Two Point Resolution: An Introspection

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Abstract: Two point resolution is not unambiguously defined, it is interpreted in many ways by many researchers. In this paper, which reviews the concept of optical resolution, a number of these interpretations are discussed. A discussion of resolution preceded by the classical approach to the study of two-point resolution are dealt. The well-known resolution criterion of Sparrow and Rayleigh resolution criterion are surveyed, Only an ideal imaging system can reproduce an infinitesimally small point object as an infinitesimally small point image. An ideal imaging system is one in which diffraction and aberrations are absent. A point-source object can be represented mathematically by a delta function called the "Point Spread Function (PSF)". This spread of light in the image is determined jointly by diffraction, aberration and also the non-uniformity of amplitude and/or phase transmission specified by the pupil function, if, particularly, the optical system is apodised. Resolution also depends on the coherence conditions of illuminance. Light waves from two distinct self-luminous point sources are incoherent, as is true for double stars imaged by a telescope. Incoherent imaging is linear in intensity. Therefore the intensity distribution produced by two incoherent point sources is obtained by adding their separate intensity diffraction patterns. Apodization processes narrowing the main lobe of the point-spread function improve the resolution in the sense of the classical criteria. However, these criteria are based on calculated images for which in principle no obvious limit to resolution exists. It remains to be seen if apodization still enhances resolution if it is applied to detect images.
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1. INTRODUCTION

Resolution has always been, and still is, an important issue in applied science. Since it is not unambiguously defined, it is interpreted in many ways. In this paper, which reviews the concept of optical resolution, a number of these interpretations are discussed.

A discussion of resolution has to be preceded by a discussion of what is actually understood by an "optical image." In a remarkable paper, Ronchi[1] distinguished ethereal images, calculated images, and detected images. The term ethereal image was introduced only to represent the physical nature of the imaging phenomenon. Attempts have been made to give a mathematical representation of this phenomenon, both geometrically and algebraically. According to Ronchi, the images that have thus been calculated are mere mathematical constructions and should therefore be called calculated images. In the past, many approaches to the concept of resolution concerned these calculated images. This resulted in the so called classical resolution criteria, such as Rayleigh's criterion and the associated reciprocal bandwidth of the image. These criteria provide resolution limits that are determined solely by the calculated shape of the point-spread function associated with the imaging aperture and the wavelength of the light. Calculated images are exactly describable by a mathematical model and thus noise free. Such images do not occur in practice. Therefore Ronchi stated that the resolution of detected images is much more important than the classical resolution, since it provides practical information about the imaging system employed. Hence one should consider primarily the resolution of detected images instead of that of calculated images. This means a necessary introduction of some new quantities of interest, such as the energy of the source and the sensitivity properties of the detector. Since Ronchi's paper, further research on resolution—concerning detected images instead of calculated ones— has shown that in the end, resolution is limited by systematic and random errors resulting in an inadequacy of the description of the observations by the mathematical model chosen. This important conclusion was independently drawn by many researchers who were approaching the concept of resolution from different points of view, which will be discussed in the subsequent sections.

The paper is organized as follows. Sections 2–5 discuss two-point resolution in the sense of classical resolution criteria, image of a point object (Section 2), its dependence on the degree of coherence (Section 3), Resolution & coherence (section-4),attempts to increase the resolution by means of apodization (Section 5) and, finally, in Section 6, conclusions have been drawn.

2. IMAGE OF A POINT OBJECT:

The image of a point is never a point. Only an ideal imaging system can reproduce an infinitesimally small point object as an infinitesimally small point image. An ideal imaging system is one in which diffraction and aberrations are absent. An imaging system is perfect, if the aberrations are absent, but, real optical imaging system are neither ideal nor perfect. Due to this, the image of a point object, formed by any real optical system is never a point. It may also be noted that no physical source is actually a mathematical point, though a point is defined as a source having definite position and no extension in space.

However**,** a point-source object can be represented mathematically by a delta function. A small bright point in the object produces an image field that is much more spread out. This image field is known as the **"Point Spread Function (PSF)"**. This spread of light in the image is determined jointly by diffraction, aberration and also the non-uniformity of amplitude and/or phase transmission specified by the pupil function, if, particularly, the optical system is apodised.

