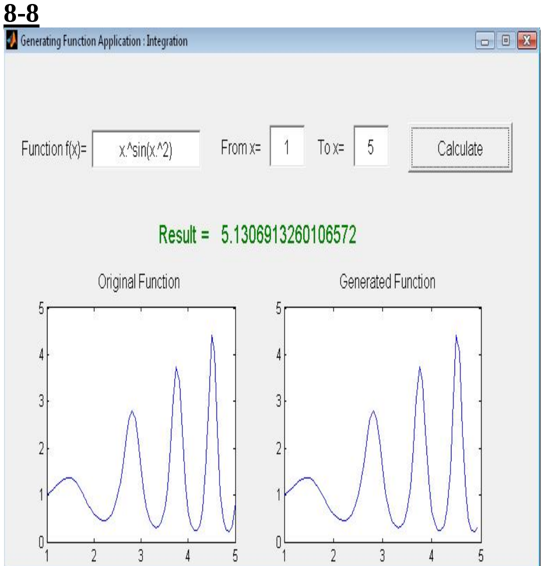
**8-6**



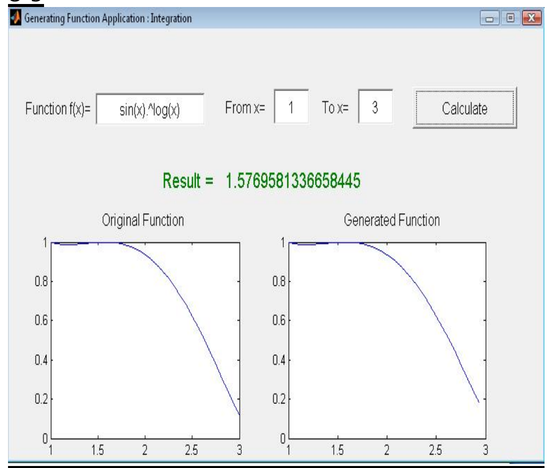

**8-7**



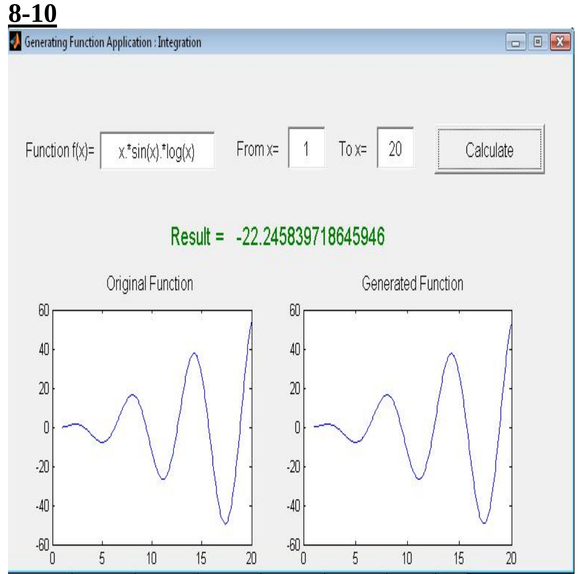

**8-8**



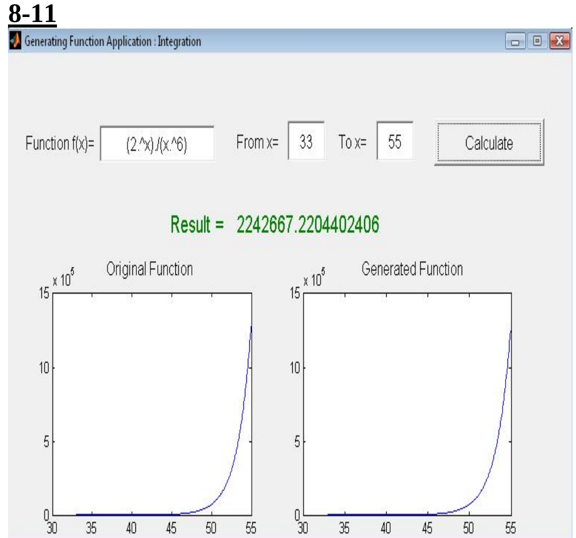

**8-9**



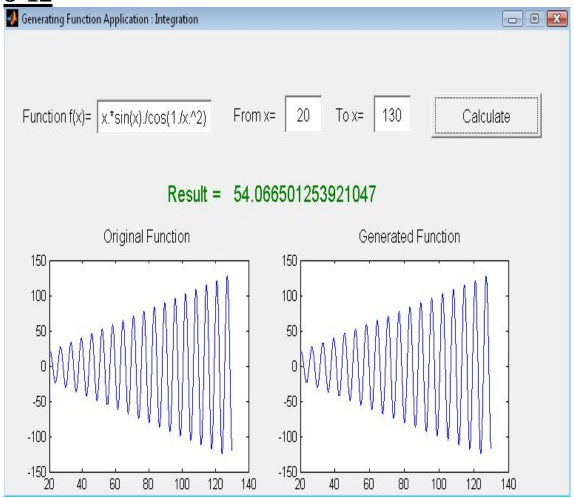

**8-10**



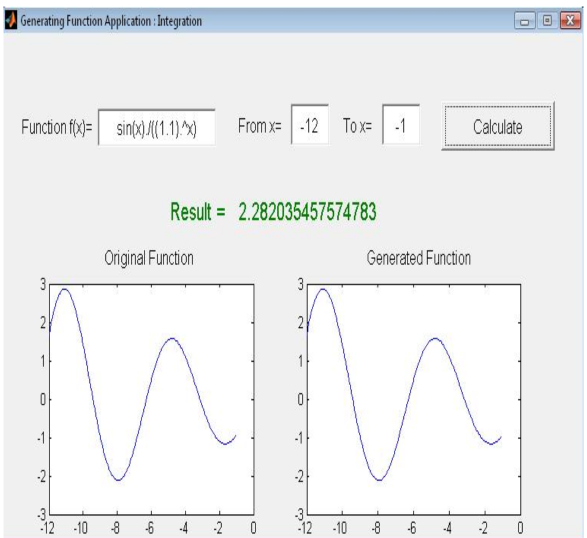

**8-11**



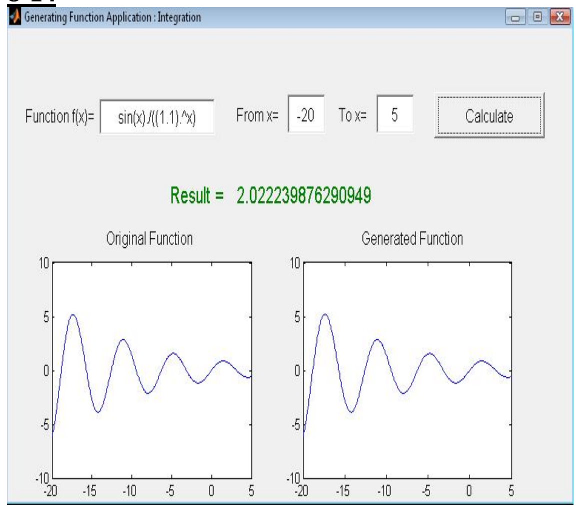
**8-12**



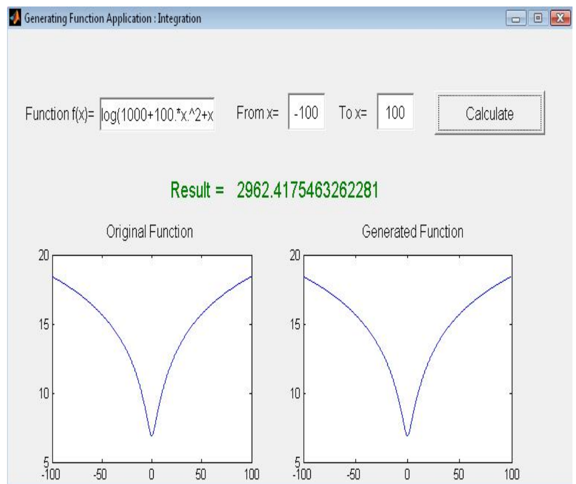

**8-13**




**8-14**

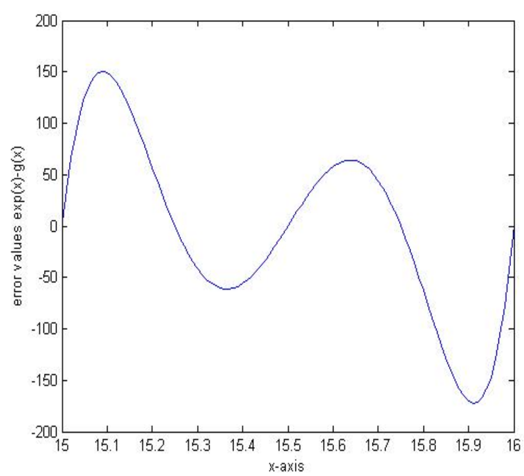



**8-15**



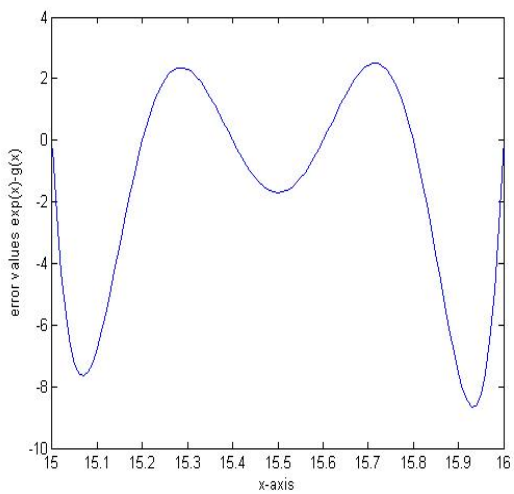

**Figures (9)**

**9-a**



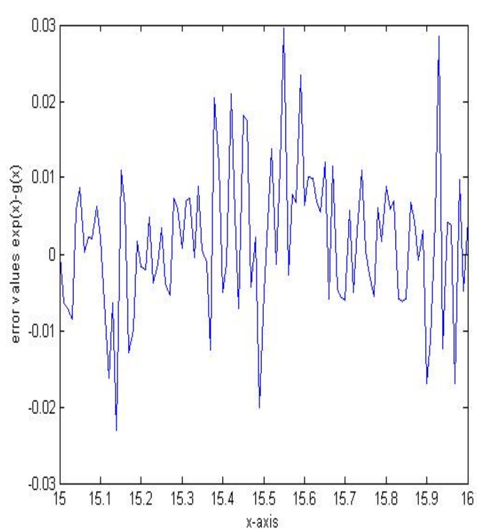
In this figure we see the error values in case of we take g (x) = $a_0 + a_1 x + a_2 x^2 + a_3 x^3$ to cover exp(x) from x=15 to x=16 so we put in y-axis exp(x)-g(x) and its clear that the error is low compared to exp(x) values where exp(15) = 3269017.372 & exp(16) = 8886110.521 where the maximum error value in the figure nearly = -175 at exp(15.9) and that in percentage = 0.0021%

