

Investigation the factors affecting the integrity of FRP-reinforced sheet resistance in bending concrete beams

Mohammad Ali Dashti Rahmatabadi¹, Alireza Mirjalili¹, Kazem Yavarinasab² (Corresponding author)

¹ Faculty member of Department of Civil Engineering, Yazd Branch, Islamic Azad University, Yazd, Iran

² Msc of Department of Civil Engineering, Yazd Branch, Islamic Azad University, Yazd, Iran
yavari_nasab@yahoo.com

Abstract: Beams retrofitted with FRP may come to failure, due to different reason; and several models of the separated FRP boundary and resistance at the FRP junction to concrete is done and factors included in the calculation of the capacity of the beams are: Compressive strength of concrete, Mechanical properties of steel, cement and FRP, width and thickness FRP, beam cross-section dimensions, sections of steel and etc. and variables for each model are different. But when retrofitting buildings, certain variables such as retrofitting buildings of steel, compressive strength of concrete, steel and mechanical properties are not specified. For future research to analysis existing models, it is necessary to obtain the sensitivity of each variable in the model. In this study, a sensitivity analysis of model variables, continuity sheet resistance of FRP-reinforced concrete beams in bending with changes in variables over a certain percentage of the variable is obtained. Graphs and sensitivity analysis are presented for each model.

[Mohammad Ali Dashti Rahmatabadi, Alireza Mirjalili, Kazem Yavari Nasab. **Investigation the factors affecting the integrity of FRP-reinforced sheet resistance in bending concrete beams.** *Researcher* 2013;5(11):115-126]. (ISSN: 1553-9865). <http://www.sciencepub.net/researcher>. 19

Keywords: Sensitivity analysis, Reinforced bending beam - FRP Sheets

Introduction:

Poor quality concrete, inappropriate reinforcement, bad performance concrete, inferior materials and etc. are the main problems of reinforced concrete structures [1]. According to the cost of structure substantially renovation, to strengthen the weak structures, and repair damaged structures in recent years, this issue have been widely discussed and many studies have been carried on [2]. In our country, there are lots of old buildings that most of them are in short and medium order with office and educational use [3]. Improve a wide range of engineering services and activities in different purposes, which may include technical, economic, social, cultural, political, aesthetic which have been implemented [1]. Strengthened means increase the strength of weak members against bearing loads and repair means increase the strength to the bearing for damaged organs [2].

For example, the following guidelines can be used singly or in combination with each other to improve the building to be used:

- Local correction of structural elements which have unsatisfactory performance in earthquake condition
- Eliminate or reduce irregularities in existing buildings
- Provision of lateral stiffness for whole structure
- Provide the necessary resistance for whole structure
- Reduced mass of buildings
- To complete the orientation of tension
- Increased integration of buildings with the coil ratings

- Change of land use to reduce the level of expected performance of buildings

- The application of seismic isolation systems.

In this regard, two aspects of retrofitting hard strength is assessed [1]. Seismic retrofitting of existing structures on three parameters: strength, hardness and ductility are essential would run: [4].

Designed to withstand the additional loads, improve deficiencies caused by aging, increased ductility structures, etc. In recent years retrofitting concrete structures using fiber reinforced polymer (FRP) has enjoyed considerable growth [5] [6]. The use of steel plates for strengthening reinforced concrete that is common and has been replaced by FRP [7]. The end of 1970, the first practical applications of repair or retrofit with external amplification was carried out in Belgium [8]. Beams for flexural strength, tensile side of the sheet is attached to the beam, the adhesive sheet funds increased vertical beams near the abutments shear capacity of the beams [9]. The use of FRP sheets as new materials are used in the construction industry for strengthening and repair of reinforced concrete structures [10]. The main advantages of this new material in front of traditional materials can lose weight, high strength; good durability, high tensile strength and elastic coefficient accept them mentioned. On the other hand, the FRP materials are highly resistant to corrosion in alkaline and saline environments are considered, so some researchers as an alternative material for steel materials in corrosive environments have been proposed [11] [12] [13]. The

experimental study of the bending failure of RC beams strengthened with FRP sheets are:

- 1- Bending failure as failure of FRP sheets
- 2- The bending failure of flexural concrete crushing in compression
- 3- Shear failure
- 4- Separation of the concrete cover of the longitudinal bars from the end of FRP sheets
- 5- Removable glue the ends of FRP sheets
- 6- Separation of FRP plate flexural cracks
- 7- Separation of FRP plate flexural cracks - shear [14] [6].

Concrete beam with FRP sheets stick increase the percentage of longitudinal reinforcement, the percentage of FRP, concrete compressive strength and mechanical properties, the rate of deterioration of concrete and etc [2] [15].

Literature Review:

Extensive research in theoretical, experimental and numerical monitoring in the field of bending beams reinforced with GFRP or CFRP sheets of researchers have taken place. Among the various methods for the recovery and re-empowerment beams (RC) damaged, it has attracted a wide range of researchers, GFRP and CFRP sheets sticking to the outer surface of the tensile zone of the beam. Two reasons, first, ease of operation and high impact strength and bending stiffness of the beam other than the application of these methods have a critical role [16]. M. Accardi & L La Mendola on stress transfer between concrete and FRP plates have been doing research [17]. Houssam Toutanji et al. showed in a study that examined the relationship of bond strength [18].

Büyükoztürk FRP-concrete system and Tzu-Yang Yu failure were studied [19]. Riadh Al-Mahaidi and colleagues conducted a study on CFRP strengthened by shear [20]. Edalati M. and Irani F., have investigated sheet of paper on the mechanical interface between the concrete and FRP, especially the stress distribution in the concrete, the FRP plate and adhesive layer at the end of their connection to each other deaf [16]. Karimi A. and Anwar S., investigated an article entitled studied the connection of external beam finite elements - concrete columns reinforced with FRP sheets under seismic loads parameters affecting the behavior of junctions reinforced with FRP sheets under seismic loads, FRP sheets and prevention of detachment were examined [21]. Mohammad Reza Ghasemi et al. in an article entitled numerical flexural behavior of reinforced concrete beams strengthened with FRP composite materials using the finite element method to investigate the flexural strengthening of reinforced concrete beams strengthened with FRP, and the

parameters flexural behavior of the components. Some of these parameters can influence the direction of the fibers relative to the longitudinal axis of the beam, number of layers, thickness, material, width and length of the composite, arrange layers and other parameters related to the properties of the concrete member concrete compressive strength, longitudinal tensile reinforcement area pointed out [22]. Mohammad Reza Esfahani and Mohammadi T., taking advantage of numerical methods and experimental results, the flexural behavior of reinforced concrete beams strengthened with FRP sheets studied [23]. Arabzadeh A. and Mahanpour H., in an article entitled laboratory studies on shear strength of CFRP sheets deep double the effect of CFRP sheets on shear capacity of deep beams have died [24]. Momeni K., and Mazloum M., FRP rebar concrete bond in an article entitled modeling software Abaqus stretch out on the strength of FRP bars made [25]. Rakhshanmehr M., et al., in a paper strength of fiber reinforced polymer reinforcement bond between FRP and concrete beams were examined by the patchy [26]. Sobuz et al. were using FRP reinforced concrete beams [27]. Behavior of RC beams strengthened with FRP Owen Arthur Rosenboom to be examined [28]. Research of Mônica Regina Garcez et al. pre-stressing FRP sheets for strengthening carried out [29].