The PSF approach and the OTF approach are both used to specify the performance of a wide variety of systems. These two approaches are just two different ways of tackling the same problem. Both contain the same information, but located in two different planes, spatial and frequency. The OTF of an optical system describes the ability of the system to transfer the spatial distribution of light in an object to its image, through the transmission of modified spatial frequency components.

The OTF of an optical system is the Fourier transforms of the PSF of the system. The physical significance of the PSF is that the output of a system due to an input is obtained by convolving the input with the PSF of the system. It can be said that every image is a superposition of weighted and shifted PSF's. In other words, it means that for an imaging system, the image is the result of the convolution of the object with the PSF of the system. That is, if we know the amplitude of each point in the object field, then we can convolve this amplitude with the PSF in order to find the amplitude at each point in the image field, if there are no phase shifts. The PSF characterizes the imaging performance of an optical system for a point object. Consequently, it may be said that the PSF is the basic **"**building block**"** for constructing images of all extended objects. In this paper, however, we will not consider the images of extended objects. We shall study the images of binary stars and a star. However big they may be, they acts as a point objects to an observer on earth.

The PSF, in general, is a three dimensional function. Keeping in view the fact that optical signals deal with two dimensional functions, the PSF considered in this dissertation is restricted to be two dimensional. Further, it should be noted that throughout this dissertation, quasi-monochromatic PSF is used. The amplitude of PSF is used for coherent illumination and the intensity or the irradiance PSF is used for incoherent illumination. We would like to mention that, ultimately, the PSF turns out to be the **"**optical analogue**"** of the more general term, the **"**impulse response function**"** used in other physical systems.

The expression for the output from a physical device, to which an impulse (delta function) signal is applied, is called the "impulse response function" of the system. In mathematics, this is called the "Green's Function" of the system. In the appropriate context, the PSF is also known as the "smoothing" or "blurring function", since the operation of convolution, in physical terms, means that the object distribution is "blurred" or "smeared out" by the Point Spread Function.

Since the point spread function represents the contribution of the optical instrument alone to the formation of the image, it can also be called the **"**instrument function**".** Since, the PSF carries information from a point object from the object space to the corresponding image space, in the field of medical imaging, it is also known as the Point Source Response Function (PSRF)**.**

From what has been stated above, the Point Spread Function can be visualized as an optical analogue of the impulse response function used in communication theory. We have also stated that only an ideal imaging system can reproduce an infinitesimally small point object as an infinitesimally small point image. Further, no physical real source is actually a point, though a point source has to be defined as a source having a definite position and no extension in space (McALLISTER HULL, [2]). However, a point source object can be represented mathematically by a δ - function (BORN and WOLF, [3]) defined as

$$\delta(x) = 0 \quad \text{when } x \neq 0$$

$$= k \quad \text{when } x = 0 \qquad \ldots (1)$$

and

$$\int \partial(x)dx = 1$$

A small bright point object produces an image field that is much more spread out. This image field we have referred to as the Point Spread Function (JONES, [4]).

The physical significance of the PSF finally amounts to as that **"**the output of a system for any input can be obtained by convolving the input with the PSF of the system". A detailed study of the PSF of the system explains the effect of the diffraction or the aberrations or both on the final image and this study can be extended further to include the effects of image motion, atmospheric turbulence and other factors external to the optical system (SURENDER et al)[5].

3. TWO-POINT RESOLUTION: CLASSICAL RESOLUTION CRITERIA

Two-point resolution, which is defined as the system' stability to resolve two point sources of equal intensity, is a widely used measure of the overall resolving capabilities of an imaging system. In astronomical applications, two point resolution is not only a resolution measure but also has direct practical significance, since in this case many objects are effectively point sources. In the past, many criteria for two-point resolution have been proposed for diffraction-limited systems. These are systems with a performance that is limited only by diffraction as a result of the finite size of the system's optical components. Limitations as a result of wave-front aberrations are left out of consideration. Traditionally, it is believed that this omission is justified if the Rayleigh wave-front criterion is satisfied: the deviation of the wave front involved from a perfect sphere nowhere exceeds one quarter of the wavelength. However, Barakat[6] has shown that this criterion should be used with caution, because, within the amplitude constraint, the aberrations as a spatial function have to meet additional conditions.[7] Owing to the diffraction, the image of a point source that the system produces is not a point but the diffraction pattern of the system's imaging aperture. This diffraction pattern, which is centered about the geometrical image point of the point source, is the well-known point-spread function of the imaging system.