**9-b**




In this figure we take g (x) = $a_0 + a_1 x + a_2 x^2$ ……..+ $a_5 x^5$ to increase the accuracy when we cover the same f(x) in the previous figure exp(x) from x= 15 to x = 16 and the error values max. is -9 at x=15.9 so the error max. Percentage =9/exp (15.9) = 0.00011%.

**9-c**




In this figure we take g(x) = $a_0 + a_1 x + a_2 x^2$ ……..+ $a_{10} x^{10}$ to cover exp(x) from x= 15 to x= 16 and the accuracy are very high here higher than the past two figures and we can deduce that from the error values in this figure compared to the past figures and the exp(x) values also.

**Figures (10)**

In these figures figure (10-1), figure (10-2), figure (10-3) …….. we see the below elements

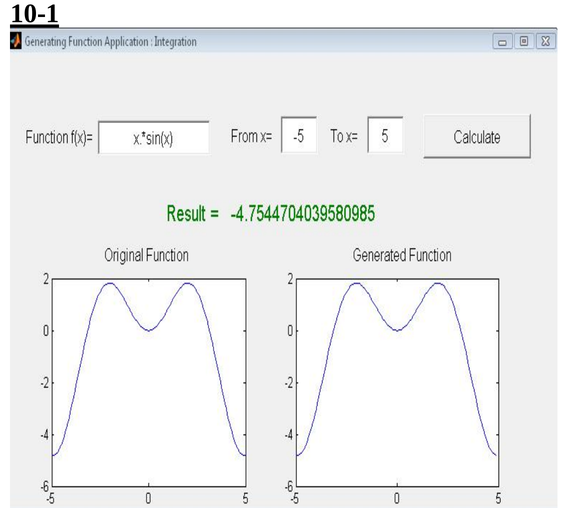
1- Function that we make to it an integration process with my method using g(x) = $a_0 + a_1 x + a_2 x^2$ ……..+ $a_{10} x^{10}$ each step = unit over x-axis ex:12 to 13.

2- In the figures it's written the limit of the integration.

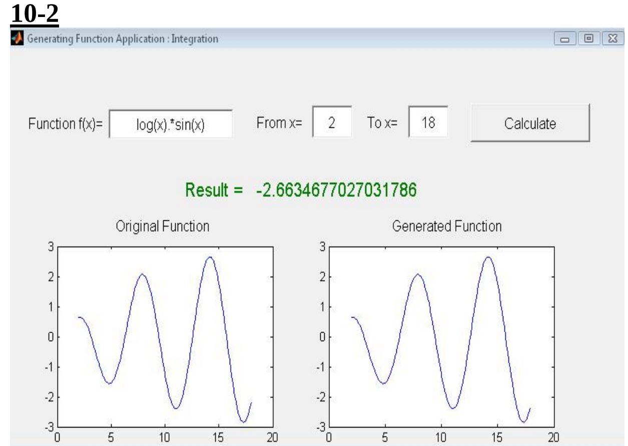
3- The result of the process is written also.

4- In the figures we will find a two graphs one of them for the function that we want to make to it the integration process in a specific interval and the other to the generated functions that cover it.