Researchers who have done research in frontier areas of strength models include: Tanaka (1996), Hioky and Wu (1997), Maada et al. (1997), Taljestin (1994), Hoolmkempfir (1994), Roustasi (1997), Yoyan and Wu (1999), Yoyan et al. (2001), Van Gemert (1980), Khalifa et al. (1998) [7] [30] [31] [32] [33] [34] [35] [36] [37] [38] [39] [40] [41] [42].

Among those who have done research on the strength of FRP sheets can be noted: Ehlers (1992), Ehlers and Moran (1996), Smith and Tang (2003), Tang and Yao (2007), Ehlers et al. (2004), Johnsez (1997), Ahmad and Vangemert (1999), Raof and Zheng (1995 - 1996 - 1997), Wang and ling (1998), Raof and Hasan (2000), Ziraba et al. (1994), Varastehpour and Hamlyn (1997), Sadatmanesh and Malek (1998) and Tamilian et al. (1999) pointed out [7] [43] [44] [45] [46] [47] [48] [49] [50] [51] [52] [51] [7] [53] [54] [55] [56] [57] [58] [59] [60] [61] [62] [63] [64] [65] [66] [67].

Methods

Sensitivity analysis to study the effects of input variables, output variables is called a statistical model. In other words, a way to change the inputs to a statistical model as an organized, systematic effects that could predict the change in model output [68] [69] [70]. Sensitivity analysis is used when the primary problem is that the final optimization

solutions obtained are then asked to analyze the problem in terms of the following:

- Changes in constraints.
- Changes in the objective functions coefficients of the variables.
- Changes in the value of the coefficient restrictions.
- Add a new variable to the objective function.
- Adding new constraints [71] [72] [73].

There are several methods for sensitivity analysis with respect to the appropriate method should be selected. The first method appears to be derived from the sensitivity analysis and comparison of changes in the variables of interest for each of the variables is the same change. This method is appropriate when the variables range similar to each other. Another method that can be used in the sensitivity analysis, changes in specific variables to be varied in the range of percent. This method is used more times. A portion of the unknown function and a variable effect on it is not exactly clear, so this method can be varied to create a real function [71] [73].

There are also issues that actually limits or constraints are very flexible and you can use this method to open, because no matter how close we were to the border restrictions or limitations have been violated and virtually no limit on how much you're supposed to do. In this study, the research results of other researchers who have been active in this field, flexural strength and adhesion between the FRP sheets have found and a sensitivity analysis for each of the existing models, the factors affecting the integrity of the sheet resistance of FRP-reinforced concrete beams in the deflection is obtained [73].

In this study, researchers found that the models linkage sheet resistance of FRP-reinforced concrete beams in bending is available. The second method, sensitivity analysis is used to model the variables of the model sensitivity analysis and the sensitivity of the model parameters is obtained. Factors affecting the integrity of the sheet resistance of FRP-reinforced concrete beams in bending are achieved.

Analysis of Results

Introducing the variables of the models linkage sheet resistance of FRP-reinforced concrete beams in bending are:

f'_c : Compressive strength of cylindrical concrete specimens

E_{frp} : Modulus of elasticity of FRP sheets

t_{frp} : Thickness of FRP sheet

E_s : Modulus of elasticity of steel bars

b : Width of the concrete beam

b_{frp} : Width of FRP sheet

h : The overall height of the beam

A_s : Sectional area of the tensile bars

A'_s : Cross bars of pressure

c : Tensile zone of the concrete cover

c' : Cover concrete in the compression zone

S : The distance between the pitchers

A_{sv} : Sectional area pitchers

f_{yv} : Failure stress pitchers

a : The distance between the fulcrums closer to the end plate

a_1 : Extended load

a_2 : Concentrated load

a_3 : Concentrated moment

B : The shear zone

E_a : Modulus adhesive

t_a : The thickness of the adhesive layer

G_a : The shear modulus of the adhesive

M_0 : Bending moment end of plate

V_0 : Shear in end of plate

q : Constant load

n : Number of tensile bars

z : Coefficient in the range of 4. 8-9. 5

Sensitivity analysis of model variables, continuity sheet resistance of FRP-reinforced concrete beams in bending moment diagrams are shown below.