Of all the diffraction-related resolution criteria, the classical Rayleigh criterion[8–11] is certainly the most famous. According to the Rayleigh criterion, two point sources are just resolved if the central maximum of the intensity diffraction pattern produced by one point source coincides with the first zero of the intensity diffraction pattern produced by the other. This means that Rayleigh's resolution limit is given by the distance between the central maximum and the first zero of the intensity point-spread function of the imaging system concerned. The criterion can be generalized to include point-spread functions that have no zero in the neighborhood of their central maximum, by taking the resolution limit as the distance for which the ratio of the value at the central dip in the composite intensity distribution to that at the maxima on either side is equal to 0.81. This corresponds to the original Rayleigh limit for a rectangular aperture. Rayleigh's choice of resolution limit, which seems rather arbitrary at first sight, is based on presumed resolving capabilities of the human visual system. This system has been employed as a sensor to detect differences in intensity at various points of the composite intensity distribution. Rayleigh said this about his criterion: ''This rule is convenient on account of its simplicity and it is sufficiently accurate in view of the necessary uncertainty as to what exactly is meant by resolution''.

Other notable examples of resolution criteria are those of Schuster,[12] Houston,[13] and Buxton.[14] Schuster's criterion states that the two point sources are just resolved if no portion of the main lobe (central band) of the diffraction pattern of one overlaps the main lobe of the other. This criterion provides a resolution limit that is twice that of Rayleigh. Houston proposed a criterion according to which the two point sources are just resolved if the distance between the central maxima of the composite intensity distribution equals the full width at half-maximum of the diffraction pattern of either point source. Buxton has proposed a criterion similar to that of Houston. However, instead of the intensity diffraction patterns, he used the amplitude diffraction patterns for his criterion. The amplitude diffraction pattern may be taken as the square root of the intensity diffraction pattern. According to Buxton's criterion, at the limit of resolution the component amplitude diffraction patterns should intersect at their points of inflection.

Since Rayleigh's days, technical progress has provided us with more and more refined sensors. Therefore, when visual inspection is replaced by intensity measurement, the natural resolution limit that is due to diffraction would be the distance between the two point sources for which the second-order derivative of the composite intensity distribution at the center of the diffraction image just vanishes. Then both central maxima and the minimum in between just coincide, and therefore even a hypothetical perfect measurement instrument would not be able to detect a central dip in the composite intensity distribution, simply because there is no such dip anymore. This resolution limit is known as the Sparrow limit.[15] Ramsay *et al.*[16] gave a clear classification and comparison of the just-mentioned classical criteria and several others. All classical criteria are to a certain extent a measure of the width of the main lobe of the point-spread function associated with the imaging aperture. Consequently, the classical criteria produce resolution limits that are independent of any condition other than the size and shape of the imaging aperture and the wavelength of the light.

As mentioned above, the classical resolution criteria concern calculated images, that is, images exactly describable by a known, two-component, mathematical model. However, if calculated images were to exist, the known two-component model could be fitted numerically to the observations with respect to the component locations and amplitudes. Then the solutions for these locations and amplitudes would be exact, a perfect fit would result, and in spite of diffraction there would be no limit to resolution no matter how closely located the two point sources; this would mean that no limit to resolution for calculated images would exist. However, imaging systems constructed without any aberration or irregularity are an ideal that is never reached in practice. Therefore the shape of the point-spread function is never known exactly. This means that systematic errors in the fitted two-component model invariably are introduced. Furthermore, the measurements are never completely noise free, which means the introduction of random errors. Consequently, calculated images do not occur in practice. It was reformulated by Goodman,[17] who stated that the ability to resolve two point sources depends fundamentally on the signal-to-noise ratio (SNR) associated with the detected image intensity pattern and that therefore criteria that do not take account of noise are subjective. It is concluded that if there is an ultimate limit to resolution, it must be a consequence of the fact that, as a result of systematic and random errors, detected images are never exactly described by the model adopted.