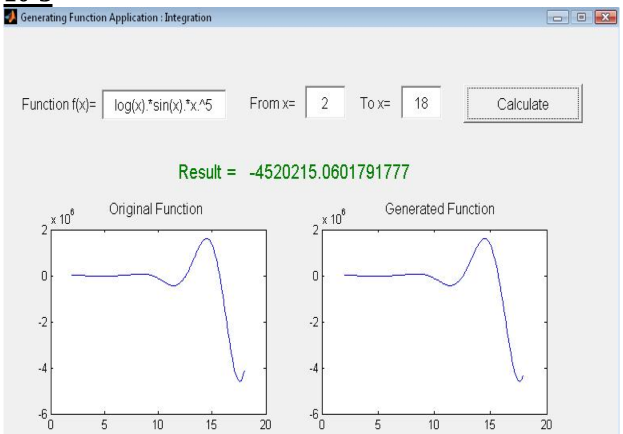
**10-1**



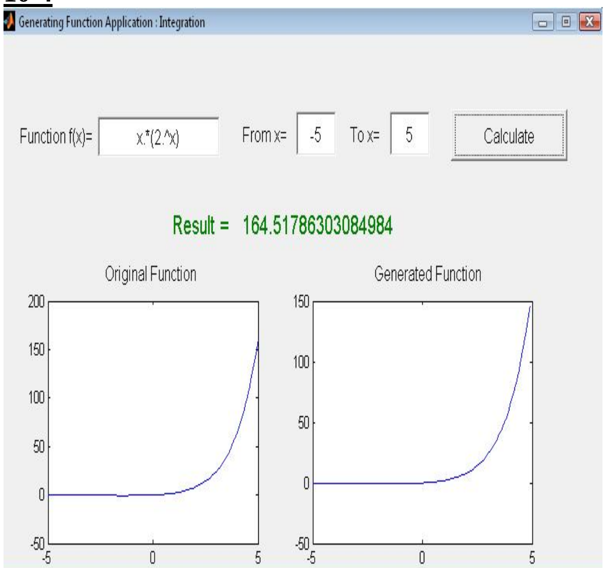

**10-2**



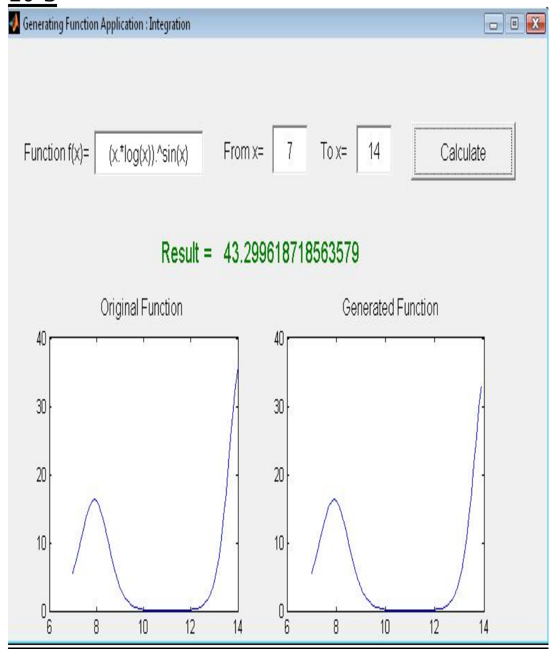
**10-3**



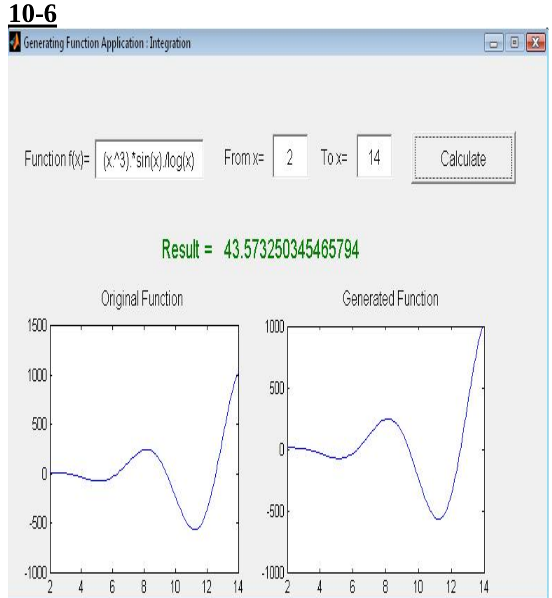

**10-4**




**10-5**




**10-6**

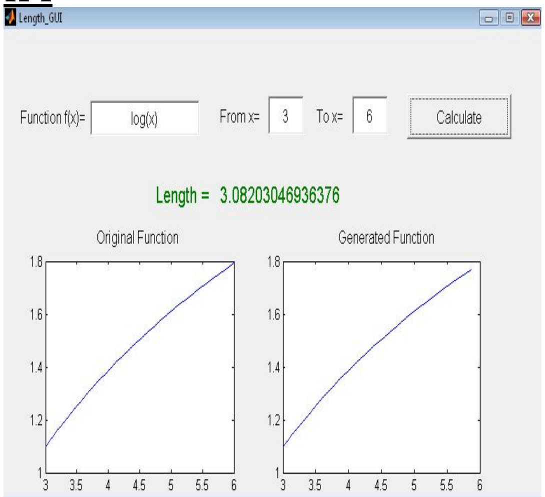



**Figures (11)**

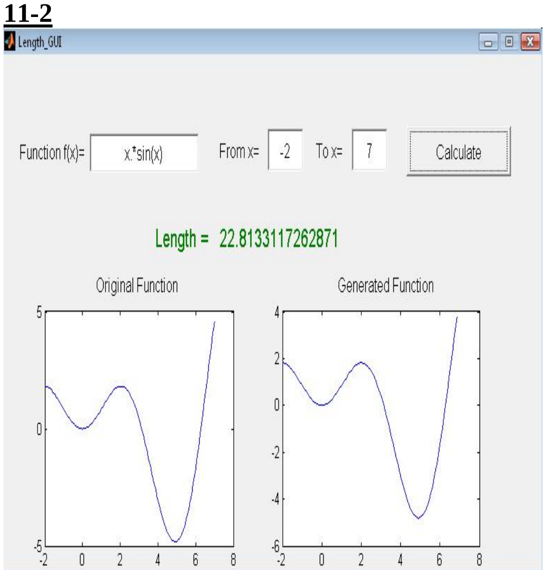
In these figures figure (11-1), figure (11-2), figure (11-3)……….we calculate the curve length using g(x) with three terms and using the equation (1.c) with step = 1/8 over x-axis to cover the curve and each figure include the below elements.
1- The function (curve) that we calculate to it its length.
2- The limits that show the start point to the end one on the curve.
3- The length value as a result.

4- Two graphs one for the function (curve) and the other include the generated functions that covering it.