Oehlers Model

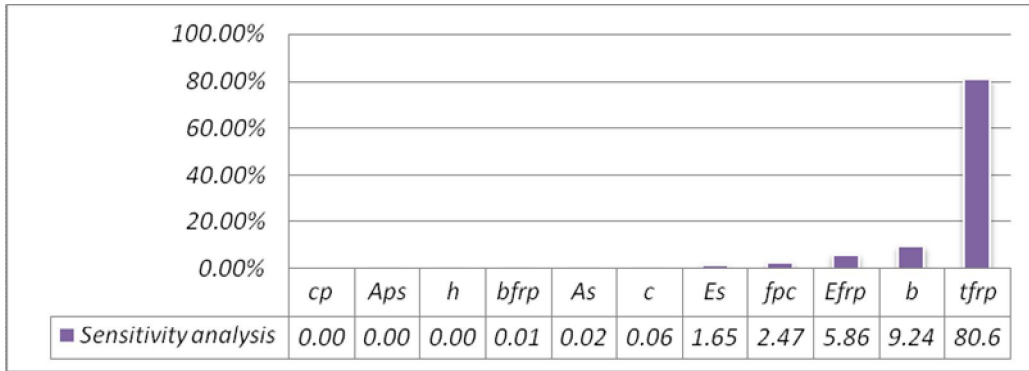


Chart 1: Sensitivity of the variables in the Ehlers model

Malek and Saadatmanesh Model

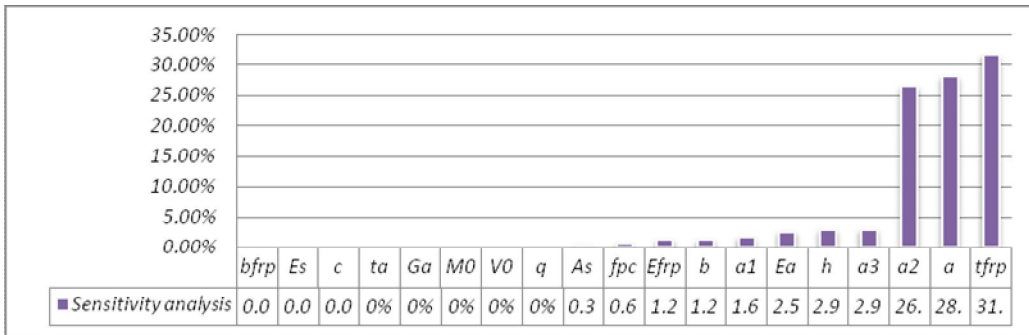


Chart 2: Sensitivity of the variables in Malek and Saadatmanesh model

Ahmed and Van Gemert Model

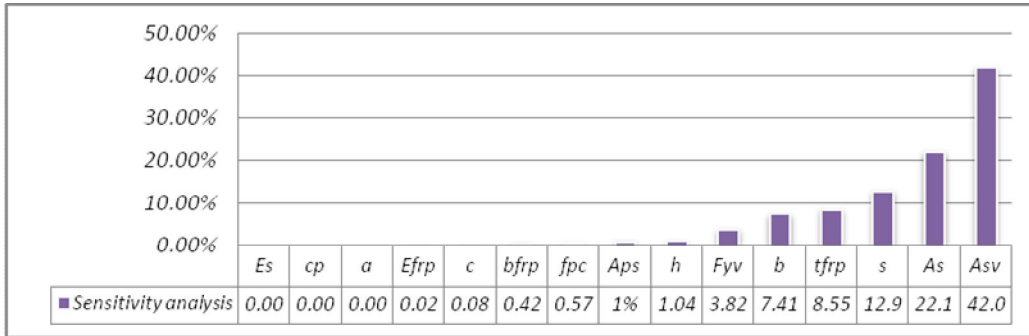


Chart 3: Sensitivity of the variables in Ahmed and Van Gemert Model

Ziraba et al. Model (model 1)

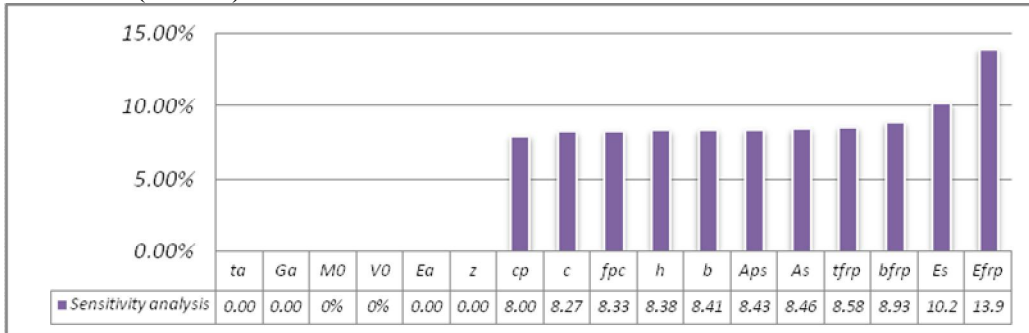


Chart 4: Percent of variables sensitivity in Ziraba et al. model (model 1)

Ziraba et al. Model (model 2)

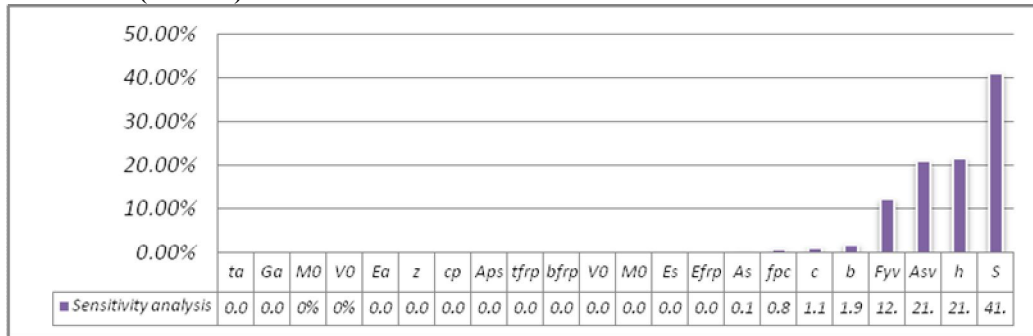


Chart 5: Percent of variables sensitivity in Ziraba et al. model (model 2)

Tomialan et al. Model

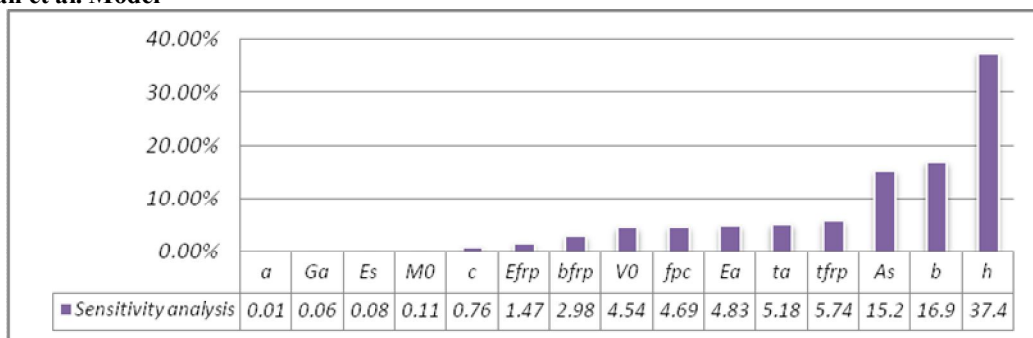


Chart 6: Sensitivity of the variables in Tomialan et al. Model

Raof and Hasanen Model (model 1)

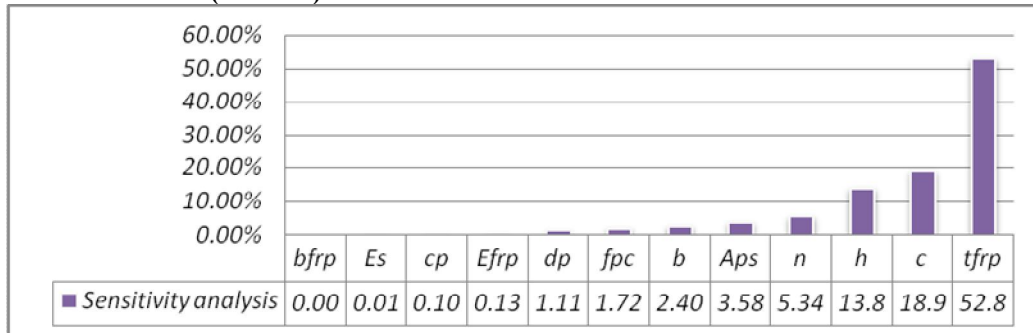


Chart 7: Sensitivity of the variables in Raof and Hasanen Model (model 1)

