

**The New Prime theorems (391) - (440)**

Jiang, Chun-Xuan

Institute for Basic Research, Palm Harbor, FL34682-1577, USA

And: P. O. Box 3924, Beijing 100854, China

[jiangchunxuan@sohu.com](mailto:jiangchunxuan@sohu.com), [cxjiang@mail.bcf.net.cn](mailto:cxjiang@mail.bcf.net.cn), [jcxuan@sina.com](mailto:jcxuan@sina.com), [Jiangchunxuan@vip.sohu.com](mailto:Jiangchunxuan@vip.sohu.com),  
[jcxxxx@163.com](mailto:jcxxxx@163.com)

**Abstract:** Using Jiang function  $J_2(\omega)$  we prove that the new prime theorems (341)- (390) contain infinitely many prime solutions and no prime solutions. Analytic and combinatorial number theory (August 29-September 3, ICM2010) is a conjecture. The sieve methods and circle method are outdated methods which cannot prove twin prime conjecture and Goldbach's conjecture. The papers of Goldston-Pintz-Yildirim and Green-Tao are based on the Hardy-Littlewood prime k-tuple conjecture (1923). But the Hardy-Littlewood prime k-tuple conjecture is false: (<http://www.wbabin.net/math/xuan77.pdf>) (<http://vixra.org/pdf/1003.0234v1.pdf>). Mathematicians do not speak advanced mathematical papers in ICM2010. ICM2010 is lower congress.

[Jiang, Chun-Xuan. **The New Prime theorems (391) - (440)** . *Researcher* 2016;8(8):65-116]. ISSN 1553-9865 (print); ISSN 2163-8950 (online). <http://www.sciencepub.net/researcher>. 12. doi:[10.7537/marsrsj080816.12](https://doi.org/10.7537/marsrsj080816.12).

**Keywords:** new; prime theorem; Jiang Chunxuan

**The New Prime theorem (391)**

$$P, jP^{702} + k - j (j = 1, \dots, k-1)$$

Chun-Xuan Jiang  
[jiangchunxuan@vip.sohu.com](mailto:jiangchunxuan@vip.sohu.com)

**Abstract**

Using Jiang function we prove that  $jP^{702} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{702} + k - j (j = 1, \dots, k-1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P-1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jP^{702} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \tag{3}$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{702} + k - j$  is a prime.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{702} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(702)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P-1)$

Example 1. Let  $k = 3, 7, 19, 79, 139$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 7, 19, 79, 139$ ,  
(1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 7, 19, 79, 139$

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 7, 19, 79, 139$ ,  
(1) contain infinitely many prime solutions

**The New Prime theorem (392)**

$$P, jP^{704} + k - j (j = 1, \dots, k-1)$$

Chun-Xuan Jiang  
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{704} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{704} + k - j (j = 1, \dots, k-1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P-1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{704} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \tag{3}$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{704} + k - j$  is a prime.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{704} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(704)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P-1)$

Example 1. Let  $k = 3, 5, 17, 23, 89, 353$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 5, 17, 23, 89, 353$ ,  
(1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 5, 17, 23, 89, 353$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 5, 17, 23, 89, 353$   
(1) contain infinitely many prime solutions

**The New Prime theorem (393)**

$$P, jP^{706} + k - j (j = 1, \dots, k-1)$$

Chun-Xuan Jiang  
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{706} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{706} + k - j (j = 1, \dots, k-1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P-1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{706} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \tag{3}$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{706} + k - j$  is a prime.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{706} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(706)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P-1)$

Example 1. Let  $k = 3$ . From (2) and(3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3$ ,  
(1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k > 3$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k > 3$ ,  
(1) contain infinitely many prime solutions

**The New Prime theorem (394)**

$$P, jP^{708} + k - j(j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{708} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{708} + k - j(j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{708} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{708} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{708} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(708)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P-1)$

Example 1. Let  $k = 3, 5, 7, 13, 709$ . From (2) and (3) we have  $J_2(\omega) = 0$

we prove that for  $k = 3, 5, 7, 13, 709$ , (1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 5, 7, 13, 709$

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 5, 7, 13, 709$  (1) contain infinitely many prime solutions

**The New Prime theorem (395)**

$$P, jP^{710} + k - j (j = 1, \dots, k-1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{710} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{710} + k - j (j = 1, \dots, k-1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P-1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{710} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \tag{3}$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{710} + k - j$  is a prime.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{710} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(710)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P-1)$

