A New Goal Programming Approach to Data Envelopment Analysis

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Abstract: Data Envelopment Analysis (DEA) has a wide range of mathematical models for measuring the relative efficiency of a set of identical units with the same inputs and outputs are used. These models, the sum of the weights for inputs and outputs of the Decision Making unit (DMU) obtain and based on the relative efficiency of each unit are computed. The freedom in the choice of weights, the DMUs put in the best position possible. The inefficient unit may for criteria in the worst efficiency, a zero weight are assigned and is recognized as an efficient unit. This issue may not be accepted by decision makers as well as analysts. It needs to define the range for controlling weight range of variation criteria in the optimal response. In this paper, a model for data envelopment analysis based on Goal Programming (GP) is proposed. In addition to limit the weight, the distinction between units greatly increases, and its application in a numerical example is provided. Linearity, wide applicability and meaningful weights are benefits of the proposed model.
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1. Introduction

In most organizations, managers have to evaluate performance homological Decision Making units (DMUs) and compare its efficiency. Many methods have been proposed for efficiency evaluation that can be divided into two group's parametric and non-parametric methods. In 1957, Farrell for the first time to proposed the non-parametric methods for estimating the efficiency [1]. Article Farrell played an important role in the study of basic, Charnes, Cooper and Rhodes called (CCR) played as the starting point for Data Envelopment Analysis (DEA) was presented. Article CCR formulation of linear programming to measure the efficiency of a decision making unit (DMU) in the case of multiple inputs and outputs extended [2]. In 1984, the paper by Banker, Cooper and Charnes known as (BCC) Model under the Variable Returns to Scale (VRS) assumption was published [3]. Also in this year, another basic model by Depress et al. introduced that called Free Disposal Hull (FDH) [4]. The model additive was introduced by Charnels et al. in 1985 [5]. In 1997, Tone introduced model Slacks-based Measure (SBM) [6]. Thereafter another basic model like Cross Efficiency (CE) and Common Set of Weight (CSW) and ... Were made [7,8,9], and according to the DEA creator's claim this method, the appearance of more than thousands of articles and books in this compilation and many research centers are working on it [9]. The use of classical DEA models often occur two problems that one of in relation to weakness of discriminate power and another unrealistic weight distribution between the inputs and output. The weakness of discriminate power problem arises that the number of units under evaluation sufficiently in comparison with the total number of inputs and outputs is not large. In this case, many classical models are identified a large number of DMUs as efficient. The problem weight of unreasonable when occurs that the model of large weight to an output or a very small weight to an input assigned that this is an unreasonable and adverse [10]. On the other hand, the methods of multi-objective decision-making is branch of operations research that have great help to the decision makers in practical situations to decide which of several different purposes, and sometimes antonym are faced. One of the methods of multi-objective decision-making is Goal Programming (GP). By Charnes and Cooper, first appeared in 1960. GP is the first technique in multi-objective planning function that has fairly wide acceptance for use in various areas of decision making in industry and services. Linear Goal Programming problem is a Linear Programming (LP) problem that seeks to meet more than one objective [11, 12]. In 1986, Sexton et al. introduced model Minsum and total deviations of all deviation variables used as the objective function in DEA models [13]. Kornbluth in 1991 stated that the DEA model can be considered as a problem of multi-objective linear fractional [14]. Troutt in 1995, Maximin efficiency ratio model has been proposed [15]. In which a common set of weights is used to distinguish between efficient units. In 1996, Stewart used from the function minimum of maximum deviations (Minimax) in DEA model [16]. Li and Reeves applied for increasing distinguish between the DMU and a multi-objective linear programming (MOLP) [17]. Another introduction to research in the field of data envelopment analysis is presented, Methods that can be used to check DEA models with weight restrictions has been noted. Among the possible ways to examine the returns to scale in DEA models with weight restrictions has been noted [18], and or Farzipoor Saen to classify different types of weight restrictions is turned [19]. Mohaghar et al. pointed out that the new method of weight restrictions introduced [20], for more information and to find other ways to measure efficiency and non-efficiency refer to [17, 21 & 22]. So can be said that the DEA model based on GP model than classical models have higher ability in discriminate power and offering real weights. The present paper, a new model of DEA based on model GP provides. Thus, this paper is formed. In the second section, a brief review of the models is discussed and the proposed model is presented in the end of second part. And the third part is to show an example of the model and its results have been evaluated. Finally, the fourth section, conclusions and summarize are stated.

