An Optimization Procedure In A Production Line Of Sokat Soap Industry, Ikotun, Lagos State.

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Abstract: This research work has been used to demonstrate the application of linear programming in optimizing the profit in a production line using SOKAT Soap Industry, Ikotun, Lagos State. Available to us are the units of profit of all the products as well as the stocks available for the production line. Using these, the objective function is formulated subject to a number of constraints based on their practices. Optimization (maximization) was obtained employing a linear programming technique. It was observed from the analysis of the data that the product which contributed mostly to the profit earned was the 1kg Salem soap. However, the establishment needed to produce others because of the need of their numerous customers (the hotels and other institutions) that needed the other smaller sizes such as the 10kg, 5kg and 2kg soap.
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1. Introduction

Company managers are often faced with decisions relating to the use of limited resources. These resources may include men, materials and money. In other sector, there are insufficient resources available to do as many things as management would wish. The problem is based on how to decide on which resources would be allocated to obtain the best result, which may relate to profit or cost or both. Linear Programming is heavily used in Micro-Economics and Company Management such as Planning, Production, Transportation, Technology and other issues. Although the modern management issues are error changing, most companies would like to maximize profits or minimize cost with limited resources. Therefore, many issues can be characterized as Linear Programming Problems (Sivarethinamohan, 2008).

A linear programming model can be formulated and solutions derived to determine the best course of action within the constraint that exists. The model consists of the objective function and certain constraints. For example, the objective of SOKAT Soap Industry, Ikotun is to produce quality soaps needed by its customers, subject to the amount of resources (raw materials) available to produce the products needed by their respective customers who should also not violate Standard Organization of Nigeria (SON). The problem then is on how to utilize limited resources to the best advantage, to maximize profit and at the sometime selecting the products to be produced out of the number of products considered for production that will maximize profit.

The research is aimed at deciding how limited resources, raw materials of SOKAT Soap Industry, Ikotun, Lagos State would be allocated to obtain the maximum contribution to profit. It is also aimed at knowing how the limited resources the industry would be allocated to achieve the best production mix at minimum cost and determining the products that contribute to such profit.

The scope of the research is to use Linear Programming on some of the soaps produced SOKAT Soap Industry, Ikotun, Lagos State. The data on which this is based are quantity of raw materials available in stock, cost and selling prices and therefore the profit of each product. The profit constitutes the objective function while raw materials available in stock are used as constraints. If demands which must be met are to be available, such can be included in the constraints. The data is secondary data collected in the year 2007 at the SOKAT Soap Industry, Ikotun, Lagos State.

The Simplex method, also called Simple technique or Simplex Algorithm, was invented by George Dantzig, an American Mathematician, in 1947. It is the basic workhorse for solving Linear Programming Problems up till today. There have been many refinements to the method, especially to take advantage of computer implementations, but the essentials elements are still the same as they were when the method was introduced (Chinneck, 2000; Gupta and Hira, 2006).

The Simplex method is a Pivot Algorithm that transverses the through Feasible Basic Solutions while Objective Function is improving. The Simplex method is, in practice, one of the most efficient algorithms but it is theoretically a finite algorithm only for non-degenerate problems (Feiring, 1986).

To derive solutions from the LP formulated using the Simplex method, the objective function and the constraints must be standardized.

The characteristics of the standard form are:

1. All the constraints are expressed in the form of equations except the non-negativity constraints which remain inequalities ≥ 0.
2. The right-hand-side of each constraint equation is non-negative.
3. All the decision variables are non-negative.
4. The Objective function is of maximization or minimization type. Before attempting to obtain the solution of the linear programming problem, it must be expressed in the standard form is then expressed in the "the table form" or "matrix form" as given below:

$$Maximize \quad Z = \sum_{j=1}^{r} C_j X_j$$

$$Subject\,to \quad \sum_{j=1}^{r} a_{ij} X_j \leq b_i,\ (b_i \geq 0),\ i = 1,2,3,\ldots,m$$

$$X_j \geq 0,\ j = 1,2,3,\ldots,m$$

In standard form (Canonical form), it is

$$Maximize \quad Z = \sum_{j=1}^{r} C_j X_j$$

$$Subject \ to \quad \sum_{j=1}^{r} a_{ij} X_j + S_i = b_i,\ i = 1,2,3,\ldots.,m$$

$$X_j \geq 0,\ j = 1,2,3,\ldots.,n$$

$$S_i \geq 0,\ i = 1,2,3,\ldots.,m$$

Any vector X satisfying the constraints of the Linear Programming Problems is called

Feasible Solution of the problem (Fogiel, 1996; Schulze, 1998; Chinneck, 2000).