Raof and Hasanen Model (model 2)

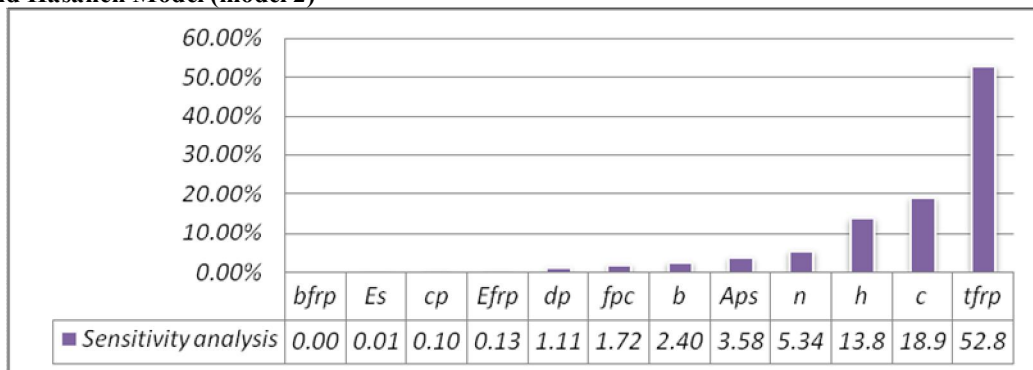


Chart 8: Sensitivity of the variables in Raof and Hasanen Model (model 2)

Wang and Ling Model

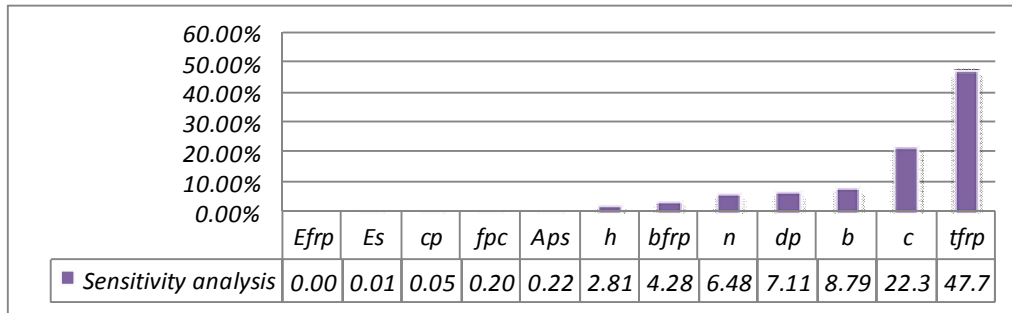


Chart 9: Sensitivity of the variables in Wang and Ling model

Hamelian and Varastehpour Model

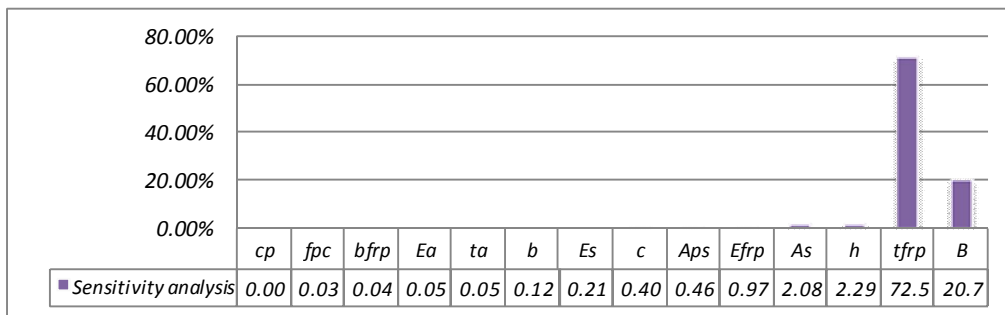


Chart 10: Sensitivity of the variables in Hamelian and Varastehpour Model

Conclusion:

Comparison of f'_c , E_{frp} , E_s in separated plate models

According to the comparison of f'_c , E_{frp} , E_s in the model in Figure 11, more sensitive models detachment pages mechanical properties of materials (steel rebar, frp panels and concrete) f'_c , E_{frp} , E_s would be low and of the sensitivity of the modulus of elasticity of steel bars has the lowest, but f'_c that the compressive strength of concrete is low too. But, E_{frp} is related to the modulus of elasticity of frp, is higher than E_s and f'_c .

Comparison of b and b_{frp} in separated plate models

According to the comparison of b and b_{frp} in the model in Figure 12, more sensitive variable representing the width of the beam is b in all models, separation pages of sensitive variables representing the width of the page is frp.

Comparison of b_{frp} and t_{frp} in separated plate

models

Based on the comparison of b_{frp} and t_{frp} in the model in Figure 13, more sensitive variable representing the width of the beam is b in all models, separation pages of sensitive variables representing the width of the page is frp. All of the above models are the highest sensitivity of the model to account for the variable sensitivity of the Ehlers with t_{frp} variable 80.60% of changes.

Comparison of h and b in separated plate models

According to the comparison of h and b in the model in figure 14, more sensitive variable representing the width of the beam is b in all models The models Hamelian, Malek and Zibara (2) and Raof (1 and 2) lower variable representing the height of the beam, but the sensitivity varied models Ahmad, Zibara (1), Wang, Ehlers and Tamilian more h variable.

Comparison of c and c' in separated plate models

The comparison between c and c' models detachment of the page in figure 15 is given, the sensitivity variable c representing the concrete cover in the area stretching in all models above the sensitivity variable c' representing the cover concrete

in the compression zone that is however in some models, there is no c' variable.

Comparison of A_s and A'_s in separated plate models

The A_s and A'_s compare the models in figure 16 are presented detached page, variable A_s , which represents the sensitivity of the cross section area of tensile reinforcement in all models. Higher sensitivity is variable A'_s , which represents a cross-section of the reinforcement in the compression zone. However, in some models, there is no variable A'_s .

Maximum sensitivity of variables in separated plate models

The maximum sensitivity of the model of separation between the plates is given in figure 17. Most models that represent the variable susceptibility t_{frp} of frp thickness are higher than other models shows in themselves.

Acknowledgement

It's necessary to thanks from the faculty of Civil Engineering, Islamic Azad University, Yazd Branch, which this article was extracted from thesis of that faculty entitled "A continuous sheet resistance factors in FRP reinforced concrete beams in bending".

