Calculation of phase shift for p+^{40}Ca elastic scattering at low energy

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Abstract: The theoretical analysis of the experimental proton elastic scattering on ^{40}Ca taken in the energy range 9 to 22 MeV has been studied within the framework of the optical model employing Woods-Saxon and its derivative forms for potentials. Using SCAT2000 FORTRAN code and selected set of optical model parameters (OMPs) the contribution of different partial waves to scattering amplitude and cross section are studied. Based on phase shift values the significant number of partial waves in terms of their corresponding angular momenta at a given projectile energy is determined.

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1. Introduction

Study of nuclear scattering is an important source that provides information about the nuclear structure. Especially the scattering of protons is sensitive to details of the nuclear wave function [1]. Therefore the phase shift of the partial waves, for the elastic scattering nucleon-nucleus interaction, is considered a significant parameter that can be used to study the contribution of partial waves to the scattering amplitude, and hence to the cross section and polarization results [2]. Previously, phase shifts were calculated by Gross et al, for p+^{40}Ca elastic scattering in the range of $30 \le E_p \le 45$ MeV to get simultaneous fit to the polarization and cross-section data with a reasonable set of optical-model parameters[3]. They concluded that the calculated phase shifts gave no indication of elastic scattering resonances in the considered energy range. The phase shifts, for proton elastic scattering on light nuclei in the range of $15 \le A \le 40$ and $14 \le E_p \le 44$ MeV, were used by Fabrici et al. for determining the partial waves that are most significant and responsible for x^2 improvement of polarization for proton-light nuclei at low energy. They used phase shift analysis for x^2 minimization for every nucleus and incident energy [4]. In addition, the phase shifts can be used to study anomalous absorption of the scattered partial waves of proton-nucleus interaction by the optical potential [5-6].

In the present work the elastic scattering of P+^{40}Ca will be discussed within optical model in energy range $9 \le E_p \le 22$ MeV. The purpose is the determination of partial waves of most contribution to the scattering amplitude or the cross section values based on the phase shift calculation. Moreover, the reflection coefficients dependence on angular momentum is studied.

2. Theory

In nucleon-nucleus elastic scattering problems, Schrodinger equation must include optical potential to get the scattering observables [7-8]. By numerical solution of Schrodinger equation one can obtain the reflection coefficient and phase shift that will be discussed in the next sections. In this section theoretical method of calculation that was used in SCAT2000 code is outlined. The radial wave equation $u_l(r)$ that includes optical potential can be written as:

$$\left[\frac{d^2}{dr^2} - U_{op}(r) + k^2 - \frac{l(l+1)}{r^2}\right] u_l(r) = 0 \qquad (1)$$

where $k^2 = 2\mu E/\hbar^2$ and $\mu = mM/(m+M)$ are the wave number and reduced mass of the system, respectively. E is the center of mass energy is in $\hbar$ being the Planck constants, l is the angular momentum, $\frac{l(l+1)}{r^2}$ is the centrifugal potential, and U_{op} is the optical potential representing the interaction.

$$U_{op}(r) = V_c(r) - [V_v(E)f(x_n) - 4a_m V_s(E)g(x_m)] \\ -i[W_v(E)f(x_n) - 4a_m W_s(E)g(x_m)] \\ +\left[\left(V_{so}(E) + iW_{so}(E)\right)H(x_{so}).\vec{s}.\vec{l}\right] \qquad (2)$$

where $V_v(E)$, $V_s(E)$, $W_v(E)$, $W_s(E)$, $V_{so}(E)$, and $W_{so}(E)$ are the potential depths of the real volume, real surface, imaginary volume, imaginary surface, real spin orbit, and imaginary spin orbit terms, respectively. The terms $f(x_n)$, $g(x_m)$, and $H(x_{so})$

are the Wood- Saxon and its derivative forms that can be defined as:

$$f(x_n) = (1 + e^{x_n})^{-1}, \quad x_n = (r - R_n)/a_n.$$
$$R_n = r_n A^{1/3} \qquad n = V_v \text{ or } W_v \qquad (3)$$
$$g(x_m) = \frac{d}{dr} f(x_m) = \frac{1}{a_n}[e^{x_m}/(1 + e^{x_m})^2],$$
$$x_m = (r - R_m)/a_m.$$
$$R_m = r_m A^{1/3}, \; m = V_s \text{ or } W_s \qquad (4)$$
$$H(x_{so}) = C_{so} \frac{4a_{so}}{r} \frac{d}{dr} (1 + e^{x_{so}})^{-1},$$
$$x_{so} = (r - R_{so})/a_{so}. \qquad R_{so} = r_{so} A^{1/3} \qquad (5)$$
$$C_{so} = \left(\frac{\hbar}{m_\pi c}\right)^2 = 2.00 fm^2 \qquad (6)$$
$$\vec{s}.\vec{l} = 1/2[j(j+1) - s(s+1) - l(l+1)] \qquad (7)$$

where r_i ($i = n, m$ *and* so) represents the constant of radius parameter R_i and a_i ($i = n, m$ *and* so) represents the diffuseness parameter. Considering the incident proton as a point charge and the target as a uniformly charged sphere with radius R_C , the coulomb energy has the form.

$$V_C(r) = \frac{Zze^2}{2R_c}\left(3 - \frac{r^2}{R_C^2}\right) for \; \le R_C,$$
$$= \frac{Zze^2}{r} \qquad for \;\; r \ge R_C.$$
$$R_C = r_C A^{1/3} \qquad (8)$$

where C_{so} is the Thomas constant, j total angular momentum, s spin of incident projectile, Z and z atomic number of target and projectile and r_C is the constant of coulomb radius parameter. Equation (1) was solved numerically by using Cowell method [7]. The obtained result of the numerical solution is the transmission coefficients defined as:

$$T_{lj} = 1 - |\eta_{lj}|^2. \qquad (9)$$

where η_{lj} is the reflection coefficient.

$$\eta_{lj} = e^{2i\delta_{lj}} \qquad (10)$$

Expressing complex phase shifts in terms of real and imaginary parts, then

$$\eta_{lj} = e^{2i(\delta_{lj}^{Re} + i\delta_{lj}^{Im})}$$
$$= e^{-2\delta_{lj}^{Im}} e^{2i\delta_{lj}^{Re}} \qquad (11)$$

where $e^{-2\delta_{lj}^{Im}}$ is the phase amplitude coefficient. If $e^{-2\delta_{lj}^{Im}} = 1$, the intensity of the outgoing wave is equal to that of the incoming wave and reflection is complete. However there are non-elastic processes (absorbed wave functions), the intensity of the outgoing elastically scattered wave must be less than that of the incident wave so that $e^{-2\delta_{lj}^{Im}} < 1$ [9-11]. The value of phase shift (δ_{lj}) vector magnitude is:

$$\|\delta_{lj}\| = \sqrt{\left(\delta_{lj}^{Re}\right)^2 + \left(\delta_{lj}^{Im}\right)^2} \qquad (12)$$

where
$$\delta_{lj}^{Im} = -\frac{1}{2} ln \sqrt{\left(\eta_{lj}^{Re}\right)^2 + \left(\eta_{lj}^{Im}\right)^2}$$
$$\delta_{lj}^{Re} = \frac{1}{2}\tan^{-1}\frac{\eta_{lj}^{Im}}{\eta_{lj}^{Re}}$$

3. Optical model parameters

Three OMPs sets are used to calculate the elastic scattering phase shifts for p+^{40}Ca interaction. Two of them are of Koning [12] and Xiaohua [13] which are obtained from literature by using the reference input parameters library (RIPL-2) [14]. These parameter sets are extracted from (RIPL-3) by FORTRAN code (MOM) [15]. This code is available on the web site of IAEA. The other set is our OMPs set which is obtained by fitting to get better agreement with experimental data. This set is presented in tables 1. The results of three OMPs angular distribution were discussed in details in previous paper [16].