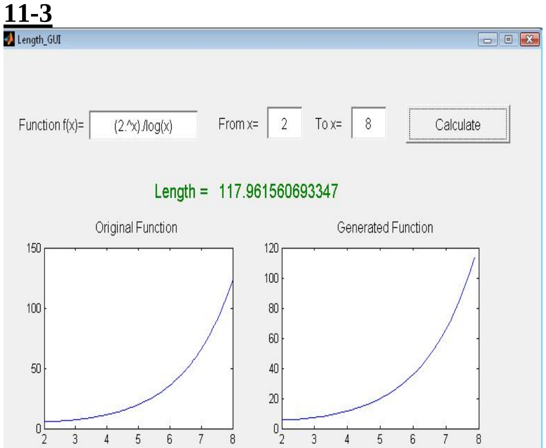
**11-1**



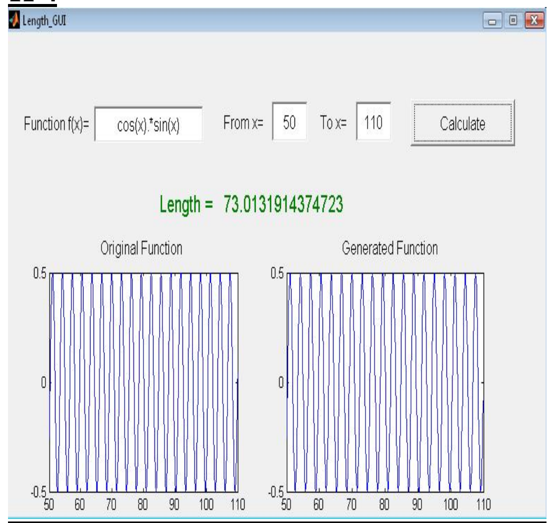

**11-2**



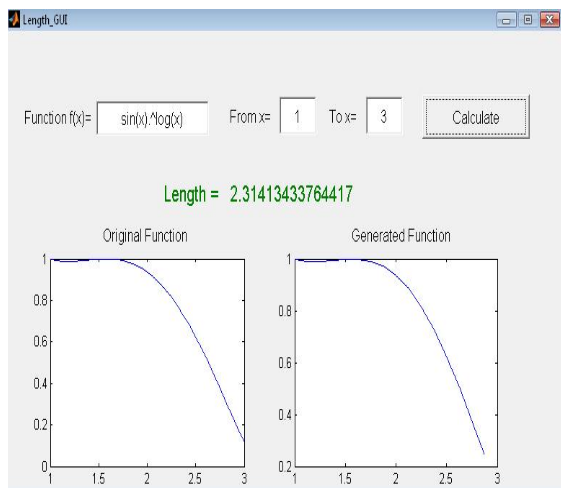

**11-3**



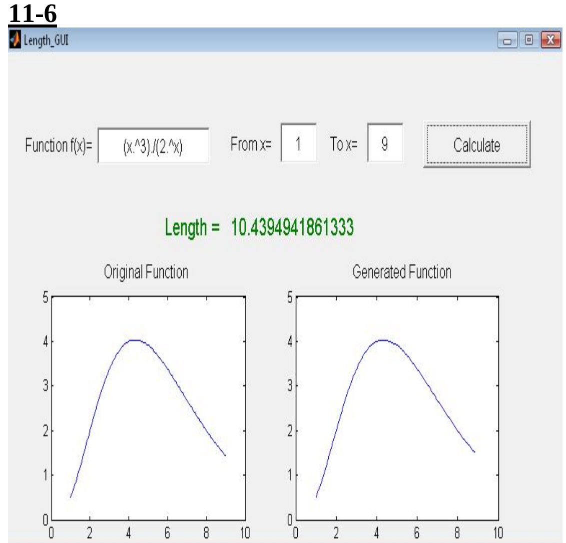

**11-4**



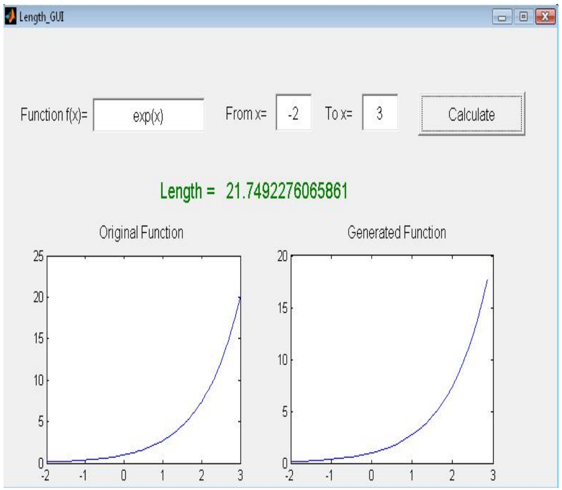

**11-5**



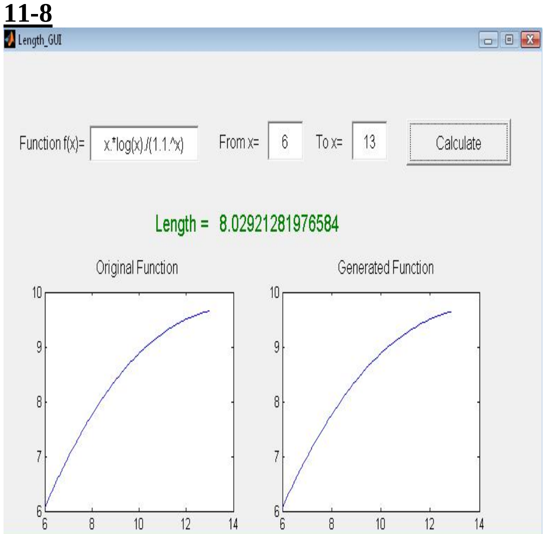

**11-6**



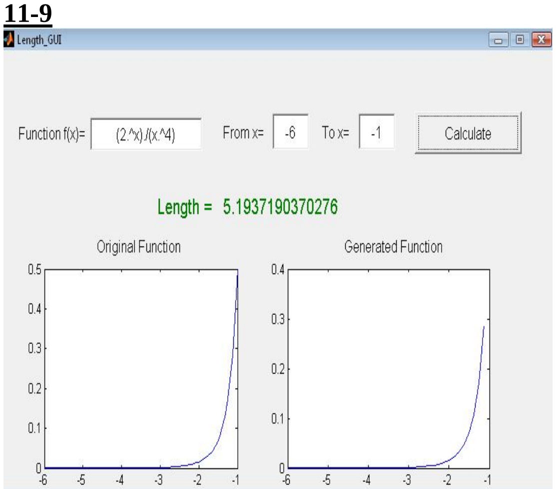
**11-7**



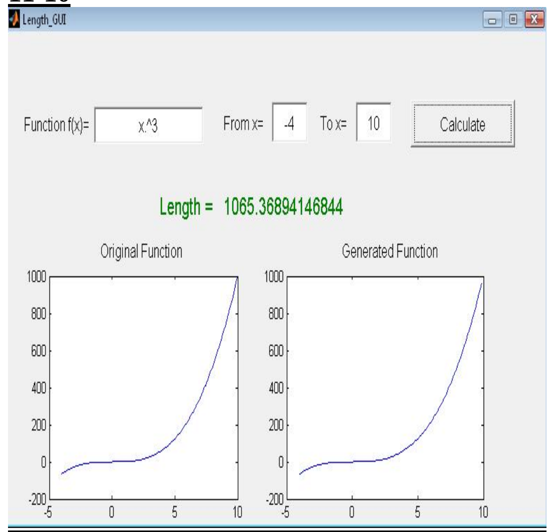

**11-8**



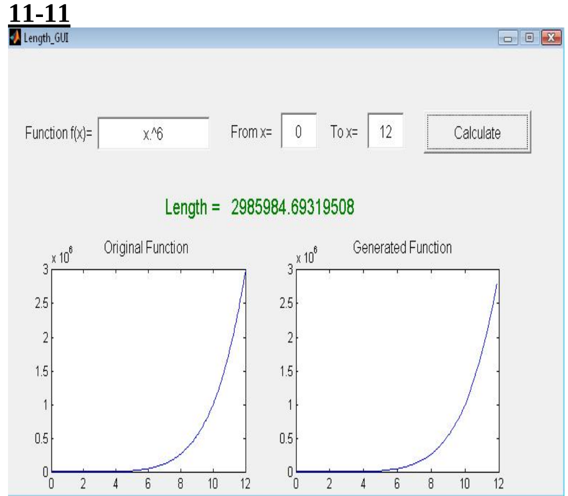

**11-9**




**11-10**




**11-11**



**11-12**

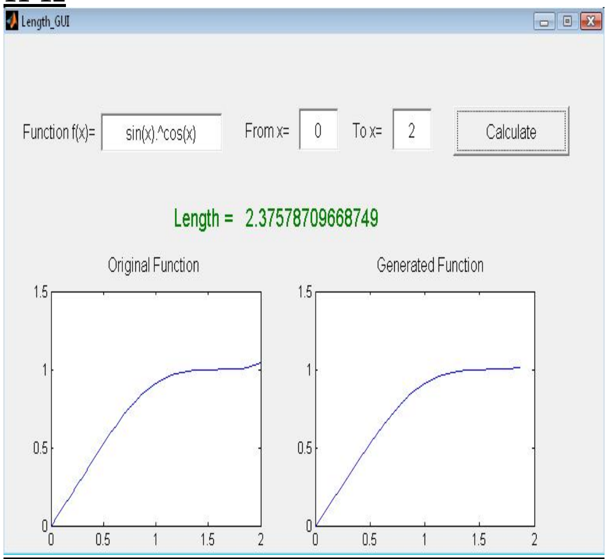
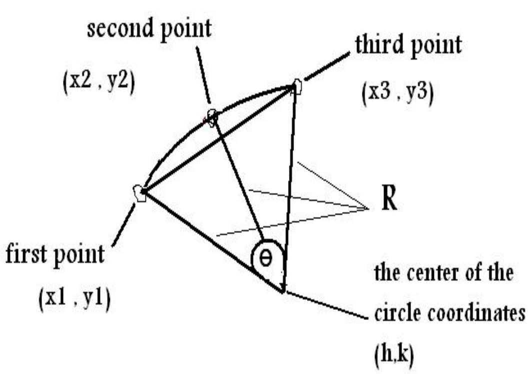
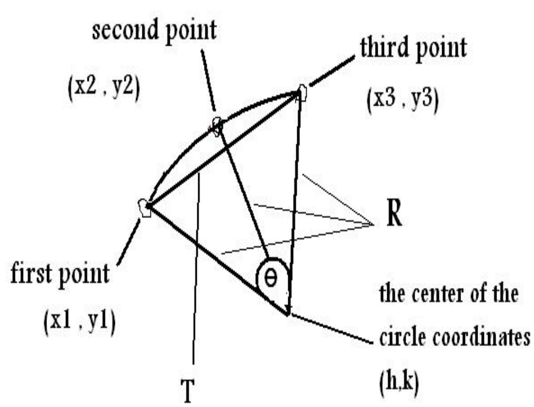



**Figure (12)**




1- The curve (x1,y1), (x2,y2 )& (x3,y3) represent a part from any function that could be represent by part of a circle if we take three close points and deduce the circle passing through them.
2- The point (h,k) represent the center point of the circle that pass through the three points on the wanted curve.
3- The line (h,k), (x1,y1) = (h,k), (x2,y2 ) = (h,k), (x3,y3 ) = R which represent the radius of the circle.
4- The angle $\theta$ which located between the lines (h,k), (x1,y1) & (h,k), (x3,y3 ) it limit the curve. In other words if it become wider the curve that we take from the function will be wider too and the distance between the three points will be increased.

**Figure (13)**



1- The notes and definitions that we hint about it in figure (12) are found in figure (13) beside the below points.
2- The character (T) in the figure refer to the straight line (x1,y1) , (x3,y3 ).
3- In the proof (2) we make our operation on the triangle (h,k), (x1,y1), (x3,y3 ) and its sides T,R,R.

If we imagine a triangle with 90 degree its bigger side is the line T then T = $\sqrt{(y_3 - y_1)^2 + (x_3 - x_1)^2}$.

## MATLAB CODES & GUI explanation followed by hardcopy of the codes

1. **Folder named (1):**
this folder include two m-files with the name (new,new2) and I will explain each separately.

**The file new:**
this file is a MATLAB program 2007 edition & the below notes describe this file.
*the target of this file is to deduce and plot the generated function (sets of generated function) that replace and represent the input function that we put it.
*we make this by programming this code to take a subinterval each step = 1/8 over x-axis and get for it g(x) with three terms and the plot it compared with the original function as a result from this code.