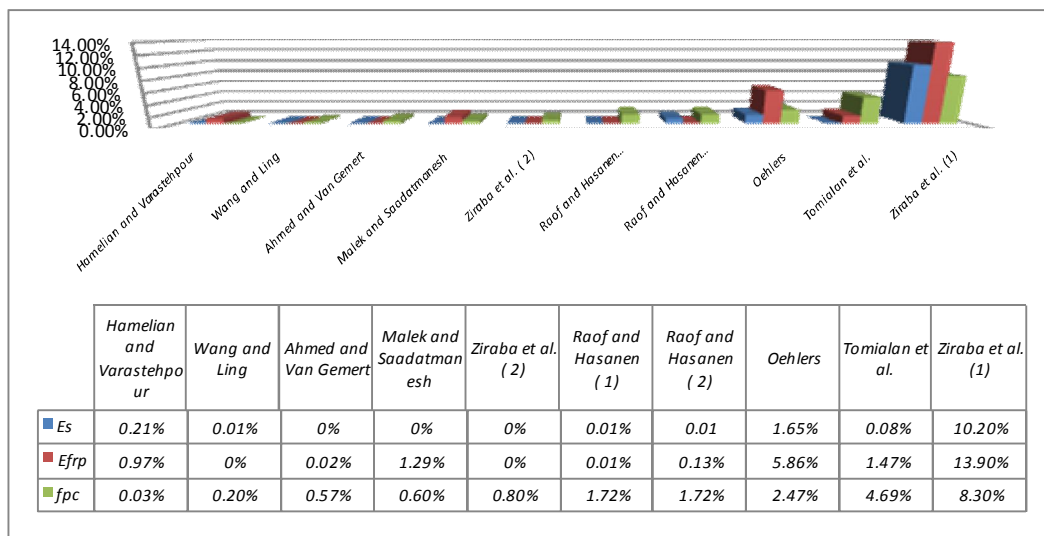


Chart 11: Comparison of f'_c , E_{frp} , E_s in separated plate models

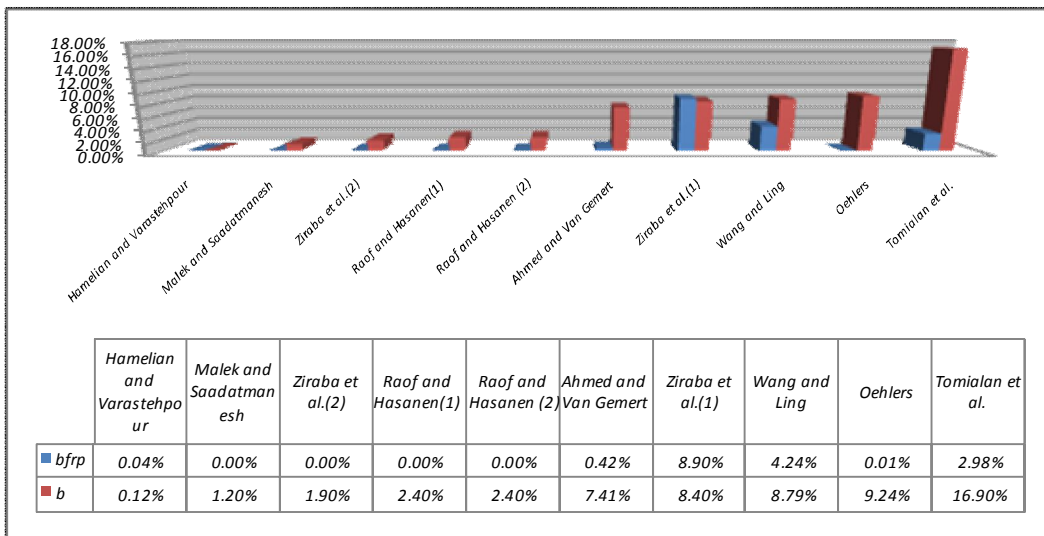


Chart 12: Comparison of b and b_{frp} in separated plate models

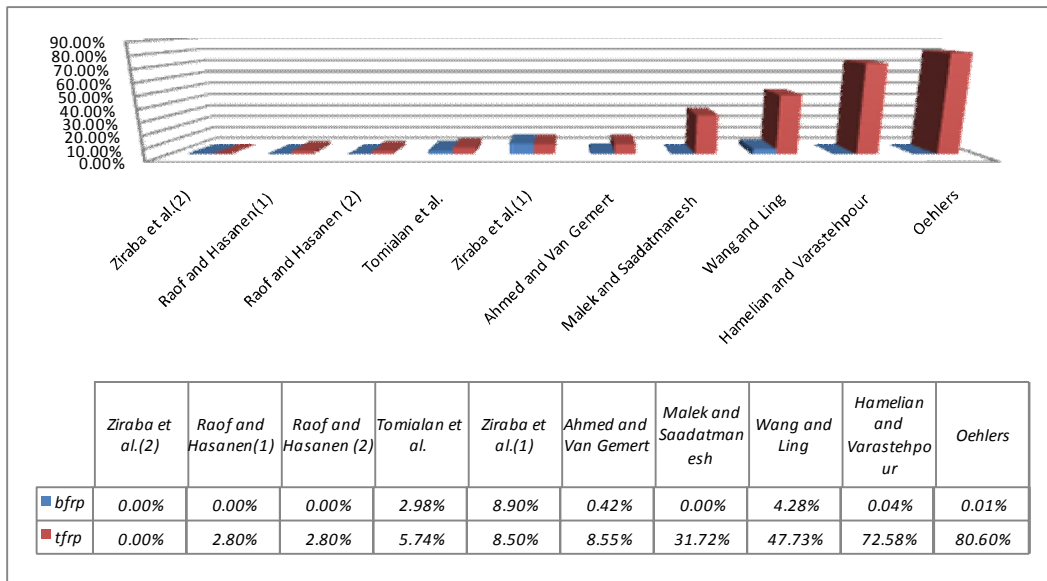


Chart 13: Comparison of b_{frp} and t_{frp} in separated plate models

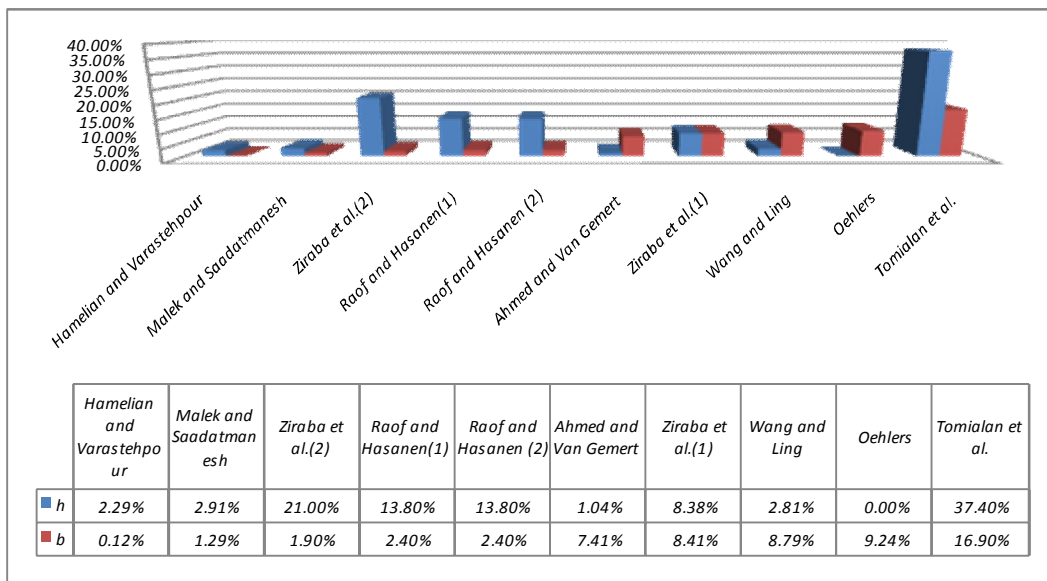


Chart 14: Comparison of h and b in separated plate models

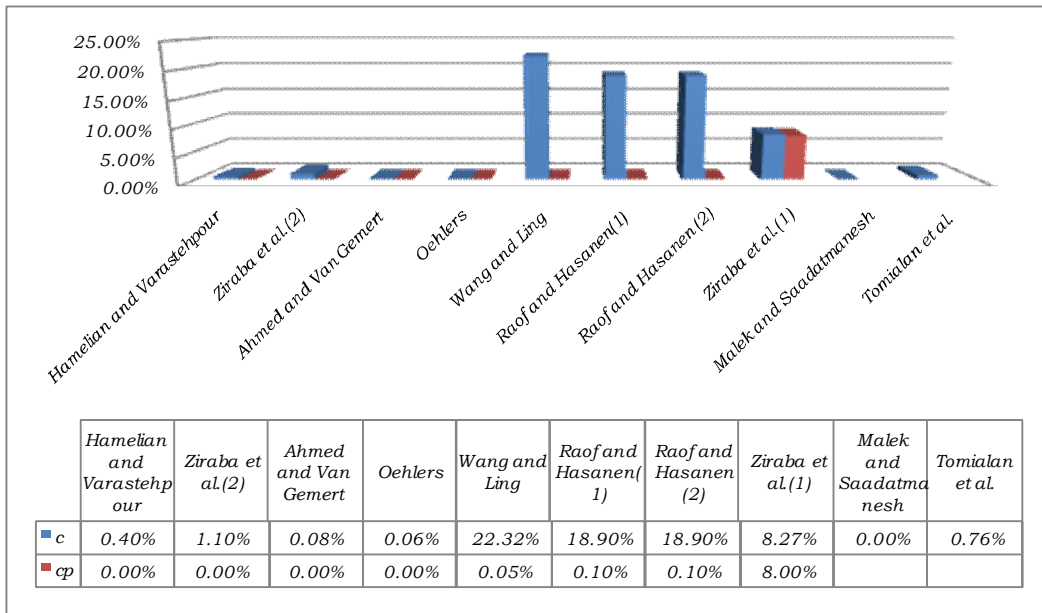


Chart 15: Comparison of c and c' in separated plate models

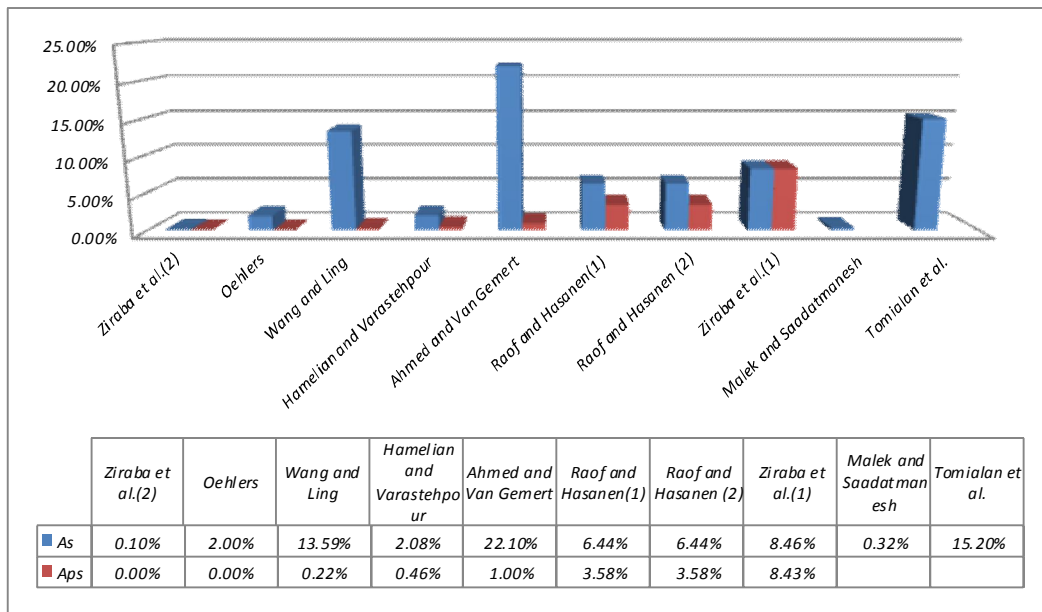


Chart 16: Comparison of A_s and A'_s in separated plate models

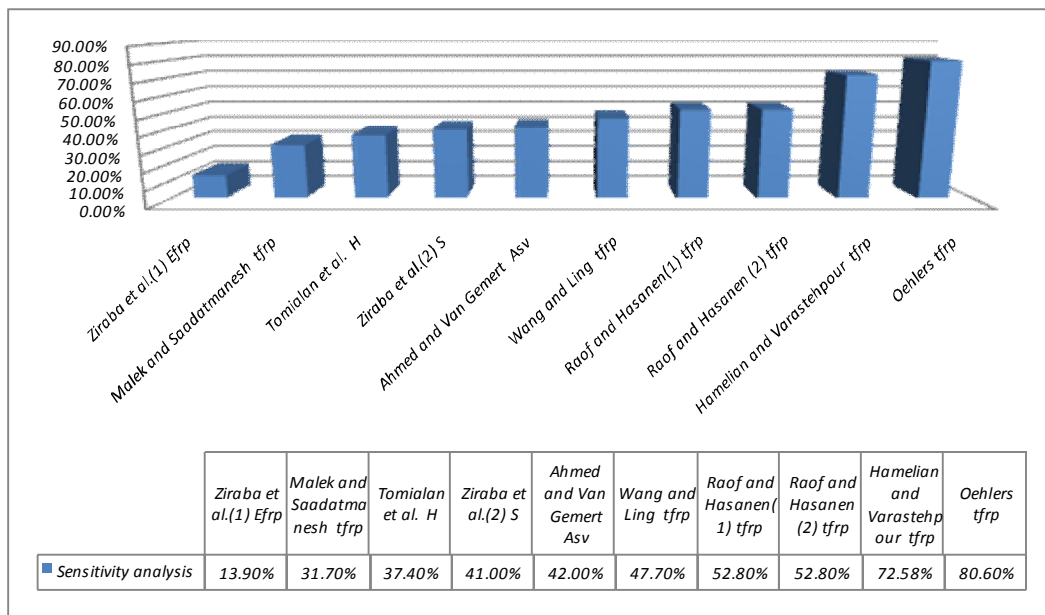


Chart 17: Maximum sensitivity of variables in separated plate models

References:

- Daryabeigi, R., Hesari Hosseinin, H., Rezaeefar, O., Strengthening and rehabilitation of concrete structures with FRP materials, and application of concepts. Tehran: Science and Literature, 2011.
- Rahaei, A., Zomorodian, A., Repair and strengthening of reinforced concrete structures with fiber reinforced polymer composite FRP. TehKaveli, S., Bakhsayesh Eghbali, N. and Taheri, E. 2008. Comparison of different methods for seismic retrofit of concrete buildings, structures and case studies. Third National Conference of Retrofitting. www.fanomran.com
- Improved flexural concrete members with fiber reinforced polymer FRP. According to ACI regulations 440. 2R-08.
- Management and Planning Department. Design Guidelines and Criteria for performance improvement of existing concrete structures using reinforcement materials FRP. Publication No. 345, 2006.
- Nateghi Elahi, F., Maleki, SH. Retrofit of concrete structures using FRP. Tehran: Noorpardazan, 2010.
- Strengthening of concrete structures with externally bonded reinforcement Case studies. Brosens, Ir. Kris. 2000, BVSM vzw, Leuven. www.7civil.com
- Interfacial stress analysis of RC. Huang, Y. C., 2008, Construction and Building Materials, doi:10.106.
- Interfacial stresses in FRP-plated RC beams: Effect of adherend shear deformations. Touns, A, 2009, International Journal of Adhesion & Adhesives .
- FRP Strengthened RC Structural. Lam, L, Teng, G. J and Chen, j. F. Published by John Wiley and Sons Ltd, 2002.
- Strengthening of Concrete Beams Using Innovative Ductile Fiber-Reinforced Polymer Fabrics. Nabil, F. Grace., Ragheb, Wael F and George, Abdel-sayed. 2009, ACI Structural Journal.
- Guide for the design and construction of externally bonded FRP systems for strengthening concrete structures. 440. 2R-02, ACI. 2002, Reported by ACI Committee 440.
- Pahlaviani, A., Retrofitting buildings repaired. Tehran: Niooshnegar, 2008.
