Solving Intuitionistic Fuzzy Linear Programming by using Metric Distance Ranking

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Abstract: This paper proposes a new method of solving Intuitionistic Fuzzy Linear Programming Problem [IFLLP], where all the parameters and variables are Intuitionistic fuzzy numbers. Here we introduce a new ranking technique called Metric Distance Ranking of Intuitionistic fuzzy numbers. Based on this new approach, we solve Intuitionistic Fuzzy Linear Programming Problem (IFLPP) with a numerical example.

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1. Introduction

The theory of Fuzzy set was introduced by Zadeh [9]. The Fuzzy set was extended to develop the Intuitionistic fuzzy set , IFS was introduced by Attanosov [1], by adding an additional non-membership degree and hesitancy degree. In real life, the information available is insufficient, vague or inexact. Therefore it is desirable to consider the knowledge of experts about the parameters as fuzzy data. Out of several higher order fuzzy sets, Intuitionistic fuzzy sets have been found to be highly useful to deal with vagueness. The Intuitionistic fuzzy number is used for qualifying an ill-known quantity. In operations research, linear programming is one of the most techniques. Intuitionistic fuzzy set is applied in many fields such as decision making, medical diagonosis and logic programming etc. Intuitionistic fuzzy optimization was introduced by Hassan Nehi[4].

Ranking fuzzy number is important in decision making, data analysis, economic systems and Operations research. Ranking of Intuitionistic fuzzy number plays an important role in Intuitionistic fuzzy clustering and Intuitionistic fuzzy decision making. Ordering method of Intuitionistic fuzzy number was proposed by Hassan Nehi [3]. V.L.G.Nayagam [8] proposed scoring method for Intuitionistic fuzzy numbers. Ratio ranking [5] was proposed by Deng Feng Li. The aim of the paper is to introduce a new ranking technique called metric distance ranking of Intuitionistic fuzzy number. Based on this approach, IFLPP will be solved. Dipti Dubey [2] solved the Linear programming with Triangular Intuitionistic fuzzy numbers in which Triangular Intuitionistic fuzzy numbers are converted to crisp set and solved. In [6] the author solved Fuzzy linear Fractional programming problems by using metric distance method and in [7] we solved Intuitionistic fuzzy linear programming problems by using similarity measures.

The organization of the paper is as follows:

In section 2 the basic definitions and operations of IFN are discussed. Existing ranking methods are discussed in section3. In section 4 we propose a new ranking method called metric distance ranking for IFN (Both TrIFN and TIFN). Section 5 defines the concepts of IFLPP and the solution procedure is explained through the Algorithm. Based on this new approach a Numerical example is illustrated in section 6. Section 7 concludes the paper.

2. Definitions and Operations

In this section some of the fundamental definitions, operations and concepts of Intuitionistic fuzzy sets theory are reviewed.

2.1 Defition: Let $X=\{x_1,x_2,.....,x_n\}$. An **Intuitionistic fuzzy set (IFS)** is defined as $A=\left(\langle x_i, t_A(x_i), f_A(x_i)\rangle / x_i \in X\right)$ which assigns to each element x_i a membership degree $t_A(x_i)$ and non-membership degree $f_A(x_i)$ under the condition $0 \le t_A(x_i)+f_A(x_i) \le 1$, for all $x_i \in X$.