Example 1. Let  $k = 3, 11$ . From (2) and(3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 11$ ,  
(1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 11$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 11$   
(1) contain infinitely many prime solutions

**The New Prime theorem (396)**

$$P, jP^{712} + k - j (j = 1, \dots, k-1)$$

Chun-Xuan Jiang  
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{712} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{712} + k - j (j = 1, \dots, k-1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P-1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{712} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \tag{3}$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{712} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have  
 $J_2(\omega) = 0$  (5)

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{712} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(712)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}$$
 (6)

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 5$ . From (2) and(3) we have  
 $J_2(\omega) = 0$  (7)

we prove that for  $k = 3, 5$ ,  
 (1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 5$ .

From (2) and (3) we have  
 $J_2(\omega) \neq 0$  (8)

We prove that for  $k \neq 3, 5$ ,  
 (1) contain infinitely many prime solutions

**The New Prime theorem (397)**

$$P, jP^{714} + k - j(j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{714} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{714} + k - j(j = 1, \dots, k - 1)$$
 (1)

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)]$$
 (2)

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{714} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1$$
 (3)

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0$$
 (4)

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes

$P$  such that each of  $jP^{714} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{714} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(714)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 7, 43, 103$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 7, 43, 103$ ,

(1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 7, 43, 103$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 7, 43, 103$ ,

(1) contain infinitely many prime solutions

**The New Prime theorem (398)**

$$P, jP^{716} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang

Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{716} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{716} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{716} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$



We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{716} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{716} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(716)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 5, 359$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 5, 359$ , (1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 5, 359$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 5, 359$ , (1) contain infinitely many prime solutions

**The New Prime theorem (399)**

$$P, jP^{718} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

**Abstract**

Using Jiang function we prove that  $jP^{718} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{718} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{718} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{718} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{718} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(718)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3$ . From (2) and(3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3$ ,  
(1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k > 3$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k > 3$ ,  
(1) contain infinitely many prime solutions

**The New Prime theorem (400)**

$$P, jP^{720} + k - j(j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{720} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{720} + k - j(j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{720} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{720} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{720} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(720)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 5, 7, 11, 13, 17, 19, 31, 37, 41, 61, 73, 181, 241$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 5, 7, 11, 13, 17, 19, 31, 37, 41, 61, 73, 181, 241$ , (1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 5, 7, 11, 13, 17, 19, 31, 37, 41, 61, 73, 181, 241$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 5, 7, 11, 13, 17, 19, 31, 37, 41, 61, 73, 181, 241$ , (1) contain infinitely many prime solutions

**The New Prime theorem (401)**

$$P, jP^{722} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{722} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{722} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{722} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{722} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{722} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(722)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3$ , (1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k > 3$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k > 3$ , (1) contain infinitely many prime solutions

**The New Prime theorem (402)**

$$P, jP^{724} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{724} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{724} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{724} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have  
 $J_2(\omega) \neq 0$  (4)

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{724} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have  
 $J_2(\omega) = 0$  (5)

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{724} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(724)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}$$
 (6)

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 5$ . From (2) and(3) we have

$$J_2(\omega) = 0$$
 (7)

we prove that for  $k = 3, 5$ ,  
 (1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k > 5$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0$$
 (8)

We prove that for  $k > 5$ ,  
 (1) contain infinitely many prime solutions

**The New Prime theorem (403)**

$$P, jP^{726} + k - j(j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

**Abstract**

Using Jiang function we prove that  $jP^{726} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{726} + k - j(j = 1, \dots, k - 1)$$
 (1)

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)]$$
 (2)

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{726} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1$$
 (3)

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{726} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{726} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(726)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 7, 23, 67, 727$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 7, 23, 67, 727$ , (1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 7, 23, 67, 727$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 7, 23, 67, 727$ , (1) contain infinitely many prime solutions

**The New Prime theorem (404)**

$$P, jP^{728} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{728} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{728} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{728} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \tag{3}$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{728} + k - j$  is a prime.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{728} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(728)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P-1)$

Example 1. Let  $k = 3, 5, 29, 53$ . From (2) and(3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 5, 29, 53$ , (1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 5, 29, 53$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 5, 29, 53$ , (1) contain infinitely many prime solutions

**The New Prime theorem (405)**

$$P, jP^{730} + k - j(j = 1, \dots, k-1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