*for example if we input ln(x) from x=4 to x=5 then it will take from x=4 to x=4.125 as a first process, then from x=4.125 to x=4.25 as a second process, then from x=4.25 to x=4.375 as a third process and so on ----------- to reach the end x=5 and plot all of these generated functions on their intervals compared with the main function.
*if we open the file we find that in the line 8 we put our function and the line 7 to specify the limits by putting the start point and the end point without change the part (1/16) we put 4:(1/16):5 or 10:(1/16):14 where the first is the start point and the last is the end point.
*After putting the input function and the limits we push the button (F5) on the keyboard then the program will be simulated and there will be a figure includes the function and its replacement generated functions.
*I make the figures (6) from figure 6-1 to 6-10 with this code and this m-file and I described these figures clearly in the figures notes.

**The file new2:**
this m-file is like the previous file (new) except the next points
*instead of take a subinterval with step = 1/8 it become = 1.
*instead of generated function with 3 terms each step it becomes generated function with 11 terms to cover each subinterval.
*for example if we have ln(x) from x = 14 to x = 16 we will take the first process from x = 14 to x = 15 and get the first generated function, then the second process from x = 15 to x = 16 and get for it also a generated function with 11 terms.
*I use this file to make the figures (6) from 6-11 to 6-20

***the mathematics process code start from the line 69 if we open the m-file in the below folders 2,3,4,5 codes**

2. **Folder named (2):**
this folder contain m-file that when it activated show a GUI its function is to determine a numerical integration using the generated functions.
*After opening the file and pushing (F5) then the GUI will be opened.
*the GUI contain a place that we put a function on it to be integrated and there are two places for input x values one of them for the start point and the other for the end point. So they specify the limits.
*there are two graphs inside the GUI one of them to plot the original function (the input function that we want to integrate) limited by the wanted interval and the second to plot the generated functions used to replace the

interval that we take from the original function.
*after pushing the button (calculate) the integration process on the function that we input with limits will be done and it will appear the result and the two graphs will be activated showing the plot.
*I use this GUI and take the figures (8) using it and I explained these figures clearly in the figures notes.