2.2 Definition: A **Triangular Intuitionistic Fuzzy Number** $\widetilde{A}^I=[(a_1,a_2,a_3)(a_1',a_2,a_3')]$ in R with membership function and non membership function is defined as follows:

$$\mu_{\tilde{A}^I}(x)=\begin{cases}\dfrac{(x-a_1)}{a_2-a_1} & \text{if } a_1 \le x \le a_2 \\ \dfrac{x-a_3}{a_2-a_3} & \text{if } a_2 \le x \le a_3 \\ 0 & \text{Otherwise}\end{cases}$$

$$\nu_{\tilde{A}^I}(x)=\begin{cases}\dfrac{(a_2-x)}{a_2-a_1'} & \text{if } a_1' \le x \le a_2 \\ \dfrac{(x-a_2)}{a_3'-a_2} & \text{if } a_2 \le x \le a_3' \\ 1 & \text{Otherwise}\end{cases}$$

Where $a_1' \le a_1 \le a_2 \le a_3 \le a_3'$ and $\mu_{\tilde{A}^I}(x)$ + $\nu_{\tilde{A}^I}(x) \le 1$

2.3 Arithmetic Operations on Trapezoidal Intuitionistic Fuzzy Number

If $\tilde{A}^I = (a_1,a_2,a_3,a_4)(a_1',a_2,a_3,a_4')$ and $\tilde{B}^I = (b_1,b_2,b_3,b_4)(b_1',b_2,b_3,b_4')$ are two TrIFN [ ]. Then

$$\tilde{A}^I \oplus \tilde{B}^I = (a_1+b_1, a_2+b_2, a_3+b_3)(a_1'+b_1', a_2+b_2, a_3'+b_3')$$

$$\tilde{A}^I \otimes \tilde{B}^I = (a_1b_1, a_2b_2, a_3b_3)(a_1'b_1', a_2b_2, a_3'b_3')$$

$$\tilde{A}^I - \tilde{B}^I = \begin{pmatrix} a_1-b_3, a_2-b_2, a_3-b_1) \\ (a_1'-b_3', a_2-b_2, a_3'-b_1') \end{pmatrix}$$

$$\tilde{A}^I / \tilde{B}^I = \begin{pmatrix} a_1/b_3, a_2/b_2, a_3/b_1) \\ (a_1'/b_3', a_2/b_2, a_3'/b_1') \end{pmatrix}$$

If $\tilde{A}^I = (a_1,a_2,a_3;\ a_1',a_2,a_3')$ is a TrIFN and y=ka with (k>0) then $\tilde{y}^I = k\tilde{A}^I$ is TrIFN $(ka_1,ka_2,ka_3;\ ka_1',ka_2,ka_3')$

If y=ka with (k<0) then $\tilde{y}^I = k\tilde{A}^I$ is TrIFN $(ka_3,ka_2,ka_1;\ ka_3',ka_2,ka_1')$

3. Existing Ranking Methods

We discuss some of the existing ranking methods:

3.1 Deng Fend Li: A Value index and Ambiguity Index for the Triangular Intuitionistic Fuzzy Number are defined as

$V(\tilde{a},\lambda)=V_\mu(\tilde{a})+\lambda[V_\nu(\tilde{a})-V_\mu(\tilde{a})]$ And

$A(\tilde{a},\lambda)=A_\nu(\tilde{a})-\lambda[A_\nu(\tilde{a})-A_\mu(\tilde{a})]$.

He proposed a new ranking method called Ratio ranking of Triangular Intuitionistic Fuzzy Number which is defined as follows

$$R(\tilde{a},\lambda)=\frac{V(\tilde{a},\lambda)}{1+A(\tilde{a},\lambda)}$$

3.2 V.L.G.Nayagam: He proposed the modified ranking of Intuitionistic Fuzzy Number.The proposed scoring method has been applied to clustering based on IFN. Let (a,b,c) be a Triangular IFN. The Total Score of M is given by

$T(M)=\dfrac{[L(M)+1-R(M)]}{2}$, where left and right score is defined as

$L(M)=\dfrac{a}{1-b+a}$ And $R(M)=\dfrac{1-c}{1-c+b}$ respectively.