4. RESOLUTION AND COHERENCE

It is known that resolution also depends on the coherence conditions of illuminance. In this section this dependence is briefly discussed. In general, light waves from two distinct self-luminous point sources are incoherent, as is true for double stars imaged by a telescope. Incoherent imaging is linear in intensity. Therefore the intensity distribution produced by two incoherent point sources is obtained by adding their separate intensity diffraction patterns. Of course, this is no longer allowed if, for example, the two point sources are created by illuminating two pinholes in an opaque screen. Then some degree of phase correlation will exist, which has to be taken into account when diffraction patterns are added in the image plane.

Point sources that radiate fully coherently may be treated as additive in complex amplitude. If neither of these extremes applies, we speak of partial coherence. Zernike[18] introduced the degree of coherence as a measure of the correlation of the waves at different places in the image plane. A thorough treatment of the concept of partially coherent imaging can be found in Refs. [19] and [20]. To decide whether coherent, incoherent, or partially coherent analysis should be applied, one must compare the width of the so-called region of coherence with the width of the point-spread function of the imaging system.[21] If the region of coherence is so wide that the degree of coherence is almost unity over the point-spread function, the system may be considered coherent. Conversely, if the region of coherence is small compared with the width of the point-spread function, the system may be considered incoherent. If the two widths are comparable, the system must be treated as partially coherent.

In their original context, classical resolution criteria such as Rayleigh's and Sparrow's tacitly assume incoherent illumination. Abbe was the first to extend two-point resolution to fully coherent illumination with special reference to the microscope.18 Later Luneberg,[22] McKechnie,[23] and Born and Wolf[20] generalized these criteria to include partially coherent illumination.

Authors discussing two-point resolution of partially coherent imaging systems are, among others, Grimes and Thompson,[24] McKechnie,[25] and Nayyar and Verma.[26] Grimes and Thompson stated that the only directly measurable quantity in the image of the two point sources is the separation between the two central maxima of the composite intensity distribution. Notice that thus the possibility of extracting analytic data from measurements by means of model fitting is disregarded. Grimes and Thompson also studied how accurately this measured separation describes the real separation between the two point sources, under varying conditions of coherence. They found that for any degree of coherence but zero the real separation and the measured separation differed systematically except for specific values of the real separation. Therefore they concluded that the correct separation of the two point sources might not be measurable even when the classical criteria predicted good resolution. Nayyar and Verma[26] investigated the dependence of the two-point resolution of a Gaussian circular aperture on the degree of coherence. They found that the resolution, according to both the Sparrow and the Rayleigh criteria, increases almost monotonously with the degree of coherence.

5. RESOLUTION AND APODIZATION

The pupil function of an imaging system is defined as the spatial distribution of transmittance in the plane containing the exit aperture.[27] When the transmittance over the aperture is uniform, the pupil function is equal to one at points in the aperture and equal to zero at points outside the aperture. This leads to conventional point-spread functions such as, for example, Airy functions and sine square functions for circular and rectangular apertures, respectively. However, it is possible to produce a varying amplitude distribution over the aperture, for instance, by placing a nonuniformly absorbing filter or screen at the aperture. Such a modification of the uniform amplitude distribution over the aperture (or pupil) is known as apodization. In order to conform with its etymology, the term apodization would have to be restricted to those modifying processes that suppress, or at least considerably decrease, the *feet* (or sidelobes) of the point-spread function.[28] However, in this paper we shall deal with apodization in the widest sense.

In the past, several apodization procedures aimed at improving the resolution have been proposed. In general, apodization procedures try to narrow the main lobe of the point-spread function, which improves resolution in the sense of the classical criteria. Most of them are based on the Rayleigh criterion or the Sparrow criterion.[30-32] One way of apodization is to expand the pupil function in some complete set of functions with arbitrary coefficients and then to adjust these coefficients to approximate a prespecified point-spread function. A different approach is to use the calculus of variations to determine the optimal pupil function.[22],[32],[33] Once the pupil function has been optimized, it can be implemented in practice by modifying the aperture with a suitable filter, using, for example, photographic or metal deposition techniques, or by electronic processing of the image signal.