3. **Folder named (3):**
like the previous folder except the process methods and I explained it in the figures notes because I used this GUI to make the figures 10

4. **Folder named (4):**
this folder contain m-file that after being activated it open a GUI its function is to calculate the curve length for any input function which represent the curve and plot two graphs one of them contain the original function (input curve that we deduce its length) and the other contain the generated functions that replace it.
*this GUI contain a place that we input the function that we want to calculate its curve length and a place to input the start value of x and the end value of x
*two graphs that contain a plot for the function that we input and the other one contain a plot for the generated functions that replace and cover it.
*After pushing (F5) the GUI start working and if we input the function ,start value of x ,end value of x and press (calculate) then the result will appear and also the two graphs will be activated one of them with a plot for the original function and the other for its replacement.
*the figures 11 made by using this GUI and I explained that clearly in the figures notes.

**folder named length2:**
this folder include a MATLAB code that simulate the second method I made for determining the curve length
*After opening the file the line 4 is to write the input function to be calculated.
*The line 3 to specify the limits by putting the start point and the end point without change the part (1/16) we put 4:(1/16):5 or 10:(1/16):14 where the first is the start point and the last is the end point.
*push (F5) to get the result.

5. **Folder named (5):**
this folder contain m-file that activate a MATLAB GUI by pushing (F5) in the keyboard. This GUI function is to make a differentiation operation for any input function to any order at instant x (get first, second, third…… derivatives).
*the GUI contain a place that we input a function that will be differentiated , place for the instant value of x we want get a result on it, there is a place that we input the order number which specify the derivative number (the differentiation order).
*After pushing (calculate) the result from the process will appear and also a graph that plot the interval that we take to do the process.

**Folder named (6):**
this folder contain an advanced m-file that active a MATLAB GUI after pushing (F5).
This GUI function is to make an integration process using the generated function with our choice from generated function with two terms to generated function with 11 terms.
this GUI contain like previous place for input functions, limits values, two graphs for plotting the function and its replacement but we add this time a place to input the max order which represent the length of the generated function (number of terms) where 1 represent g(x) with two terms for each step and 10 represent 11 terms for each step. Click the button (calculate) to start the process.

# The hardcopy of the codes

the code (new) in folder 1:

```
format long
clc
clear all
close all

j=1:(1/16):150;
input=log(x);
x=j(1);
gen=[];
for i=1:2:length(input)-2
      m=[1 x x^2;
      1 x+(1/16) (x+(1/16))^2;
      1 x+(1/8) (x+(1/8))^2 ;
      ];


f=[input(i);input(i+1);input(i+2)];

   sol=m\f;
    a0=sol(1);
    a1=sol(2);
    a2=sol(3);

    for k=x:1/64:x+(1/8-1/64);
    gen =[ gen (a0 + a1*k +
a2*k^2)];
    end
x=x+1/8;


end
subplot(1,2,1);plot(j(1):1/64:j(end)
-1/64,gen)
subplot(1,2,2);plot(j,input)
```

the code (new2) in folder 1:

```
format long
clc
clear all
close all

j=-10:0.1:30;
input=sin(exp(j/8));
x=j(1);
gen=[];
for i=1:10:length(input)-10
   m=[1 x x^2 x^3 x^4 x^5 x^6 x^7
x^8 x^9 x^10;
      1 x+0.1 (x+0.1)^2 (x+0.1)^3
(x+0.1)^4 (x+0.1)^5 (x+0.1)^6
(x+0.1)^7 (x+0.1)^8 (x+0.1)^9
(x+0.1)^10;
      1 x+0.2 (x+0.2)^2 (x+0.2)^3
(x+0.2)^4 (x+0.2)^5 (x+0.2)^6
(x+0.2)^7 (x+0.2)^8 (x+0.2)^9
(x+0.2)^10;
      1 x+0.3 (x+0.3)^2 (x+0.3)^3
(x+0.3)^4 (x+0.3)^5 (x+0.3)^6
(x+0.3)^7 (x+0.3)^8 (x+0.3)^9
(x+0.3)^10;
      1 x+0.4 (x+0.4)^2 (x+0.4)^3
(x+0.4)^4 (x+0.4)^5 (x+0.4)^6
(x+0.4)^7 (x+0.4)^8 (x+0.4)^9
(x+0.4)^10;
      1 x+0.5 (x+0.5)^2 (x+0.5)^3
(x+0.5)^4 (x+0.5)^5 (x+0.5)^6
(x+0.5)^7 (x+0.5)^8 (x+0.5)^9
(x+0.5)^10;
      1 x+0.6 (x+0.6)^2 (x+0.6)^3
(x+0.6)^4 (x+0.6)^5 (x+0.6)^6
(x+0.6)^7 (x+0.6)^8 (x+0.6)^9
(x+0.6)^10;
      1 x+0.7 (x+0.7)^2 (x+0.7)^3
(x+0.7)^4 (x+0.7)^5 (x+0.7)^6
(x+0.7)^7 (x+0.7)^8 (x+0.7)^9
(x+0.7)^10;
      1 x+0.8 (x+0.8)^2 (x+0.8)^3
(x+0.8)^4 (x+0.8)^5 (x+0.8)^6
(x+0.8)^7 (x+0.8)^8 (x+0.8)^9
(x+0.8)^10;
      1 x+0.9 (x+0.9)^2 (x+0.9)^3
(x+0.9)^4 (x+0.9)^5 (x+0.9)^6
(x+0.9)^7 (x+0.9)^8 (x+0.9)^9
(x+0.9)^10;
      1 x+1 (x+1)^2 (x+1)^3 (x+1)^4
(x+1)^5 (x+1)^6 (x+1)^7 (x+1)^8
(x+1)^9 (x+1)^10
      ];

f=[input(i);input(i+1);input(i+2);in
put(i+3);input(i+4);input(i+5);input
(i+6);input(i+7);input(i+8);input(i+
9);input(i+10)];
```

```
   sol=m\f;

    a0=sol(1);
    a1=sol(2);
    a2=sol(3);
    a3=sol(4);
    a4=sol(5);
    a5=sol(6);
    a6=sol(7);
    a7=sol(8);
    a8=sol(9);
    a9=sol(10);
    a10=sol(11);

    for k=x:0.01:x+0.99
    gen=[ gen (a0 + a1*k + a2*k^2 +
a3*k^3 + a4*k^4 + a5*k^5 + a6*k^6 +
a7*k^7 + a8*k^8 + a9*k^9 +
a10*k^10)];
    end
x=x+1;

end
subplot(1,2,1);plot(j(1):0.01:j(end)
-0.01,gen)
subplot(1,2,2);plot(j,input)
```

code in folder 2:

```
x=begin:(1/16):finish;
input=eval(get(handles.func,'String'
));
j=begin;
s=[];
gen=[];

for i=1:2:length(input)-2

   m=[1 j j^2 ;
      1 j+1/16 (j+1/16)^2 ;
      1 j+1/8 (j+1/8)^2 ;
      ];

f=[input(i);input(i+1);input(i+2)];
```

```
   sol=m\f;

    a0=sol(1);
    a1=sol(2);
    a2=sol(3);

    for k=j:1/16:j+1/16;
    gen=[ gen (a0 + a1*k + a2*k^2)];
    end
    s= [ s ((a0)/8 + (a1 *
(((j+1/8))^2 - (j)^2))/2 + (a2 *
((j+1/8)^3 - (j)^3))/3)];

   j=j+1/8;

end

set(handles.result,'String',num2str(
sum(s),50));

axes(handles.axes1)
plot(x,input)

axes(handles.axes2)
plot([begin:1/16:finish-1/16],gen)
```