1.1 Algorithm to solve linear programming problem:

1. See that all b_i's are positive, if a constraint has negative b_i multiply it by -1 to make b_i positive.
2. Convert all the inequalities by the addition of slack or by subtraction of surplus variable as the case may be.
3. Find the starting Basic Feasible Solution.
4. Construct the Simplex table as follows:

$Basic\ Variable$	E_j	X_1	X_2	$X_3\ldots.$	X_n	y_1	$y_2\ldots$	y_m	X_b
y_1	0	a_{11}	a_{12}	$a_{13}\ldots$	a_{1m}	1	0	0	b_1
y_2	0	a_{21}	a_{22}	$a_{23}\ldots$	a_{2m}	0	1	0	b_2
y_3	0	a_{31}	a_{32}	$a_{33}\ldots$	a_{3m}	0	0	1	b_m
	Z_j	Z_1	Z_2	Z_3	Z_4	0	0	0	$Z = Z_0$
	E_j	E_1	E_2	E_3	E_4	0	0	0	
	.	.	.	.	.	.	.	.	
$\Delta b_j = Z_j - E_j$		ΔX_1	ΔX_2	$\Delta X_3\ldots$	ΔX_n	Δy_1	$\Delta y_2\ldots$	Δy_m	

5. Testing for optimality of Basic Feasible Solution by computing $\Delta Z_j - E_j\ if\ Z_j - E_j \geq 0$ the solution is optimal; otherwise, we proceed to the next step.
6. To improve on the Basic Feasible Solution, we find the basic matrix. The variable that corresponds to the most negative of $Z_j - E_j$ is the INCOMING VECTOR while the variable that corresponds to the minimum ratio b_i / a_{ij} for a particular $j, and\ a_{ij} \geq 0, i = 1,2,3,\ldots,m$ is the OUTGOING VECTOR.
7. The key element or the pivot element is determined by considering the intersection between the arrows that corresponds to both incoming and outgoing vectors. The key element is used to generate the next table. In the next table, pivot element is replaced by UNITY, while all other elements of the pivot column are replaced by zero. To calculate the new values for all other elements in the remaining rows of that first column, we use the relation.
8. New row = Former element in old rows $-$ (intersection element in the old row) $\times$
9. (Corresponding element of replacing row).
10. Test of this new Basic Feasible Solution for optimality as (6) it is not optimal; repeat the process till optimal solution is obtained. This