**Abstract**

Using Jiang function we prove that  $jP^{730} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{730} + k - j(j = 1, \dots, k-1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P-1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{730} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \tag{3}$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{730} + k - j$  is a prime.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{730} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(730)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P(P-1)$

Example 1. Let  $k = 3, 11$ . From (2) and(3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 11$ ,

(1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 11$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 11$ ,

(1) contain infinitely many prime solutions

**The New Prime theorem (406)**

$$P, jP^{732} + k - j(j = 1, \dots, k-1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

**Abstract**

Using Jiang function we prove that  $jP^{732} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{732} + k - j(j = 1, \dots, k-1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]



$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{732} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \tag{3}$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{732} + k - j$  is a prime.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{732} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(732)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P-1)$

Example 1. Let  $k = 3, 5, 7, 13, 367, 733$ . From (2) and(3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 5, 7, 13, 367, 733$ , (1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 5, 7, 13, 367, 733$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 5, 7, 13, 367, 733$ , (1) contain infinitely many prime solutions

**The New Prime theorem (407)**

$$P, jP^{734} + k - j(j = 1, \dots, k-1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

**Abstract**

Using Jiang function we prove that  $jP^{734} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{734} + k - j(j = 1, \dots, k-1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{734} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \tag{3}$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{734} + k - j$  is a prime.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{734} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(734)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P-1)$

Example 1. Let  $k = 3$ . From (2) and(3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3$ ,  
(1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k > 3$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k > 3$ ,  
(1) contain infinitely many prime solutions

**The New Prime theorem (408)**

$$P, jP^{736} + k - j(j = 1, \dots, k-1)$$

Chun-Xuan Jiang  
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{736} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{736} + k - j(j = 1, \dots, k-1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P-1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{736} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \tag{3}$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{736} + k - j$  is a prime.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{736} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(736)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P-1)$

Example 1. Let  $k = 3, 5, 17, 47$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 5, 17, 47$ , (1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 5, 17, 47$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 5, 17, 47$ , (1) contain infinitely many prime solutions

**The New Prime theorem (409)**

$$P, jP^{738} + k - j (j = 1, \dots, k-1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{738} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{738} + k - j (j = 1, \dots, k-1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{738} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \tag{3}$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{738} + k - j$  is a prime.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{738} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(738)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P-1)$

Example 1. Let  $k = 3, 7, 19, 739$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 7, 19, 739$ , (1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 7, 19, 739$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 7, 19, 739$ , (1) contain infinitely many prime solutions

**The New Prime theorem (410)**

$$P, jP^{740} + k - j (j = 1, \dots, k-1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{740} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{740} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{740} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{740} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{740} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(740)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 5, 11, 149$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 5, 11, 149$ , (1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 5, 11, 149$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 5, 11, 149$ , (1) contain infinitely many prime solutions

### The New Prime theorem (411)

$$P, jP^{742} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{742} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{742} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{742} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{742} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{742} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(742)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 107, 743$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 107, 743$ , (1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 107, 743$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 107, 743$ , (1) contain infinitely many prime solutions

**The New Prime theorem (412)**

$$P, jP^{744} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{744} + k - j$  contain infinitely many prime solutions and no prime

solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{744} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{744} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{744} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{744} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(744)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 5, 7, 13, 373$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 5, 7, 13, 373$ , (1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 5, 7, 13, 373$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 5, 7, 13, 373$ , (1) contain infinitely many prime solutions

**The New Prime theorem (413)**

$$P, jP^{746} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{746} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{746} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{746} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{746} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{746} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(746)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3$ ,

(1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k > 3$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k > 3$ ,

(1) contain infinitely many prime solutions

**The New Prime theorem (414)**

$$P, jP^{748} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

Abstract



Using Jiang function we prove that  $jP^{748} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{748} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{748} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{748} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{748} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(748)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 5, 23$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 5, 23$ ,

(1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 5, 23$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 5, 23$ ,

(1) contain infinitely many prime solutions

**The New Prime theorem (415)**

$$P, jP^{750} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{750} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{750} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{750} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{750} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{750} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(750)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 7, 11, 31, 151, 751$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 7, 11, 31, 151, 751$ , (1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 7, 11, 31, 151, 751$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 7, 11, 31, 151, 751$ , (1) contain infinitely many prime solutions

**The New Prime theorem (416)**

$$P, jP^{752} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{752} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{752} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{752} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{752} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \{P \leq N : jP^{752} + k - j = \text{prime}\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(752)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 5, 17$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 5, 17$ , (1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 5, 17$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 5, 17$ , (1) contain infinitely many prime solutions

