Copula Density Estimation between Norwegian and MSCI World Stock Index Using Wavelets Analysis

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Abstract: Wavelet analysis has been used for copula density estimation. This paper highlights the usefulness of the compact support and orthogonal wavelet in order to approximate copula density functions. Our method involves high approximation order properties rather than other previous methods such as kernel and orthonormal series method. Finally, we apply our proposed method to approximate the copula density between Norwegian stock index and MSCI world stock index.
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1. Introduction

In a special case, a copula is simply a joint distribution on the unit square to build bivariate distribution and to investigate dependency structure. We can uniquely determine a joint distribution for 2 random variables by specifying the univariate distribution for each variable and, in addition, specifying the copula. Following Sklar's theorem (Sklar, 1959) the joint distribution function H can be written as

$$H(x,y)=C(F(x),G(y)), \qquad (1)$$

where C is a copula distribution function, and F,G are the univariate, or marginal distribution functions. Given that F,G and C are differentiable, the joint density $h(x,y)$ can be easily concluded from (1) as follows

$$h(x,y)=c(F(x),G(y)).f(x).g(y),$$

where $f(x)$ and $g(y)$ are the density corresponding to $F(x)$ and $G(y)$, respectively, and $c= \ C/ \ F \ G$ is called the copula density. In practice c is unknown and should be determined. More information on copula can be found in Joe (1997) and Nelson (2006). There are mainly two ways to estimate the copulas. The first one is parametric method that relies on the assumption of parametric marginal distributions. A drawback of this method is its need for intensive computations and sometimes optimization problems hard to solve (Gui, 2009). If we don't want to assume a parametric form for the copula, nonparametric estimation methods must be used. Deheuvels (1979) introduced empirical copula as one of first nonparametric copulas. Scaillet and Fermanian (2003) proposed a kernel technique. Gui (2009) proposed a nonparametric method to copula density functions based on two dimensional orthonormal series expansion.

Wavelet method enhances the use of orthonormal series for copula density estimation. Genest et al. (2009) proposed copula density estimation through wavelets method. In this paper, we estimate copula density through wavelet using compact support and orthogonal property in wavelets.

The rest of the present paper is organized as follows: In the second section, we follow the presentation of copula density estimation by wavelet. In Section 3, we apply wavelet to copula density estimation between Norwegian stock index and MSCI world stock index and compared to the other previous methods. Finally in the last section, some conclusions are stated.

2. Development of dynamic composting of processes simulation model

Wavelets dates back to the 1980s, and have found many applications in signal and image processing, numerical analysis, operator theory, geophysics, and other field of science as well as statistics. More details on wavelet can be found in Chui (1992) and Daubechies (1992). Wavelet analysis of a two-place function $h(x,y)$ is a procedure by which this mapping can be decomposed simultaneously at an infinite number of resolution levels $j=0,1,\ldots$. The decomposition at arbitrary level $j_0 \ N$ is given by

$$h(x,y)=h_{j_0}(x,y)+D_{j_0}h(x,y) \qquad (2)$$

where

$$h_{j_0}(x,y) \square \sum_{k_1,k_2=0}^{2^{j_0} \quad 1} a_{j_0,k_1,k_2} \quad {}_{j_0,k_1,k_2}(x,y),$$

is an approximation and

$$D_{j_0}h(x,y) = \sum_{\gamma=1}^{3}\sum_{j=j_0}^{J-1}\sum_{k_1,k_2=0}^{2^j-1} b_{j,k_1,k_2}^{(\gamma)}\psi_{j,k_1,k_2}^{(\gamma)}(x,y),$$

is a sum of details of three types: vertical (1), horizontal (2), and oblique (3). Also, the unique coefficients a_{j_0,k_1,k_2} and $b_{j,k_1,k_2}^{(\ )}$ for $j=j_0, \quad ,J \quad 1$, $k_1,k_2=0, \quad ,2^{j(j_0)} \quad 1$, and =1,2,3 should be determined such that compact support property leads to sum over k_1 and k_2 be finite. Note that the functions ${}_{j_0,k_1,k_2}$, ${}_{j,k_1,k_2}^{(1)}$, ${}_{j,k_1,k_2}^{(2)}$, and ${}_{j,k_1,k_2}^{(3)}$ will be defined as follows:

$$\phi_{j_0,k_1,k_2}(x,y) = \phi_{j_0,k_1}(x)\phi_{j_0,k_2}(y)$$

$$\psi_{j,k_1,k_2}^{(2)}(x,y) = \psi_{j,k_1}(x)\phi_{j,k_2}(y)$$

$$\psi_{j,k_1,k_2}^{(1)}(x,y) = \phi_{j,k_1}(x)\psi_{j,k_2}(y).$$

$$\psi_{j,k_1,k_2}^{(3)}(x,y) = \psi_{j,k_1}(x)\psi_{j,k_2}(y),$$

in terms of a specific scaling function , an associated wavelet , and their location-scale transforms given by for all t R, and k_3 Z. The functions and must satisfy a number of technical conditions which ensure that the location-scale families they generate form an orthonormal system of L_2, the collection of square-integrable functions. Each choice of pair (,) leads to a different approximation of the copula density. Classical examples include the Adelson, coiflet, Daubechies, Haar, Meyer and symlet families of compact support of wavelets.