3.3 Debashre Guha: A new distance measure for IFN is proposed on interval difference.

Let $A=(m;\alpha,\beta;\alpha',\beta')$. The distance measure between two Triangular IFN is defined as $d_{IFN}=(d,\theta_1,\tau_1,\theta_2,\tau_2)$ where

$$\theta_1 = d_u^l(1)-\max\left\{\int_0^1 d_u^l(\varepsilon)d\varepsilon, 0\right\},$$

$$\tau_1=\int_0^1 d_u^R(\varepsilon)d\varepsilon - d_u^R(1)$$

$$\theta_2 = d_{1-\nu}^l(1)-\max\left\{\int_0^1 d_{1-\nu}^l(\varepsilon)d\varepsilon, 0\right\},$$

$$\tau_2=\int_0^1 d_{1-\nu}^R(\varepsilon)d\varepsilon - d_{1-\nu}^R(1)$$

The above ranking methods are used only to rank the Triangular Intuitionistic Fuzzy Numbers.

3.4 Hassan Mishmast Nehi: He proposed a method of ranking IFNs based on the characteristic Values of the membership and non-membership of IFN.

Let $A=(b_1,a_1,b_2,a_2,a_3,b_3,a_4,b_4)$

$$C_{\mu}^{k}(A)=\frac{a_2+a_3}{2}+\frac{(a_1-a_2)+(a_4-a_3)}{2(k+2)}$$

$$C_{\nu}^{k}(A)=\frac{b_1+b_4}{2}+\frac{(b_2-b_1)+(b_3-b_4)}{2(k+2)}.$$

Using Nehi's ordering method; we can rank only Trapezoidal IFN.

Our proposed Ranking method is applicable to rank both Trapezoidal and Triangular Intuitionistic Fuzzy Number.

4. Proposed Ranking Method

An efficient approach for ordering the elements is to define a ranking function

D : T (R) $\rightarrow$R which maps for each intuitionistic fuzzy number into the real line. We define T (R) as follows:

$\tilde{A}^I \geq \tilde{B}^I$ if and only if $D(\tilde{A}^I) \geq D(\tilde{B}^I)$

$\tilde{A}^I \leq \tilde{B}^I$ if and only if $D(\tilde{A}^I) \leq D(\tilde{B}^I)$

$\tilde{A}^I = \tilde{B}^I$ if and only if $D(\tilde{A}^I) = D(\tilde{B}^I)$

$\tilde{A}^I$ and $\tilde{B}^I$ be two intuitionistic fuzzy numbers which are defined as follows: –

$$f_{\tilde{A}^I}(x)=\begin{cases} f_{\tilde{A}^I}{}^{L}(x), x < m_{\tilde{A}^I} \\ f_{\tilde{A}^I}{}^{R}(x), x \geq m_{\tilde{A}^I} \end{cases} \quad (1)$$

$$f_{\tilde{B}^I}(x)=\begin{cases} f_{\tilde{B}^I}{}^{L}(x), x < m_{\tilde{B}^I} \\ f_{\tilde{B}^I}{}^{R}(x), x \geq m_{\tilde{B}^I} \end{cases} \quad (2)$$

Where $m_{\tilde{A}^I}$ and $m_{\tilde{B}^I}$ are the mean of $\tilde{A}^I$ and $\tilde{B}^I$.

The metric distance between $\tilde{A}^I$ and $\tilde{B}^I$ can be calculated as follows:

$$D(\tilde{A}^I,\tilde{B}^I)=\left[\int_0^1 (g_{\tilde{A}^I}^{L}(y)-g_{\tilde{B}^I}^{L}(y))^2\,dy + \int_0^1 g_{\tilde{A}^I}^{R}(y)-g_{\tilde{B}^I}^{R}(y))^2\,dy\ \right]^{1/2} \quad (3)$$

Where $g_{\tilde{A}^I}^{L}, g_{\tilde{A}^I}^{R}, g_{\tilde{B}^I}^{L}$ and $g_{\tilde{B}^I}^{R}$ are the inverse functions of $f_{\tilde{A}^I}^{L}, f_{\tilde{A}^I}^{R}, f_{\tilde{B}^I}^{L}$ and $f_{\tilde{B}^I}^{R}$ respectively.