- RC beams strengthened with FRP sheets flexural design equations based on assumptions governing the ABA. Justice, Mahmoud. 2010, Book of Abstracts of the International Congress on Civil Engineering, Volume II, Shiraz University, Shiraz.
- Stress transfer at the interface of bonded joints between FRP and calcarenite natural stone. Accardi, M and La Mendola, L. 2001, Department of Structural and Geotechnical Engineering, University of Palermo, Italy.
- Verifications of Design Equations of Beams Externally Strengthened with FRP Composites. Toutanji, Houssam, et al. 2006, Databases; Design; Fiber reinforced polymers; Concrete; Reinforced; Beams.
- Understanding and Assessment of Debonding Failures in FRP-Concrete Systems. Büyüköztürk, Oral and Yang Yu, Tzu. 2006, Seventh

- International Congress on Advances in Civil Engineering.
17. Strengthening of Shear-Damaged Reinforced Concrete T-Beam Bridges with .Al-Mahaidl, Riadh, Taplin, Geoff ,Susa, John .2005 , Department of Civil Engineering, Monash University, Clayton, Vic., Australia 3168 .
 18. Karimi, S., 2010. Vertical shear stresses at the interface between the RC beam and flat reinforcing hybrid GFRP and CFRP. Fifth National Congress on Civil Engineering.
 19. Ghasemi, M. R., 2010. Numerical analysis of flexural behavior of reinforced concrete beams strengthened with frp composite materials using finite element method.
 20. Isfahani, M. R., Mohammadi, T., 2011. The use of numerical analysis and sensitivity, the flexural behavior of reinforced concrete beams strengthened with sheets of FRP. Journal of Civil Engineering.
 21. Arabzadeh, A., Mahanpour, H., 2011. Experimental Study on Shear Strength of CFRP sheets deep tangly ends. Journal of Science - Civil Engineering Research Lecturer.
 22. Momeni, K., and Mazloun, M. 2011. FRP rebar concrete continuum modeling software Abaqus. First National Conference on Development.
 23. Rakhshanimehr M., et al. 2011. Cohesive strength of the relationship depends on the parameters of FRP reinforcement strain out the results.
 24. Use of carbon fiber laminates for strengthening reinforced concrete beams in bending .Sobuz, Habibur Rahman ,2011 ,International Journal Of Civil And Structural Engineering .
 25. Behavior Of Frp Repair/Strengthening Systems For Prestressed Concrete. Rosenboom, Owen Arthur. 2006, A dissertation submitted to the Graduate Faculty of North Carolina State University in partial fulfillment of the requirements for the degree of Doctor of Philosophy.
 26. Applying Post-Tensioning Technique to Improve the Performance of FRP Post-Strengthening. Garcez, Mônica Regina, Meneghetti, Leila Cristina and Carlos, Luiz. 2003, Additional information is available at the end of the chapter.
 27. Analysis of debonding fracture properties of CFS strengthened member subject to tension. Hiroyuki, Y , Wu, Z. 1997, Proceedings of the 3rd International Symposium on Non-Metallic (FRP) Reinforcement for Concrete Structures.