**The New Prime theorem (417)**

$$P, jP^{754} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{754} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{754} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{754} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{754} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{754} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(754)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 59$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 59$ ,

(1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 59$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 59$ ,

(1) contain infinitely many prime solutions

**The New Prime theorem (418)**

$$P, jP^{756} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang

Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{756} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{756} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{756} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{756} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{756} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(756)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 5, 7, 13, 19, 29, 37, 43, 127, 379, 757$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 5, 7, 13, 19, 29, 37, 43, 127, 379, 757$ ,

(1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 5, 7, 13, 19, 29, 37, 43, 127, 379, 757$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 5, 7, 13, 19, 29, 37, 43, 127, 379, 757$ ,

(1) contain infinitely many prime solutions

**The New Prime theorem (419)**

$$P, jP^{758} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{758} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{758} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{758} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{758} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{758} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(758)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3$ . From (2) and(3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3$ ,

(1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k > 3$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k > 3$ ,

(1) contain infinitely many prime solutions

**The New Prime theorem (420)**

$$P, jP^{760} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{760} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{760} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{760} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{760} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{760} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(760)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 5, 11, 41, 191, 761$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 5, 11, 41, 191, 761$ , (1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 5, 11, 41, 191, 761$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 5, 11, 41, 191, 761$ , (1) contain infinitely many prime solutions

**The New Prime theorem (421)**

$$P, jP^{762} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{762} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{762} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{762} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{762} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{762} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(762)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 7$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 7$ ,

(1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 7$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 7$ ,

(1) contain infinitely many prime solutions



$$P, jP^{764} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{764} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{764} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{764} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{764} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{764} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(764)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 5, 383$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 5, 383$ , (1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 5, 383$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 5, 383$ , (1) contain infinitely many prime solutions

**The New Prime theorem (423)**

$$P, jP^{766} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{766} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{766} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{766} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{766} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{766} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(766)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3$ ,

(1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k > 3$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k > 3$ ,

(1) contain infinitely many prime solutions

**The New Prime theorem (424)**

$$P, jP^{768} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{768} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{768} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{768} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{768} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{768} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(768)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 5, 7, 13, 17, 97, 193, 257, 769$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 5, 7, 13, 17, 97, 193, 257, 769$ ,

(1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 5, 7, 13, 17, 97, 193, 257, 769$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 5, 7, 13, 17, 97, 193, 257, 769$ ,

(1) contain infinitely many prime solutions

**The New Prime theorem (425)**

$$P, jP^{770} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

**Abstract**

Using Jiang function we prove that  $jP^{770} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{770} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{770} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{770} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{770} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(770)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 11, 23, 71$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 11, 23, 71$ , (1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 11, 23, 71$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 11, 23, 71$ , (1) contain infinitely many prime solutions

**The New Prime theorem (426)**

$$P, jP^{772} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

**Abstract**

Using Jiang function we prove that  $jP^{772} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{772} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{772} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{772} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{772} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(772)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 5, 773$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 5, 773$ , (1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 5, 773$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 5, 773$ , (1) contain infinitely many prime solutions

**The New Prime theorem (427)**

$$P, jP^{774} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{774} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{774} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{774} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{774} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{774} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(774)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 7, 19$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 7, 19$ , (1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 7, 19$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 7, 19$ ,

(1) contain infinitely many prime solutions

**The New Prime theorem (428)**

$$P, jP^{776} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{776} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{776} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{776} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{776} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{776} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(776)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 5, 389$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 5, 389$ ,

(1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 5, 389$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 5, 389$ ,  
 (1) contain infinitely many prime solutions

**The New Prime theorem (429)**

$$P, jP^{778} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

**Abstract**

Using Jiang function we prove that  $jP^{778} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{778} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{778} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{778} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{778} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(778)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3$ . From (2) and(3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3$ ,  
 (1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k > 3$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$



We prove that for  $k > 3$ ,  
 (1) contain infinitely many prime solutions

**The New Prime theorem (430)**

$$P, jP^{780} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

**Abstract**

Using Jiang function we prove that  $jP^{780} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{780} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{780} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{780} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{780} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega) \omega^{k-1}}{(780)^{k-1} \phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 5, 7, 11, 13, 31, 53, 61, 79, 131, 157$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 5, 7, 11, 13, 31, 53, 61, 79, 131, 157$ ,