code in folder 3:

```
x=begin:(1/10):finish;
input=eval(get(handles.func,'String'
));
j=begin;
s=[];
gen=[];

for i=1:10:length(input)-10
         m=[1 j j^2 j^3 j^4 j^5 j^6
j^7 j^8 j^9 j^10;
      1 j+0.1 (j+0.1)^2 (j+0.1)^3
(j+0.1)^4 (j+0.1)^5 (j+0.1)^6
(j+0.1)^7 (j+0.1)^8 (j+0.1)^9
(j+0.1)^10;
      1 j+0.2 (j+0.2)^2 (j+0.2)^3
(j+0.2)^4 (j+0.2)^5 (j+0.2)^6
```

```
(j+0.2)^7 (j+0.2)^8 (j+0.2)^9
(j+0.2)^10;
      1 j+0.3 (j+0.3)^2 (j+0.3)^3
(j+0.3)^4 (j+0.3)^5 (j+0.3)^6
(j+0.3)^7 (j+0.3)^8 (j+0.3)^9
(j+0.3)^10;
      1 j+0.4 (j+0.4)^2 (j+0.4)^3
(j+0.4)^4 (j+0.4)^5 (j+0.4)^6
(j+0.4)^7 (j+0.4)^8 (j+0.4)^9
(j+0.4)^10;
      1 j+0.5 (j+0.5)^2 (j+0.5)^3
(j+0.5)^4 (j+0.5)^5 (j+0.5)^6
(j+0.5)^7 (j+0.5)^8 (j+0.5)^9
(j+0.5)^10;
      1 j+0.6 (j+0.6)^2 (j+0.6)^3
(j+0.6)^4 (j+0.6)^5 (j+0.6)^6
(j+0.6)^7 (j+0.6)^8 (j+0.6)^9
(j+0.6)^10;
      1 j+0.7 (j+0.7)^2 (j+0.7)^3
(j+0.7)^4 (j+0.7)^5 (j+0.7)^6
(j+0.7)^7 (j+0.7)^8 (j+0.7)^9
(j+0.7)^10;
      1 j+0.8 (j+0.8)^2 (j+0.8)^3
(j+0.8)^4 (j+0.8)^5 (j+0.8)^6
(j+0.8)^7 (j+0.8)^8 (j+0.8)^9
(j+0.8)^10;
      1 j+0.9 (j+0.9)^2 (j+0.9)^3
(j+0.9)^4 (j+0.9)^5 (j+0.9)^6
(j+0.9)^7 (j+0.9)^8 (j+0.9)^9
(j+0.9)^10;
      1 j+1 (j+1)^2 (j+1)^3 (j+1)^4
(j+1)^5 (j+1)^6 (j+1)^7 (j+1)^8
(j+1)^9 (j+1)^10
      ];

f=[input(i);input(i+1);input(i+2);in
put(i+3);input(i+4);input(i+5);input
(i+6);input(i+7);input(i+8);input(i+
9);input(i+10)];



   sol=m\f;


    a0=sol(1);
    a1=sol(2);
    a2=sol(3);
    a3=sol(4);
    a4=sol(5);
    a5=sol(6);
    a6=sol(7);
    a7=sol(8);
    a8=sol(9);
    a9=sol(10);
    a10=sol(11);

    for k=j:0.1:j+0.9
    gen=[ gen (a0 + a1*k + a2*k^2 +
a3*k^3 + a4*k^4 + a5*k^5 + a6*k^6 +
a7*k^7 + a8*k^8 + a9*k^9 +
a10*k^10)];
    end
    s= [ s ((a0) + (a1 * (((j+1))^2
- (j)^2))/2 + (a2 * ((j+1)^3 -
(j)^3))/3 + (a3 * (((j+1))^4 -
(j)^4))/4 + (a4 * (((j+1))^5 -
(j)^5))/5 + (a5 * (((j+1))^6 -
(j)^6))/6 + (a6 * (((j+1))^7 -
(j)^7))/7 + (a7 * (((j+1))^8 -
(j)^8))/8 + (a8 * (((j+1))^9 -
(j)^9))/9 + (a9 * (((j+1))^10 -
(j)^10))/10 + (a10 * (((j+1))^11 -
(j)^11))/11)];

   j=j+1;



end



set(handles.result,'String',num2str(
sum(s),50));

axes(handles.axes1)
plot(x,input)

axes(handles.axes2)
plot([begin:0.1:finish-0.1],gen)
```

code (length) in folder 4:

```
x=begin:(1/16):finish;
input=eval(get(handles.func,'String'
));
j=begin;
s=[];
gen=[];
for i=1:2:length(input)-2

   m=[1 j j^2;
      1 j+(1/16) (j+(1/16))^2;
      1 j+(1/8) (j+(1/8))^2 ;
      ];

f=[input(i);input(i+1);input(i+2)];
   sol=m\f;

    a0=sol(1);
```

```
    a1=sol(2);
    a2=sol(3);

    gen= [gen (a0 + a1*j + a2*j^2)];

    s= [ s ( (1/(4*a2))*(
sec(atan(a1+2*a2*(j+1/8)))*(a1+2*a2*
(j+1/8))+log(
sec(atan(a1+2*a2*(j+1/8))) +
(a1+2*a2*(j+1/8))))-((1/(4*a2))*(
sec(atan(a1+2*a2*j))*(a1+2*a2*j)+log
( sec(atan(a1+2*a2*j)) +
(a1+2*a2*j))))) ];

   j=j+(1/8);

end

set(handles.result,'String',num2str(
sum(s),15));

axes(handles.axes1)
plot(x,input)

axes(handles.axes2)
plot([begin:1/8:finish-1/16],gen)
```

code (length2) in folder 4:

```
clc
    t=3:1/16:8;
    input =  log(t);
    step=1/8;
    j=1;
    L=[];
    for i=3:step:8-step

        ['(-(' num2str(i,8) '-h)^2 +
R^2)=(' num2str(input(j),8) '-
k)^2'];
        ['(-(' num2str(i+1/16,8) '-
h)^2 + R^2)=(' num2str(input(j+1),8)
'-k)^2'];
        ['(-(' num2str(i+1/8,8) '-
h)^2 + R^2)=(' num2str(input(j+2),8)
'-k)^2'];


        s=solve(['(-(' num2str(i,8)
'-h)^2 + R^2)=(' num2str(input(j),8)
'-k)^2'],['(-(' num2str(i+1/16,8) '-
h)^2 + R^2)=(' num2str(input(j+1),8)
'-k)^2'],['(-(' num2str(i+1/8,8) '-
h)^2 + R^2)=(' num2str(input(j+2),8)
'-k)^2']);
        R=double(s.R);
        double(s.h);
        double(s.k);
        R=abs(R(1));
        Q=sqrt((step^2)+(input(j+2)-
input(j))^2);

        theta=2*asin(Q/(2*R));

        L = [L R*theta];

        j=j+2;


    end

sum(L)

end
```