Generally, apodization achieves an improvement only of certain qualities at the expense of others. This is illustrated as follows. Luneberg [22]found that, among all point-spread functions of equal energy, the one with the highest central maximum corresponds to uniform transmittance. Therefore any point-spread function that gives improved resolution must have a lower central maximum. This may be undesirable. Wilkins[34] has shown that there is no minimum width of the main lobe of the point-spread function below which it is impossible to go. That is, the main lobe can be narrowed indefinitely by means of apodization. Therefore it is theoretically possible to attain unlimited resolution in the Rayleigh sense. However, there is no practical interest in taking the apodization process to extremes, since narrowing the main lobe of the point-spread function will generally have the secondary effect of a considerable rise in the level of the sidelobes of the modified point-spread function. For example, it is well known that an obstruction in the central part of the aperture of an imaging system enhances the resolution in the Rayleigh sense. However, this apodization technique is not often used, because there is an increasing loss of light in the image as more of the aperture is obstructed. Furthermore, there is a deterioration in image quality owing to the larger amount of light diffracted from its proper geometrical position, or, stating the latter more explicitly, the point-spread function produced by the obstructed system contains more energy in its sidelobes. [23] In spectral analysis such as leak of energy from the main lobe to the sidelobes is known as leakage. Leakage makes the image more difficult to interpret.

With recognition of the practical limits mentioned above, later work on apodization focused on the problems of finding for a specified Rayleigh limit the pupil function (and associated point-spread function) having maximum central irradiance[35–37] and the pupil function corresponding to a point-spread function having as much of its energy as possible concentrated in a circle of a specified radius.[38]

In conclusion, apodization procedures that narrow the point-spread function result in an improvement of the resolution in the sense of the classical resolution criteria. However, these criteria are based on calculated images for which, as explained in before, in principle no obvious limit to resolution exists. It remains to be seen whether apodization still results in an improvement if applied to detected images. These images are always corrupted by errors which ultimately limit resolution.

6. CONCLUSIONS

The classical resolution criteria, such as Rayleigh's and the associated spectral bandwidth, concern calculated images. These images are by definition noise free and exactly describable by a known mathematical model. The corresponding resolution limits are a measure of the width of the main lobe of the point-spread function and therefore independent of any condition other than the size and the shape of the imaging aperture and the wavelength of the light. More recently it has been recognized that if calculated images were to occur in practice, there would be no limit to resolution at all. It follows from model-fitting theory and the work on superresolution that then one could attain as high a resolution as desired. Therefore the classical criteria certainly do not represent the ultimate limit to resolution. Limits to resolution stem from the fact that in practice, detected instead of calculated images are encountered. These detected images are always disturbed by noise, that is, nonsystematic errors. Furthermore, the point-spread function will never be exactly known. This introduces systematic errors. It is these errors, both systematic and nonsystematic, that prevent unlimited resolution. This main conclusion follows from considerations starting from different points of view, such as information theory, linear filter theory, decision theory, and parameters timation theory. It has inspired researchers to propose new resolution criteria that, unlike the classical criteria, take the measurement errors into account. It has been found that if the system's transfer function is known with sufficient accuracy and the noise level is low, super resolution procedures can provide resolution beyond the classical limits. With respect to two-point resolution, the results discussed in this paper indicate that if the model is properly specified, there is no basic obstacle to resolve two point sources, even when the separation is significantly less than the classical limits. Apodization processes narrowing the main lobe of the point-spread function improve the resolution in the sense of the classical criteria. However, these criteria are based on calculated images for which in principle no obvious limit to resolution exists. It remains to be seen if apodization still enhances resolution if it is applied to detected images. In any case, apodization procedures often have the secondary unfavorable effect of a loss of light and an increasing level of the side lobes of the point spread function and should therefore not be taken to extremes.

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