2. Proposed method

Classical models of DEA to evaluate relative units by Charnels et al. to assess the unit under evaluation were presented as follows [2]:

$$\begin{aligned}
&Max\, Z. = \sum_{r=1}^{s} u_r y_{ro}, \\
&St: \\
&\sum_{i=1}^{m} v_i x_{io} = 1, \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} x_{ij} v_i \le 0, \qquad (j = 1,2,.....,n), \\
&u_r, v_i \ge 0, \qquad (r = 1,2,....,s),(i = 1,2,...,m).
\end{aligned} \qquad \text{Model (1)}$$

The j number of units under consideration $(j = 1,2,.....,n)$, r the number of outputs $(\text{r} = 1,2,.....,s)$, I the number of inputs $(\text{i} = 1,2,.....,m)$, y_{rj} the r th output for the j th unit, x_{ij} input value i am for unit j th, u_r weight allocated to the r th output and v_i the weight given to the i th input. $Z.$ The relative efficiency of the unit under evaluation will tell. On issues related to GP addition to the common variables in Linear Programming, other variables as “variables, deviations from ideal” are defined. These variables represent the difference between the determined ideal and Earned Value. Model of classical DEA (1) can be a DEA model with the goal of minimizing the deviation variables and the Goal Programming model is presented as follows:

$$\begin{aligned}
&Min = d., \\
&St: \\
&\sum_{i=1}^{m} v_i x_{io} = 1, \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} x_{ij} v_i + d_j = 0, \qquad (j = 1,2,.....,n), \\
&u_r, v_i, d_j \ge 0, \qquad (r = 1,2,....,s),(i = 1,2,...,m),(j = 1,2,.....,n).
\end{aligned} \qquad \text{Model (2)}$$

That $d.$ deviation variable is for the unit under evaluation and d_j deviation variable for unit j (that appearing in the inequality constraints j).In the model under consideration when the unit is efficient $Z.=1$ or $d.=0$ (model 2-1). If the unit under evaluation is not efficient; therefore, its efficiency scores are equal $Z.=1-d.$. Here, the model is the same model of classical DEA. Value $d.$ in the range of $[0,1)$ states inefficiency. Whatever, $d.$ is less than, inefficiency under evaluation for lower is lower (and so more efficiency). Therefore, can be said that the classical model to minimize inefficiency of the DMU under evaluation and measuring with $d.$ and the constraint that the sum of weighted outputs is less than or equal to sum of weighted inputs [17, 22]. In 1986, Sexton used total deviations of deviation variables as a function in the DEA model. This model is called (MinSum) and the general form of this model is as follows:

$$
\begin{aligned}
& Min = \sum_{j=1}^{n} d_j, \\
& St: \\
& \sum_{i=1}^{m} v_i x_{io} = 1, \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} x_{ij} v_i + d_j = 0, \qquad (j=1,2,\ldots,n), \\
& u_r, v_i, d_j \geq 0, \qquad (r=1,2,\ldots,s),(i=1,2,\ldots,m),(j=1,2,\ldots,n).
\end{aligned}
$$

Model (3)

Value efficiency for j is obtained from ($Z.=1-dj$) [17, 22].

As earlier mentioned, the use of classical DEA models usually occur two problems that one in relation to the lack of discrimination power and the other is unrealistic weight distribution of the other's inputs and output criteria. Therefore, the following model is introduced to solve these problems:

$$
\begin{aligned}
& Min = W_1\left(\sum_{j=1}^{n} d_j\right) + W_2 (d' + d''), \\
& St: \\
& \sum_{i=1}^{m} v_i x_{io} = 1, \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} x_{ij} v_i + d_j = 0, \qquad (j=1,2,\ldots,n), \\
& P \leq u_r \leq 1-P, \qquad (r=1,2,\ldots,s), \\
& Q \leq v_i \leq 1-Q, \qquad (i=1,2,\ldots,m), \\
& P + d' = 0.5, \\
& Q + d'' = 0.5, \\
& d_j, d', d'' \geq 0, \qquad (j=1,2,\ldots,n).
\end{aligned}
$$

Model (4)

This model (4) is used for two purposes: 1) minimum of sum of deviations $\sum_{j=1}^{n} d_j$, 2) to minimize the weight range of inputs and outputs. Parameters $W1$ and $W2$ represent Prioritizing Goals. Variables P and Q are outputs and inputs respectively limit range of output variables u_r and input variables v_i , and the values are between ranges

$[0,0.5]$. If value deviation variables d' and d'' is less than variables value P and Q respectively increases and as previously noted, with increasing value P and Q, the weight range outputs u_r and inputs v_i respectively becomes limited.