(1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 5, 7, 11, 13, 31, 53, 61, 79, 131, 157$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 5, 7, 11, 13, 31, 53, 61, 79, 131, 157$ ,  
 (1) contain infinitely many prime solutions

**The New Prime theorem (431)**

$$P, jP^{782} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

**Abstract**

Using Jiang function we prove that  $jP^{782} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{782} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{782} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{782} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{782} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(782)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 47$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 47$ ,  
 (1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 47$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 47$ ,  
(1) contain infinitely many prime solutions

**The New Prime theorem (432)**

$$P, jP^{784} + k - j (j = 1, \dots, k-1)$$

Chun-Xuan Jiang  
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{784} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{784} + k - j (j = 1, \dots, k-1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{784} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \tag{3}$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{784} + k - j$  is a prime.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{784} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(784)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P-1)$

Example 1. Let  $k = 3, 5, 17, 29, 113, 197$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 5, 17, 29, 113, 197$ ,  
(1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 5, 17, 29, 113, 197$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 5, 17, 29, 113, 197$ ,  
 (1) contain infinitely many prime solutions

**The New Prime theorem (433)**

$$P, jP^{786} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

**Abstract**

Using Jiang function we prove that  $jP^{786} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{786} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{786} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{786} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{786} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(786)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 7, 263, 787$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 7, 263, 787$ ,  
 (1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 7, 263, 787$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 7, 263, 787$ ,  
 (1) contain infinitely many prime solutions

**The New Prime theorem (434)**

$$P, jP^{788} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{788} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{788} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{788} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{788} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{788} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(788)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 5$ . From (2) and(3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 5$ ,  
 (1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 5$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 5$ ,

(1) contain infinitely many prime solutions

**The New Prime theorem (435)**

$$P, jP^{790} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang

Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{790} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{790} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{790} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{790} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{790} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega) \omega^{k-1}}{(790)^{k-1} \phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 11$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 11$ ,

(1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 11$ .  
From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 11$ ,  
(1) contain infinitely many prime solutions

**The New Prime theorem (436)**

$$P, jP^{792} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{792} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{792} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{792} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{792} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{792} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(792)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 5, 7, 13, 19, 37, 67, 73, 199, 397$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 5, 7, 13, 19, 37, 67, 73, 199, 397$ ,  
 (1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 5, 7, 13, 19, 37, 67, 73, 199, 397$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 5, 7, 13, 19, 37, 67, 73, 199, 397$ ,  
 (1) contain infinitely many prime solutions

**The New Prime theorem (437)**

$$P, jP^{794} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{794} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{794} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{794} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{794} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{794} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(794)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3$ . From (2) and (3) we have



$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3$ ,  
 (1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k > 3$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k = 3$ ,  
 (1) contain infinitely many prime solutions

**The New Prime theorem (438)**

$$P, jP^{796} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{796} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{796} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{796} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{796} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{796} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(796)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 5, 797$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 5, 797$ ,  
 (1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 5, 797$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 5, 797$ ,  
 (1) contain infinitely many prime solutions

**The New Prime theorem (439)**

$$P, jP^{798} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{798} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{798} + k - j (j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{798} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{798} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{798} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(798)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$ .

Example 1. Let  $k = 3, 7, 43$ . From (2) and(3) we have  
 $J_2(\omega) = 0$  (7)

we prove that for  $k = 3, 7, 43$ ,  
 (1) contain no prime solutions. 1 is not a prime.

Example 2. Let  $k \neq 3, 7, 43$ .  
 From (2) and (3) we have  
 $J_2(\omega) \neq 0$  (8)

We prove that for  $k \neq 3, 7, 43$ ,  
 (1) contain infinitely many prime solutions

**The New Prime theorem (440)**

$$P, jP^{800} + k - j(j = 1, \dots, k - 1)$$

Chun-Xuan Jiang  
 Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{800} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{800} + k - j(j = 1, \dots, k - 1) \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{800} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \tag{3}$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any  $k$  there are infinitely many primes  $P$  such that each of  $jP^{800} + k - j$  is a prime.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{800} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(800)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P - 1)$

Example 1. Let  $k = 3, 5, 11, 17, 41, 101, 401$ . From (2) and(3) we have

$$J_2(\omega) = 0 \tag{7}$$

we prove that for  $k = 3, 5, 11, 17, 41, 101, 401$ ,  
(1) contain no prime solutions. 1 is not a prime.