In order to rank the intuitionistic fuzzy numbers, the metric distance between A and O is calculated as follows.

$$D(\tilde{A}^I,0)=\left(\int_0^1 (g_{\tilde{A}^I}^{L}(y))^2\,dy+\int_0^1 (g_{\tilde{A}^I}^{R}(y))^2\,dy\right)^{1/2} \quad (4)$$

A triangular intuitionistic fuzzy number $\tilde{A}^I=[(a_1,a_2,a_3)(a_1',a_2,a_3')]$ can be approximated as a symmetry intuitionistic fuzzy number $S(\mu,\sigma)$, μ denotes the mean of $\tilde{A}^I$, σ denotes the standard deviation of A and the membership function of $\tilde{A}^I$ is defined as follows:

$$f_{\tilde{A}^I}(x)=\begin{cases} \dfrac{x-(\mu-\sigma)}{\sigma}, & \text{if } \mu-\sigma \leq x \leq \mu \\ \dfrac{(\mu+\sigma)-x}{\sigma}, & \text{if } \mu \leq x \leq \mu+\sigma \end{cases} \quad (5)$$

Where μ and σ are calculated as follows.

For membership function

$$\sigma=\frac{a_3 - a_1}{2} \text{ and } \mu=\frac{a_1+2a_2+a_3}{3} \quad (6)$$

For non-membership function

$$\sigma=\frac{a_3' - a_1'}{2} \text{ and } \mu=\frac{a_1'+2a_2+a_3'}{3} \quad (7)$$

To rank the membership function of intuitionistic fuzzy numbers, the metric distance between A and O is calculated as follows.

$$D_{\mu}(\tilde{A}^I,0)=\left(\int_0^1 (g_{\tilde{A}^I}^{L}(y))^2\,dy+\int_0^1 (g_{\tilde{A}^I}^{R}(y))^2\,dy\right)^{1/2}$$

To rank the non- membership function of intuitionistic fuzzy numbers, the metric distance between A and O is calculated as follows.

$$D_{\nu}(\tilde{A}^I,0)=\left(\int_0^1 (g_{\tilde{A}^I}^{L}(y))^2\,dy+\int_0^1 (g_{\tilde{A}^I}^{R}(y))^2\,dy\right)^{1/2}$$

The rank is defined as

$$D(\tilde{A}^I,0)=\frac{D_{\mu}(\tilde{A}^I,0)}{1+D_{\nu}(\tilde{A}^I,0)}$$

The inverse functions $g_{\tilde{A}^I}^{L}$ and $g_{\tilde{A}^I}^{R}$ of $f_{\tilde{A}^I}^{L}$ and $f_{\tilde{A}^I}^{R}$ respectively are shown as follows:

$$g_{\tilde{A}^I}^{L}(y)=(\mu-\sigma)+\sigma y \quad (8)$$

$$g_{\tilde{A}^I}^{R}(y)=(\mu+\sigma)-\sigma y \quad (9)$$

5. Intuitionistic Fuzzy Linear Programming

LPP: Consider the crisp LPP Max

$$\sum_{j=1}^{n} c_j x_j$$

Subject to the constraints

$$\sum_{j=1}^{n} a_{ij} x_j = b_i, \qquad i = 1,2,\ldots,m$$

$$x_j \geq 0$$

Where c_j is an $m \times n$ real matrix.