was implemented by the software Tora Window Version 2.0

1.2 SIMPLEX METHOD FLOWCHART

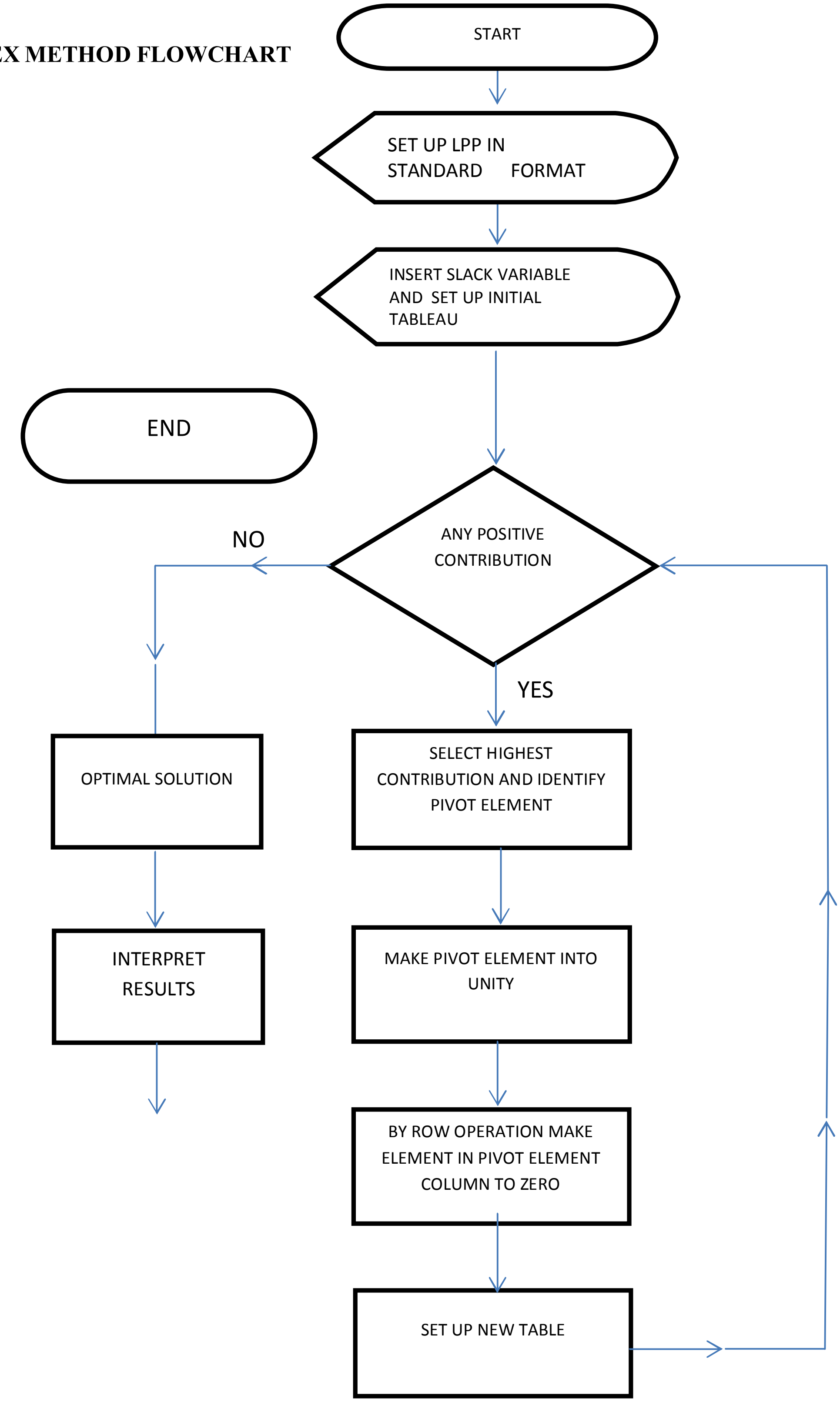

Table 1: Quantity of raw materials available in stock

	Raw Material	Maximum Quantity available in Stock(kg)
1.	P.K.O	1500
2.	Tallow	800
3.	Caustic Soda	1200
4.	Chalk	1000
5.	Silicate	200

Table 2: Quantity of raw materials needed to produce various sizes of Salem Soap

	Raw Material	10kg	5kg	2kg	1kg
1.	P.K.O	3.6	1.8	0.7	0.36
2.	Tallow	2.14	1.07	0.428	0.214
3.	Caustic Soda	0.96	0.48	0.192	0.096
4.	Chalk	1.2	0.60	0.24	0.12
5.	Silicate	0.32	0.15	0.06	0.03

Table 3: Average Cost and Selling of a crate of each product

Sizes	Average Cost price(₦)	Average Selling price(₦)	Profit(₦)
10kg	1000	1170	170
5kg	500	585	85
2kg	200	234	34
1kg	100	117	17

1.3 Model Formulation

$$Maximize\ Z = 170X_1 + 85X_2 + 34X_3 + 17X_4$$

$$Subject\ to$$

$$3.6X_1 + 1.8X_2 + 0.72X_3 + 0.36X_4 \le 1500$$

$$2.14X_1 + 1.07X_2 + 0.438X_3 + 0.21X_4 \le 800$$

$$0.96X_1 + 0.48X_2 + 0.19X_3 + 0.09X_4 \le 1200$$

$$1.20X_1 + 0.60X_2 + 0.24X_3 + 0.21X_4 \le 1000$$

$$0.30X_1 + 0.15X_2 + 0.06X_3 + 0.03X_4 \le 200$$

$$For\ \ X_i \ge 0, i = 1, 2, 3, 4$$

Now, introducing the slack variable to convert inequalities to equations, it gives:

$$Maximize\ Z = 170X_1 + 85X_2 + 34X_3 + 17X_4$$

$$Subject\ to$$

$$3.6X_1 + 1.8X_2 + 0.72X_3 + 0.36X_4 + X_5 = 1500$$

$$2.14X_1 + 1.07X_2 + 0.438X_3 + 0.21X_4 + X_6 = 800$$

$$0.96X_1 + 0.48X_2 + 0.19X_3 + 0.09X_4 + X_7 = 1200$$

$$1.20X_1 + 0.60X_2 + 0.24X_3 + 0.21X_4 + X_8 = 1000$$

$$0.30X_1 + 0.15X_2 + 0.06X_3 + 0.03X_4 + X_9 = 800$$

$$For\ X_i \ge 0, i = 1, 2, 3, \ldots, 9$$

Where,
X_1 = 10kg of Salem soap
X_2 = 5kg of Salem soap
X_3 = 2kg of Salem soap
X_4 = 1kg of Salem soap

2.0 Analysis and Result
Optimal Solution
Objective function value = 64,761.90

Variables	Value
X_1	0.000
X_2	0.000
X_3	0.0000
X_4	3809.52

The Tora Window Version 2.0 gives:
Z = 64,761.90 and X_4 = 3809.52

3. Interpretation of Result

Based on the data collected the optimum results derived from the model indicate that one of their products should be produced 1kg of Salem Soap. The production quantity should be 3810 units. This will produce a maximum profit of ₦64,762.

4. Conclusion

Based on the analysis carried out in this research and the result shown, SOKAT Soap Industry, Ikotun should produce 10kg, 5kg and 2kg of Salem soap but more of 1kg of Salem soap in order to satisfy their customers. Also, more of 1kg of Salem soap should be produced in order to attain maximum profit because it contribute mostly to the profit earned.

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