 28. Shear reinforcing effect of carbon fibre sheet attached to side of reinforced concrete beams. Proceedings of the 2nd International Conference on Advanced Composite Materials in Bridges and Structures .Sato, Y ,Montreal, Canada 1996 .
 29. A study on bond mechanism of carbon fibre sheet . Maeda, T .,Japan :Proceedings of the 3rd International Symposium, 1997 .Non-Metallic (FRP) Reinforcement for Concrete Structures .
 30. A study on bond mechanism of carbon fibre sheet . Maeda, T .,Japan :Proceedings of the 3rd International Symposium, 1997 .Non-Metallic (FRP) Reinforcement for Concrete Structures .
 31. Adam, N., Ramazanpour, A., Arabi, N. Concrete technology. Tehran: the author's knowledge, 2011.
 32. Kiniana, A. Analysis and design of reinforced concrete structures. Isfahan: Isfahan University of Jihad, 1989.
 33. Strengthening of existing concrete structures with epoxy bonded plates of steel or fibre reinforced plastics .Täljsten, B .1994 ,Doctoral thesis, Luleå University of Technology Sweden .
 34. Theoretical solution on interfacial stress transfer of externally bonded steel/composite laminates . Yuan, H, Wu, Z , Yoshizawa, H .2001 ,Journal of Structural Mechanics and Earthquake Engineering .
 35. Design aspects of concrete structures strengthened with externally bonded CFRP plates .Neubauer, U ,Rostásy, F. S .Edinburgh, UK :Engineering Technics Press, 1997 .Proceedings of the 7th International Conference on Structural Faults and Repairs .
 36. Force transfer in epoxy-bonded steel-concrete joints .van Gemert, D .1980 ,International Journal of Adhesion and Adhesives
 37. Strengthening of reinforced concrete beams with externally bonded fibre reinforced plastic plates: design guidelines for shear and flexure .Challal, O, Nollet, M. J , Perraton .1998 ,Canadian Journal of Civil Engineering .
 38. Contribution of externally bonded FRP to shear capacity of RC flexural members. Khalifa, A. 1998, Journal of Composite for construction.
 39. Anchorage strength model for FRP and steel plates attached to concrete. Chen, J. F and Teng, J. G. 2001, Journal of Structural Engineering.
 40. Shear debonding failure of FRP-plated RC beams . Smith, S. T , Teng, J. G .2003 ,Advances in Structural Engineering .
 41. Premature Failure of Externally Plated Reinforced Concrete Beams .Oehlers, D. J , Moran, J. P . 1996 ,Journal of Structural Engineering .
 42. Reinforced concrete beams with plates glued to their soffits. Oehlers, D. J. 1992, Journal of Structural Engineering.
 43. Effects of freeze-thaw cycling on anchorage of fibre-reinforced polymer sheets bonded to reinforced concrete beams. Luke alexander blsby, luke alexander. 1999, Queen's University Kingston, Ontario, Canada.