FLPP: Fuzzy Linear Programming Problem can be defined as

$$\text{Max} \sum_{j=1}^{n} \tilde{c}_j \tilde{x}_j$$

Subject to the constraints

$$\sum_{j=i}^{n} \tilde{a}_{ij} \tilde{x}_j = \tilde{b}_i \qquad i = 1,2,\ldots,m$$

$$\tilde{x}_j \geq 0$$

Where $\tilde{c}_j, \tilde{a}_{ij}$ are fuzzy numbers

IFLPP: Corresponding IFLPP is defined as

$$\text{Max} \sum_{j=1}^{n} \tilde{c}_j^I \tilde{x}_j^I$$

Subject to the constraints

$$\sum_{j=i}^{n} \tilde{a}_{ij}^I \tilde{x}_j^I = \tilde{b}_i^I \qquad i = 1,2,\ldots,m$$

$$\tilde{x}_j^I \geq 0$$

Where

$$\tilde{c}_j^I = \left\{ \left(c_{1j}, c_{2j}, c_{3j};\ c'_{1j}, c_{2j}, c'_{3j} \right) \right\}$$

$$\tilde{b}_i^I = \left(b_{1i}, b_{2i}, b_{3i};\ b'_{1i}, b_{2i}, b'_{3i} \right)$$

$$\tilde{a}_{ij}^I = \left(a_{1ij}, a_{2ij}, a_{3ij};\ a'_{1ij}, a_{2ij}, a'_{3ij} \right)$$

i=1………… m, j=1-------n are Trapezoidal Intuitionistic Fuzzy Numbers

5. 1 Definition: A set of variables $\tilde{x}_i^I = \left\langle x_1, x_2, x_3;\ x'_1, x_2, x'_3 \right\rangle$ satisfies the above constraints of a general IFLPP is called a **Intuitioistic fuzzy solution(IFS)**.

5.2 Definition: Any solution to a general FLPP which also satisfies the non negative restriction of the problem, is a called a **Intuitioistic Fuzzy Feasible solution(IFFS)**.

5.3 Definition: An Intuitionistic Fuzzy feasible solution to a IFLPP which is also a basic solution to the problem is called a **Intuitioistic Fuzzy Basic Feasible solution(IFBFS)** to the IFLPP.

5.4 Definition: An Intuitionistic Basic Fuzzy feasible solution that also optimizes the objective function is called **Intuitioistic Fuzzy Optimum Feasible solution(IFOFS).**

5.5 Algorithm to solve Intuitionistic Fuzzy Linear Programming Problem:

We propose a new algorithm to solve IFLPP as follows:

Step1: Take all the values as Trapezoidal Intuitionistic Fuzzy numbers. Apply simplex method procedure to obtain initial IFBFS.

Step2: To find most positive of $\tilde{c}_j^I - \tilde{z}_j^I$, we use the proposer ranking method. Taking the minimum value of $R(\tilde{A}_i^I)$ and which enters the basis $\tilde{y}_b$.

Step 3: Compute $\left\{ \frac{\tilde{x}_{Bi}^I}{\tilde{y}_{ir}^I}, i = 1,2,,,,m \right\}$ and choose minimum of them using proposed metric distance ranking method. Then the vector $\tilde{y}_k$ will leave the basis. This element is called Trapezoidal Intuitionistic Fuzzy pivotal number or leading element.

Step 4: Convert pivotal into unit Trapezoidal Intuitionistic Fuzzy Number and all other elements in its column to zero Trapezoidal Intuitionistic Fuzzy Number using the arithmetic operations defined in chapter 2.

Step 5: Repeat the procedure until an Intuitioistic Fuzzy Optimum Feasible solution is obtained.

6. Numerical Example

We illustrate this method with a numerical example.

Solve the following IFLPP:

Max Z = (6, 9, 11; 5, 9, 14) $\tilde{x}_1^I$ + (9, 10, 15; 7, 10, 17) $\tilde{x}_2^I$

Subject to the constraints

(2, 3, 6; 1, 3, 8) $\tilde{x}_1^I$ + (4, 6, 9; 2, 6, 15) $\tilde{x}_2^I$ $\leq$ (15, 18, 24; 11, 18, 27)

(4, 8, 9; 2, 8, 14) $\tilde{x}_1^I$ + (6, 7, 10; 5, 7, 13) $\tilde{x}_2^I$ $\leq$ (14, 20, 26; 10, 20, 29).