Table 1. OMPs set of our work that was used for fitting the experimental data [16]

E (MeV)	V_v (MeV)	V_s (MeV)	W_v (MeV)	W_s (MeV)	$V_{s.o}$ (MeV)	$W_{s.o}$ (MeV)
9.860	49.413	2.400	0.525	2.610	5.000	-0.044
10.370	49.112	2.332	0.544	2.801	5.000	-0.045
11.420	48.492	2.112	0.586	3.199	5.000	-0.048
12.440	47.890	1.898	0.628	3.587	5.000	-0.052
13.950	46.999	1.581	0.694	4.161	5.000	-0.061
14.520	46.663	1.461	0.720	4.378	5.000	-0.065
15.570	46.044	1.240	0.770	4.777	5.000	-0.074
15.970	45.810	1.151	0.789	4.929	5.000	-0.077
16.570	45.454	1.030	0.820	5.157	5.000	-0.083
17.570	44.864	0.820	0.870	5.537	5.000	-0.094
18.570	44.274	0.613	0.922	5.917	5.000	-0.110
19.570	43.678	0.398	0.977	6.300	5.000	-0.120
20.570	43.094	0.190	1.032	6.677	5.000	-0.130
21.680	42.440	0.000	1.093	7.100	5.000	-0.150

r_{V_v}(fm) = 1.246 $\quad r_{V_s}$(fm) = 1.280 $\quad r_{W_v}$(fm) = 1.246

a_{V_v}(fm) = 0.650 $\quad a_{V_s}$(fm) = 0.700 $\quad a_{W_v}$(fm) = 0.570

r_{W_s}(fm) = 1.246 $\quad r_{V_{s.o}}$(fm) = 1.280 $\quad r_{W_{s.o}}$(fm) = 1.280

a_{W_s}(fm) = 0.570 $\quad a_{V_{s.o}}$(fm) = 0.742 $\quad a_{W_{s.o}}$(fm) = 0.742

r_C(fm) = 1.300

4. Results and Discussion

The reflection coefficients $\eta_{lj}^{\pm}$ are plotted in figure 1. The figure represents the value of $\eta_{lj}^{\pm}$ versus angular momentum l at energies 9.86, 15.57, and 21.57 MeV.

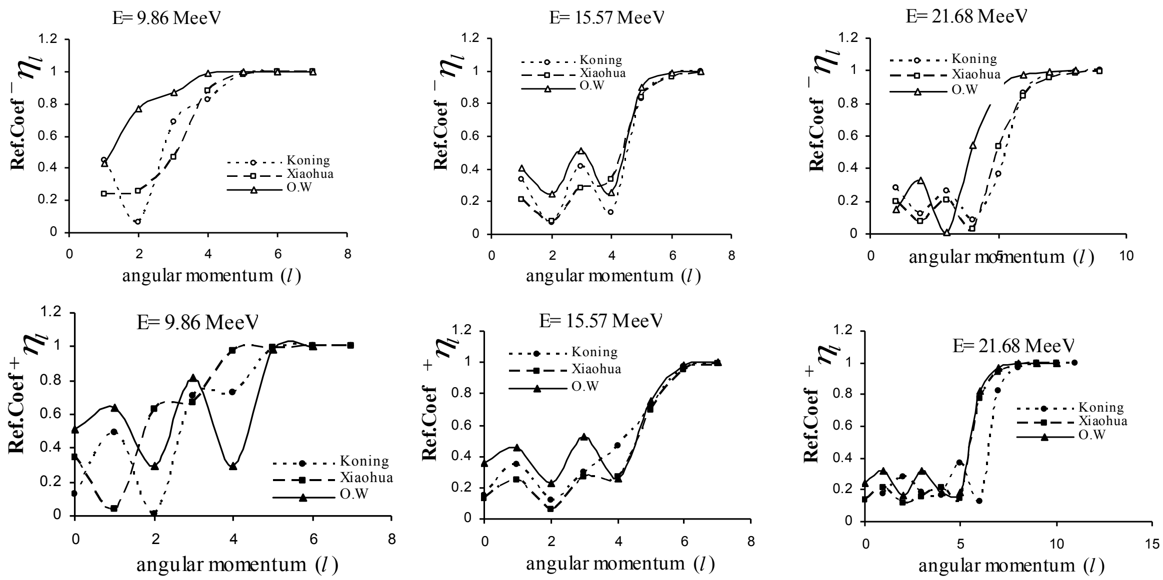

Figure 1. Reflection coefficients ($\eta_l^{\pm}$) for Koning, Xiaohua, and Our OMPs as function of angular momentum (l) at different energy, the plus sign refers to η_l^{+} at $j = l+1/2$ and minus sign refers to η_l^{-} at $j = l-1/2$.