3. Numerical Example

To evaluate the proposed model, its application is investigated in a numerical example. To this purpose, here's an example offering of Kao and Hung's 2005 have been used. These example is on measuring the efficiency of 17 of Planting areas with 4 inputs budgeting, initial inventory, labor and land, and 3 outputs main product, soil conservation and reconstruction. In Table 1 is presented the primary data [23]:

Table 1: Primary data

DMU (District)	Budget (dollars)	Initial stocking (m^3)	Labor (person)	Land (ha)	Main product (m^3)	Soil conservation (m^3)	Recreation (visits)
DMU1	51.62	11.23	49.22	33.52	40.49	14.89	3166.71
DMU2	85.78	123.98	55.13	108.46	43.51	173.93	6.45
DMU3	66.65	104.18	257.09	13.65	139.74	115.96	0
DMU4	27.87	107.6	14	146.43	25.47	131.79	0
DMU5	51.28	117.51	32.07	84.5	46.2	144.99	0
DMU6	36.05	193.32	59.52	8.23	46.88	190.77	822.92
DMU7	25.83	105.8	9.51	227.2	19.4	120.09	0
DMU8	123.02	82.44	87.35	98.8	43.33	125.84	404.69
DMU9	61.95	99.77	33	86.37	45.43	79.6	1252.62
DMU10	80.33	104.65	53.3	79.06	27.28	132.49	42.67
DMU11	205.92	183.49	144.16	59.66	14.09	196.29	16.15
DMU12	82.09	104.94	46.51	127.28	44.87	108.53	0
DMU13	202.21	187.74	149.39	93.65	44.97	184.77	0
DMU14	67.55	82.83	44.37	60.85	26.04	85	23.95
DMU15	72.6	132.73	44.67	173.48	5.55	135.65	24.13
DMU16	84.83	104.28	159.12	171.11	11.53	110.22	49.09
DMU17	71.77	88.16	69.19	123.14	44.83	74.54	6.14

This example is provided by using the original CCR model solved and the weights assigned to inputs and outputs in Table 2. Accordingly, each of the inputs and outputs of decision making units are assigned different weights. For example, the 16 decision-making units for increasing Value their efficiency on four of the seven criteria zero weight is assigned, it cannot be acceptable. As can be seen in the last column Table 2, value efficiency nine unit of seventeen is equal one and discriminate power is very low.

Table 2: The results of CCR model

DMU	V1	V2	V3	V4	U1	U2	U3	Efficiency
DMU1	0.015424	0.002655	0.001996	0.002259	0.011060	0.002615	0.000162	1.000000
DMU2	0.000778	0.005438	0.003312	0.000705	0.001146	0.005461	0.000051	1.000000
DMU3	0.008343	0.001436	0.001080	0.001222	0.005982	0.001414	0.000088	1.000000
DMU4	0.027310	0.000203	0.013697	0.000173	0.038228	0.000200	0.000012	1.000000
DMU5	0.012915	0.000563	0.007817	0.000247	0.020748	0.000286	0.000018	1.000000
DMU6	0.014839	0.000110	0.007442	0.000094	0.020771	0.000109	0.000007	1.000000
DMU7	0.000778	0.000829	0.076979	0.000705	0.046495	0.000816	0.000051	1.000000

DMU8	0.000324	0.009942	0.001275	0.000294	0.000478	0.007714	0.000021	1.000000
DMU9	0.001549	0.001650	0.018730	0.001404	0.016386	0.001625	0.000101	1.000000
DMU10	0	0.006771	0.000271	0.003503	0	0.007097	0	0.940277
DMU11	0	0.004543	0.000182	0.002351	0	0.004762	0	0.934635
DMU12	0	0.007689	0.004153	0	0.005155	0.005507	0	0.829028
DMU13	0	0.004129	0.000166	0.002137	0	0.004328	0	0.799690
DMU14	0	0.007685	0.002716	0.003992	0.004783	0.007632	0	0.773269
DMU15	0	0.006462	0.003186	0	0	0.005614	0.000046	0.762683
DMU16	0.001041	0.008742	0	0	0	0.006745	0	0.743471
DMU17	0.005829	0.001824	0.006083	0	0.015331	0.000000	0	0.687298