**Example 2.** Let  $k \neq 3, 5, 11, 17, 41, 101, 401$ .

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for  $k \neq 3, 5, 11, 17, 41, 101, 401$ ,  
(1) contain infinitely many prime solutions

**Remark.** The prime number theory is basically to count the Jiang function  $J_{n+1}(\omega)$  and Jiang prime  $k$ -tuple

singular series  $\sigma(J) = \frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)} = \prod_P \left(1 - \frac{1 + \chi(P)}{P}\right) \left(1 - \frac{1}{P}\right)^{-k}$  [1,2], which can count the number of prime numbers. The prime distribution is not random. But Hardy-Littlewood prime  $k$ -tuple singular series

$\sigma(H) = \prod_P \left(1 - \frac{\nu(P)}{P}\right) \left(1 - \frac{1}{P}\right)^{-k}$  is false [3-17], which cannot count the number of prime numbers[3].

**References**

1. Chun-Xuan Jiang, Foundations of Santilli’s isonumber theory with applications to new cryptograms, Fermat’s theorem and Goldbach’s conjecture. Inter. Acad. Press, 2002, MR2004c:11001, (<http://www.i-b-r.org/docs/jiang.pdf>) (<http://www.wbabin.net/math/xuan13.pdf>)(<http://vixra.org/numth/>).
2. Chun-Xuan Jiang, Jiang’s function  $J_{n+1}(\omega)$  in prime distribution.(<http://www.wbabin.net/math/xuan2.pdf>.) (<http://wbabin.net/xuan.htm#chun-xuan>.)(<http://vixra.org/numth/>)
3. Chun-Xuan Jiang, The Hardy-Littlewood prime  $k$ -tuple conjecture is false.(<http://wbabin.net/xuan.htm#chun-xuan>)(<http://vixra.org/numth/>).
4. G. H. Hardy and J. E. Littlewood, Some problems of “Partio Numerorum”, III: On the expression of a number as a sum of primes. Acta Math., 44(1923)1-70.
5. W. Narkiewicz, The development of prime number theory. From Euclid to Hardy and Littlewood. Springer-Verlag, New York, NY. 2000, 333-353.
6. B. Green and T. Tao, Linear equations in primes. Ann. Math, 171(2010) 1753-1850.
7. D. Goldston, J. Pintz and C. Y. Yildirim, Primes in tuples I. Ann. Math., 170(2009) 819-862.
8. T. Tao. Recent progress in additive prime number theory, preprint. 2009. <http://terrytao.files.wordpress.com/2009/08/prime-number-theory1.pdf>.
9. J. Bourgain, A. Gamburd, P. Sarnak, Affine linear sieve, expanders, and sum-product, Invent math, 179 (2010)559-644.
10. K. Soundararajan, The distribution of prime numbers, In: A. Granville and Z. Rudnik (eds), Equidistribution in number theory, an Introduction, 59-83, 2007 Springer.
11. B. Kra, The Green-Tao theorem on arithmetic progressions in the primes: an ergodic point of view, Bull. Amer. Math. Soc., 43(2006)3-23.
12. K. Soundararajan, Small gaps between prime numbers: The work of Goldston-Pintz-Yildirim, Bull. Amer. Math. Soc., 44(2007)1-18.
13. D. A. Goldston, S. W. Graham, J. Pintz and C. Y. Yildirim, Small gaps between products of two primes, Proc. London Math. Soc., 98(2009)741-774.
14. B. Green and T. Tao, The primes contain arbitrarily long arithmetic progressions, Ann. Math., 167(2008) 481-547.
15. D. A. Goldston, J. Pintz and C. Y. Yildirim, Primes in tuples II, Acta Math.,204(2010),1-47.
16. B. Green, Generalising the Hardy-Littlewood method for primes, International congress of mathematicians, Vol, II, 373-399, Eur. Math. Soc., Zurich, 2006.
17. T. Tao, The dichotomy between structure and randomness, arithmetic progressions, and the primes, International congress of mathematicians Vol. I, 581-608, Eur. Math. Soc., Zurich 2006.
18. Jiang, Chun-Xuan (蒋春暄). The New Prime theorems (391) - (440) . *Academ Arena* 2016;8(1s): 141-193. (ISSN 1553-992X). doi:<http://www.sciencepub.net/academia>. 4. doi:[10.7537/marsaaj0801s16.04](https://doi.org/10.7537/marsaaj0801s16.04).