Now our objective is to approximate a copula density function $c(u,v)$ using wavelet basis given only an i.i.d. sample of two dimensional data $(X,Y)=(X_i,Y_i)_{i=1}^{n}$. We rewrite (2) as

$$\tilde{c_J}(u,v) \simeq \sum_{k_1,k_2=0}^{2^{j_0}-1} \tilde{a}_{j_0,k_1,k_2}\phi_{j_0,k_1,k_2}(u,v)$$

$$+\sum_{\gamma=1}^{3}\sum_{j=j_0}^{J-1}\sum_{k_1,k_2=0}^{2^j-1} \tilde{b}_{j,k_1,k_2}^{(\gamma)^T}\psi_{j,k_1,k_2}^{(\gamma)}(u,v). \qquad (3)$$

(3)

where

$$\tilde{a}_{j_0,k_1,k_2} = \frac{1}{n}\sum_{i=0}^{n} \quad {}_{j_0,k_1,k_2}(F_n(X_i),G_n(Y_i)),$$

$$\tilde{b}_{j,k_1,k_2}^{(\ )} = \frac{1}{n}\sum_{i=0}^{n} \quad {}_{j,k_1,k_2}^{(\ )}(F_n(X_i),G_n(Y_i)).$$

Note that $c_{\tilde{J}}$ may sometimes be negative on parts of its domain and fail to integrate to 1. If in applications, an intrinsic copula density estimate is deemed necessary, it can be derived from $c_{\tilde{J}}$ by truncation and normalization (Genest et al., 2009). In the general case, we must decide up on which resolution level to take to use in an application. Genest et al. (2009) stated that the resolution level J such that $2^J \quad \sqrt{n} \quad 2^{J+1}$ is the optimal resolution level.

3 Application

Now, we use wavelet for copula density estimation and to construct bivariate distribution. We consider the Norwegian stock index (T), the MSCI world stock index (M) that records from January 1, 1999 to July 8, 2003. Now, we are going to estimate copula density between interested variables using

wavelet method for the optimal degree of approximation. Using presented method in Genest et al. (2009) resolution level 5 has been chosen to achieve acceptable approximation level. The AIC criteria for copula density approximation between T and M using different wavelet reported in Table 1. The finding results show better fit for wavelet rather than Kernel and Fourier method. The AIC for Kernel and Fourier are -88.64 and -90.21, respectively. The estimated copula density for these variables using Daubechies (2) wavelet is plotted in Figure 1.

Table 1: Comparison between estimated c_{TM} using different wavelets.

Type of wavelet	AIC
Adelson wavelet	-99.66
Coiflet (2) wavelet	-100.44
Daubechies (2) wavelet	-110.22
Haar wavelet	-91.32
Meyer wavelet	-103.3
Symlet (2) wavelet	-107.82

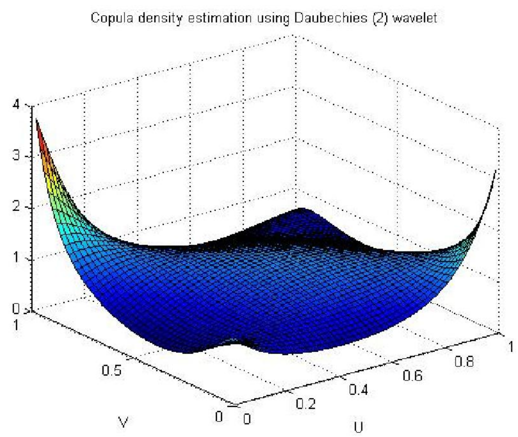

Figure 1: Copula density estimation using Daubechies (2) wavelet.

4. Conclusions

The main goal of this paper was to present the use and usefulness of wavelet for estimating copula density in comparison with previous method in the literature, Kernel and Fourier. Compact support and orthogonal property in wavelet analysis was so helpful for our approximation. Our proposed method was used to analyze the dependency structure between Norwegian stock index and MSCI world stock index.

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References

1. Chui, C. K. (1992). *An Introduction to Wavelets,* Academic Press.
2. Daubechies, I. (1992). Orthonormal Bases of Compactly Supported Wavelets, *Comm. Pure Appl. Math.*, 41: 909-996.
3. Deheuvels., P. (1979). La fonction de dependence empirique et ses proprietes: Un test nonparametrique dindependance. *Bulletin de lAcademie royale de Belgique: Classe des sciences*, 65: 274-292.
4. Genest, C., Masiellob, E. and Tribouley, K. (2009). Estimating copula densities through wavelets. *Insurance: Mathematics and Economics*, 44, 170–181.
5. Gui, W. (2009). *Adaptive Series Estimators for Copula Densities*. PhD thesis, Florida State University College of Arte and Sciences.
6. Joe, H. (1997). *Multivariate Models and Dependence Concepts*. Chapman & Hall, London.
7. Scaillet, O. and Fermanian, J. D. (2003). Nonparametric Estimation of Copulas for Time Series, *Journal of Risk.,* 11: 25-54.
8. Sklar, A. (1959). Fonctions de r'epartition 'a n dimensions et leurs marges. *Publications de lâ™Institut de Statistique de lâ™UniversitÂ´ e de Paris,* 8, 229–231.
9. Nelsen, R. B. (2006). *An Introduction to Copulas*. 2nd ed., Springer, New York.