The initial feasible solution is given in table I

Table I. The initial feasible solution

C_B	Y_B	X_B	Y_1	Y_2	S_1	S_2
(0,0,0;0,0,0)	S_1	(15,18,24; 11,18,27)	(2 3,6; 1,3,8)	(4,6,9; 2,6,15)	(1,**1**,1;1,**1**,1)	(0,**0**,0; 0,**0**,0)
(0,0,0; 0,0,0)	S_2	(14,20,26; 10,20,29)	(4,8,9; 2,8,14)	**(6,7,10; 5,7,13)**	(0,**0**,0; 0,**0**,0)	(1,**1**,1;1,**1**,1)
$\tilde{C}_j^I - \tilde{Z}_j^I$			(6,9,11;5,9,14)	(9,10,15;7,10,17)	(0,**0**,0; 0,**0**,0)	(0,**0**,0; 0,**0**,0)

Using the proposed metric distance ranking method, we find the most positive of $\tilde{C}_j^I - \tilde{Z}_j^I$.

The most positive occurs at Y_2. To find the leaving variable, Again we use proposed Ranking,

Max $\left\{ \frac{\tilde{x}_{B1}^I}{\tilde{Y}_{11}^I}, \frac{\tilde{x}_{B1}^I}{\tilde{Y}_{21}^I} \right\}$ = Min {0.204, 0.82} = 0.82, occurs at S_2. Therefore S_2 leaves the basis and Y_2 enters the basis.**(6, 7, 10; 5,7,13)** is the leading element. Convert the leading element to unity and the reaming elements in the column to zero using the above defined arithmetic operations.

Table II. The leading element to unity and the reaming elements

C_B	Y_B	X_B	Y_1	Y_2	S_1	S_2
(0,0,0;0,0,0)	S_1	(-18.4,5.2,23.7; -25.4,5.2,70.2)	(-7.4,1.2,9.2; -14.6,1.2,40)	(-7.6,**0**,11; -11,0,26.2)	(1,**1**,1;1,**1**,1)	(0.4,1.1,1.8; 0.14,1.1,2.6)
(9,10,15; 7,10,17)	Y_2	(1.4,2.9,4.3; 0.8,2.9,5.8)	(0.4,0.9,1.5; 0.2,0.9,3)	(0.6,1,**1**.7; 0.4,1,2.4)	(0,**0**,0; 0,**0**,0)	(0.1,0.1,0.2; 0.07,0.1,0.2)
$\tilde{C}_j^I - \tilde{Z}_j^I$			(-16.5,0,7.4; -16.8,0,12.6)	(-16.5,0,11.4; -33.8,0,22.7)	(0,0,0; 0,0,0)	(-3,-1.4,-0.9; -3.4,-1.4,-0.5)

Here all the $\tilde{C}_j^I - \tilde{Z}_j^I \leq 0$ and R($\tilde{C}_j^I - \tilde{Z}_j^I$) $\leq$ 0, the current Intuitionistic Fuzzy Basic Feasible solution is obtained.

The Intuitionistic Fuzzy Optimum Basic Feasible solution is

Maximum $Z = (12.6, 29, 64.5; 5.6, 29, 98.6)$ **at**

X_1 = (0, 0, 0; 0, 0, 0) and X_2 = (1.4, 2.9, 4.3; 0.8, 2.9, 5.8)

7. Conclusion

This paper proposes a new ranking method based on the metric distance ranking that ranks Intuitionistic Fuzzy Numbers which is simple and compact. Fuzzy linear fractional programming problems with fuzzy variables and fuzzy constraints are discussed. We have proposed a new technique and algorithm to solve FLFPP.The solution methodology is illustrated through a numerical example. Thus the method is very useful in the real world problems where the product is uncertain.

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