code in folder 5:

```
step=0.1;
j=d_point-step:step:d_point+step;
x=[d_point-5*step d_point-4*step
d_point-3*step d_point-2*step
d_point-step d_point d_point+step
d_point+2*step d_point+3*step
d_point+4*step d_point+5*step];
input=eval(get(handles.func,'String'
));

j=d_point;

m=[
      1 j-5*step (j-5*step)^2 (j-
5*step)^3 (j-5*step)^4 (j-5*step)^5
(j-5*step)^6 (j-5*step)^7 (j-
5*step)^8 (j-5*step)^9 (j-
5*step)^10;
      1 j-4*step (j-4*step)^2 (j-
4*step)^3 (j-4*step)^4 (j-4*step)^5
(j-4*step)^6 (j-4*step)^7 (j-
4*step)^8 (j-4*step)^9 (j-
4*step)^10;
      1 j-3*step (j-3*step)^2 (j-
3*step)^3 (j-3*step)^4 (j-3*step)^5
(j-3*step)^6 (j-3*step)^7 (j-
3*step)^8 (j-3*step)^9 (j-
3*step)^10;
      1 j-2*step (j-2*step)^2 (j-
2*step)^3 (j-2*step)^4 (j-2*step)^5
(j-2*step)^6 (j-2*step)^7 (j-
2*step)^8 (j-2*step)^9 (j-
2*step)^10;
```

```
      1 j-step (j-step)^2 (j-step)^3
(j-step)^4 (j-step)^5 (j-step)^6 (j-
step)^7 (j-step)^8 (j-step)^9 (j-
step)^10;
      1 j j^2 j^3 j^4 j^5 j^6 j^7
j^8 j^9 j^10;
      1 j+step (j+step)^2 (j+step)^3
(j+step)^4 (j+step)^5 (j+step)^6
(j+step)^7 (j+step)^8 (j+step)^9
(j+step)^10;
      1 j+2*step (j+2*step)^2
(j+2*step)^3 (j+2*step)^4
(j+2*step)^5 (j+2*step)^6
(j+2*step)^7 (j+2*step)^8
(j+2*step)^9 (j+2*step)^10;
      1 j+3*step (j+3*step)^2
(j+3*step)^3 (j+3*step)^4
(j+3*step)^5 (j+3*step)^6
(j+3*step)^7 (j+3*step)^8
(j+3*step)^9 (j+3*step)^10;
      1 j+4*step (j+4*step)^2
(j+4*step)^3 (j+4*step)^4
(j+4*step)^5 (j+4*step)^6
(j+4*step)^7 (j+4*step)^8
(j+4*step)^9 (j+4*step)^10;
      1 j+5*step (j+5*step)^2
(j+5*step)^3 (j+5*step)^4
(j+5*step)^5 (j+5*step)^6
(j+5*step)^7 (j+5*step)^8
(j+5*step)^9 (j+5*step)^10;
      ];

f=[input(1);input(2);input(3);input(
4);input(5);input(6);input(7);input(
8);input(9);input(10);input(11);];

   sol=m\f;

    a0=sol(1);
    a1=sol(2);
    a2=sol(3);
    a3=sol(4);
    a4=sol(5);
    a5=sol(6);
    a6=sol(7);
    a7=sol(8);
    a8=sol(9);
    a9=sol(10);
    a10=sol(11);


    switch d_order
        case 1
             diff_value=a1+(2*a2*j)
+ 3*a3*j^2 + 4*a4*j^3 + 5*a5*j^4 +
6*a6*j^5 + 7*a7*j^6 + 8*a8*j^7 +
9*a9*j^8 + 10*a10*j^9;
        case 2
             diff_value=(2*a2) +
3*2*a3*j + 4*3*a4*j^2 + 5*4*a5*j^3 +
6*5*a6*j^4 + 7*6*a7*j^5 + 8*7*a8*j^6
+ 9*8*a9*j^7 + 10*9*a10*j^8;
        case 3
             diff_value=3*2*a3 +
4*3*2*a4*j + 5*4*3*a5*j^2 +
6*5*4*a6*j^3 + 7*6*5*a7*j^4 +
8*7*6*a8*j^5 + 9*8*7*a9*j^6 +
10*9*8*a10*j^7;
        case 4
             diff_value=4*3*2*a4 +
5*4*3*2*a5*j + 6*5*4*3*a6*j^2 +
7*6*5*4*a7*j^3 + 8*7*6*5*a8*j^4 +
9*8*7*6*a9*j^5 + 10*9*8*7*a10*j^6;
        case 5
             diff_value= 5*4*3*2*a5
+ 6*5*4*3*2*a6*j + 7*6*5*4*3*a7*j^2
+ 8*7*6*5*4*a8*j^3 +
9*8*7*6*5*a9*j^4 +
10*9*8*7*6*a10*j^5;
        case 6
             diff_value=6*5*4*3*2*a6
+ 7*6*5*4*3*2*a7*j +
8*7*6*5*4*3*a8*j^2 +
9*8*7*6*5*4*a9*j^3 +
10*9*8*7*6*5*a10*j^4;
        case 7

diff_value=7*6*5*4*3*2*a7 +
8*7*6*5*4*3*a8*2*j +
9*8*7*6*5*4*3*a9*j^2 +
10*9*8*7*6*5*4*a10*j^3;
        case 8

diff_value=8*7*6*5*4*3*2*a8 +
9*8*7*6*5*4*3*2*a9*j +
10*9*8*7*6*5*4*3*a10*j^2;
        case 9

diff_value=9*8*7*6*5*4*3*2*a9 +
10*9*8*7*6*5*4*3*2*a10*j;
    end
set(handles.result,'String',num2str(
diff_value,15));

axes(handles.axes1)
plot(x,input)
```

code in folder 6:

```
x=begin:(1/10):finish;
```

```
input=eval(get(handles.func,'String'
));
j=begin;
s=[];
gen=[];
maxorder=str2num(get(handles.max_ord
er,'String'));
for i=1:10:length(input)-10

    for p=1:1:maxorder+1

        for q=1:1:maxorder+1
   m (p,q)= (j+(p-1)/10)^(q-1);
        end
    end

    for p=1:1:maxorder+1

        f(p)=input(i+p-1);
    end
   f= f(:);

   sol=m\f;
   for p=1:1:maxorder+1
    a(p)=sol(p);
   end

    for k=j:0.1:j+0.9
        b=0;
        for p=1:1:maxorder+1

      b= b+ (a(p) * k^(p-1)) ;
        end

        gen=[gen b];
    end

  b=0;
  for p=1:1:maxorder+1

  b= b +( a(p)* ((j+1)^p - j^p)/p);
  end

  s= [s b];

   j=j+1;

end

set(handles.result,'String',num2str(
sum(s),50));

axes(handles.axes1)
plot(x,input)

axes(handles.axes2)
plot([begin:0.1:finish-0.1],gen)…
```

## References:

**Advanced engineering mathematics for Erwin kreyszig the university Columbus, Ohio (seventh edition)**

### Author information :

- **Born on 2/8/1988**
- **Start researching at 17 years old**
- **Had a contacts with USPTO**
- **Finished a lot of researches in mathematics branches and digital communications and trying to publish them**
- **Graduated from Alexandria University in Egypt**
- **Department of communication & electronics in the faculty of engineering**