44. Plate end debonding in FRP-plated RC beams-II: strength model. Teng, J. G and Yao, J. 2007, Structural Engineering.
45. Plate end debonding in FRP-plated RC beams-I: Experiments .Yao, J ȳ Teng, J. G .2007 ,Structural Engineering .
46. Prestress model for shear deformation debonding of FRP- and steel-plated RC beams. Oehlers, D. J, 2004, Magazine of Concrete Research.
47. Strengthening of reinforced concrete members in bending by externally bonded steel plates .Jansze, W .1997 ,PhD thesis, Delft University of Technology .
48. Effect of longitudinal carbon fibre reinforced plastic laminates on shear capacity of reinforced concrete beams .Ahmed, O ȳ van Gemert, D . 1999 ,Proceedings of the 4th International Symposium on Fibre Reinforced Polymer Reinforcement for Reinforced Concrete Structures .
49. Effect of beam size and FRP thickness on interfacial shear stress concentration and failure mode in FRP-strengthened beams. Kok sang, leong. 2003, national university of singapore.
50. Prediction of peeling failure of reinforced concrete beams with externally bonded steel plates ȳ M ,Raouf ,S ,Zhang .Wood, L. A .1995 , Proceedings of the Institute of Civil Engineers: Structures and Buildings .
51. Analysis of plate peeling Raouf, M ȳ Zhang, S . 1996 ,failure of RC beams with externally bonded plates, Proceedings of the International Conference on Concrete in the Service of Mankind: Concrete Repair Rehabilitation and Protection University of Dundee, Scotland .
52. An insight into the structural behaviour of reinforced concrete beams with externally bonded plates. Raouf, M ȳ Zhang, S. 1997, Structures and Buildings.
53. Effect of freeze–thaw cycles on the bond Effect of freeze–thaw cycles on the bond .GAO, BO, LEUNG, Christopher ȳ Kyo kim, Jang .This is Per-Published Version .
54. Analysis of plate peeling failure of RC beams with externally bonded plates. Raouf, M ȳ Zhang, S. 1996, Proceedings of the International Conference on Concrete in the Service of Mankind: Concrete Repair Rehabilitation and Protection University of Dundee, Scotland .
55. An insight into the structural behaviour of reinforced concrete beams with externally bonded plates ȳ M ,Raouf .Zhang, S .1997 ,Proceedings of the Institution of Civil Engineering: Structures and Buildings .
56. Prediction models for debonding failure of cracked RC beams with externally bonded FRP sheets .Wang, C. Y ȳ Ling, F. S .1998 , Proceedings of the 2nd International Conference of Composites in Infrastructures (ICCI 98).
57. Peeling failure of reinforced concrete beams with fibre reinforced plastics or steel plates glued to their soffits .Raouf, M ȳ Hassanen, M. A. H .2000 , Proceedings of the institution of Civil Engineers: Structures and Buildings .
58. Guidelines toward the design of reinforced concrete beams with external plates .Ziraba, Y. N , 1994 ,ACI Structural Journal .
59. Interfacial stress analysis and strength prediction of plated RC beams. Yang, Jian. 2005, the university of Leeds, the school of civil engineering.
60. Approximate analysis of shear normal stress concentration in the adhesive layer of plated RC beams. Roberts, T. M. 1989, The Structural Engineer.
61. Strengthening of concrete beams using fibre reinforced plastics .Varastehpour, H ȳ Hamelin . 1997 ,Materials and Structures .
62. Design guidelines for flexure strengthening of RC beams with FRP plates .Sadadatmanesh, H Malek, A. M .1998 ,Journal of Composites for Construction .
63. Reinforced Concrete beams strengthened with CFRP composites: failure due to concrete cover delamination. Tumialan, G, Belarbi, A ȳ Nanni, A. 1999, Department of Civil Engineering, Centre for infrastructure Engineering Studies Report No. CEIS 99-01, University of Missouri Rolla.
64. Statistical descriptions of the strength of concrete . Mirza, S, Hatzinikolas, M MacGregor, J .1979 , Journal of Structural Engineering. www.fa.wikipedia.org, www.spss-iran.com, www.ieun.ir
65. Azar, A., Operations research concepts and applications of linear programming. Tehran: SAMT, 2003.
66. Khorram, E. Linear programming and network flows. Tehran: university books, 2012.
67. Sajjadi, J., Arianejad, M., Operations Research. Tehran: Iran University of Science and Technology, 1999.