For solving the mentioned problems, proposed model (model 4) is used and the results are presented in table 3. Values of the relative efficiency of DMUs and ranking based on CCR model and proposed method are presented in Table 4. With reviewing and comparing the proposed model results are commented the credit of results. Compared with the original CCR model, the result of model has more discriminate power between DMUs. Based on the result of CCR model, 9 DMUs are efficient and have a point 1. While the proposed model, this situation is only 3 DMUs. Accordingly, instead of 9 units of point 1, in the proposed model are 3 units of point 1. Therefore, the main advantage of this model is the full ranking relative efficiency of DMUs. Moreover, as is shown in table 3, the proposed model in this study for all the criteria DMUs are assigned a non-zero weight. So, all of the criteria in assessing all decision units have been effective. In addition, the range of weight changes in the proposed model is more normal than the CCR model.

Table 3: The results of proposed model

DMU	V1	V2	V3	V4	U1	U2	U3
DMU1	0.006869	0.006869	0.006869	0.006869	0.016453	0.006328	0.000076
DMU2	0.002678	0.002678	0.002678	0.002678	0.006416	0.002468	0.000030
DMU3	0.002265	0.002265	0.002265	0.002265	0.005425	0.002086	0.000025
DMU4	0.003380	0.003380	0.003380	0.003380	0.008095	0.003114	0.000037
DMU5	0.003504	0.003504	0.003504	0.003504	0.008394	0.003229	0.000039
DMU6	0.003366	0.003366	0.003366	0.003366	0.008062	0.003101	0.000037
DMU7	0.002715	0.002715	0.002715	0.002715	0.006503	0.002501	0.000030
DMU8	0.002554	0.002554	0.002554	0.002554	0.006117	0.002353	0.000028
DMU9	0.003558	0.003558	0.003558	0.003558	0.008522	0.003278	0.000039
DMU10	0.003151	0.003151	0.003151	0.003151	0.007548	0.002903	0.000035
DMU11	0.001686	0.001686	0.001686	0.001686	0.004038	0.001553	0.000019
DMU12	0.002771	0.002771	0.002771	0.002771	0.006639	0.002553	0.000031
DMU13	0.001580	0.001580	0.001580	0.001580	0.003784	0.001455	0.000017
DMU14	0.003912	0.003912	0.003912	0.003912	0.009372	0.003605	0.000043
DMU15	0.002361	0.002361	0.002361	0.002361	0.005656	0.002176	0.000026
DMU16	0.001926	0.001926	0.001926	0.001926	0.004612	0.001774	0.000021
DMU17	0.002839	0.002839	0.002839	0.002839	0.006800	0.002615	0.000031

Table 4: Results of the CCR model and proposed model and ranked according to their

DMU	CCR	Rank	Inefficiency	Efficiency	Rank
DMU1	1.000000	1	0.000000	1.000000	1
DMU2	1.000000	1	0.291445	0.708555	5

DMU3	1.000000	1	0.000000	1.000000	1
DMU4	1.000000	1	0.383472	0.616529	7
DMU5	1.000000	1	0.144067	0.855933	4
DMU6	1.000000	1	0.000000	1.000000	1
DMU7	1.000000	1	0.573461	0.426539	14
DMU8	1.000000	1	0.427521	0.572480	10
DMU9	1.000000	1	0.302865	0.697135	6
DMU10	0.940277	10	0.407949	0.592051	8
DMU11	0.934635	11	0.637959	0.362041	15
DMU12	0.829028	12	0.424999	0.575002	9
DMU13	0.799690	13	0.560890	0.439110	13
DMU14	0.773269	14	0.448545	0.551455	11
DMU15	0.762683	15	0.672862	0.327138	16
DMU16	0.743471	16	0.750247	0.249753	17
DMU17	0.687298	17	0.500004	0.499996	12

4. Concluding remarks

In this study, the concept of Data Envelopment Analysis and Goal Programming model for relative efficiency of similar Decision Making Units were proposed. Proposed model in this study can answer some questions about DEA models and use these models for managers to increase the attractiveness. In this model, the decision variables were input and output weights that the proposed model is assigned non-zero weight to them. The two man characteristics of this model are: 1) The results of the present model in comparison with the result of DEA model makes more differentiated between Decision Making Units and 2) The final ranking model based on adjusted weights is acceptance of results among decision makers will increase. Another advantage of this model is simple logic and clear. As well form of linear programming this model in comparison of nonlinear programming some other models like the model of Kao and Hung are easy to use. In summary, simplicity, broad applicability, validity and consistency of the results with the standard model of DEA is considered the main advantage of model developed in this study.

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