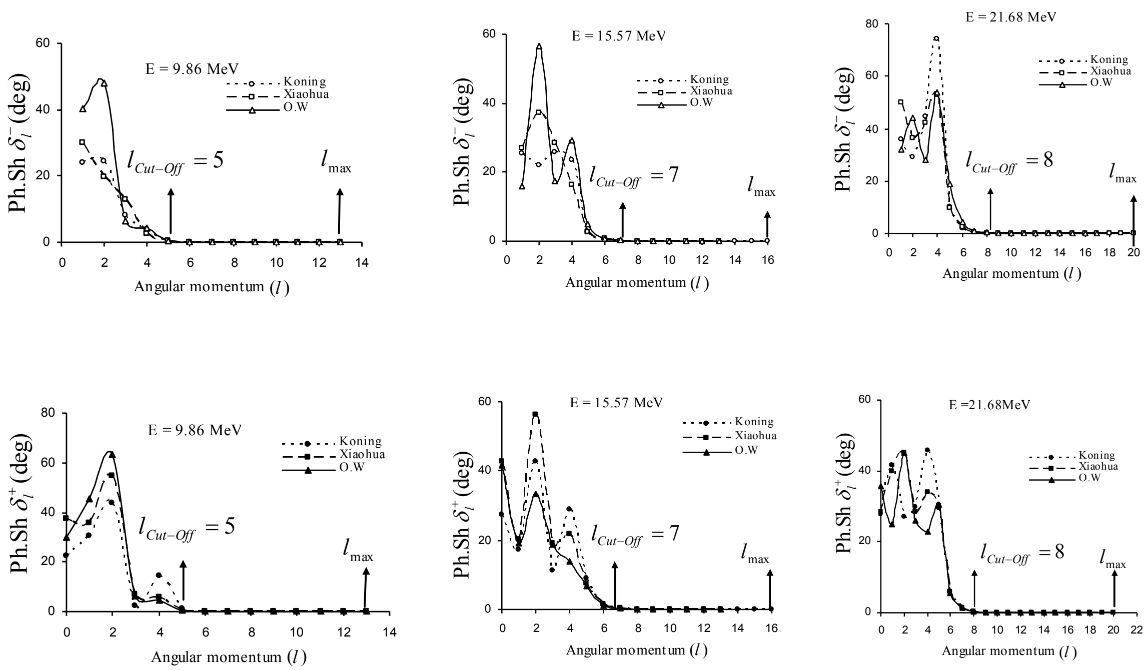

Figure 2. Phase shifts ($\delta_l^{\pm}$) for Koning, Xiaohua, and Our OMPs as function of angular momentum (l) at different energy, the plus sign refers to δ_l^{+} at $j = l+1/2$ and minus sign refers to δ_l^{-} at $j = l-1/2$.

The minus sign refers to the spin down of the incident protons, i.e. $j = l - \frac{1}{2}$. The plus sign refers to the spin up of the incident protons, i.e. $j = l + \frac{1}{2}$. The reflection coefficient $\eta_{lj}^{\pm}$ as mentioned is the ratio of the reflected partial waves to the total incident partial waves. The value of the reflection coefficient is restricted between $0 \leq \eta_{lj}^{\pm} \leq 1$. As seen in figure 1 the magnitudes of $\eta_{lj}^{\pm}$, at $E = 9.86$ for three OMPs, display large fluctuation between 0.1 and 0.8 at small values of angular momentum l . This is due to an anomalous absorption of the partial waves [5-6]. Then the fluctuation of $\eta_{lj}^{\pm}$ becomes small at $E = 15.57$ MeV and the absorbed partial waves become larger as E increases. Finally at $E = 21.68$ MeV the fluctuation of $\eta_{lj}^{\pm}$ becomes smaller with value between 0.1 and 0.3 at small value of l, this means large absorption of partial waves is occurred, then it tends to unity i.e. ≈ 1 at large value of l indicating that total reflection of partial waves occurred. If the value of $\eta_{lj}^{\pm} = 0$ the nucleus will become black and it can absorb all incident partial waves. However this cannot occur due to the strong reflection by the centrifugal potential.[10-11].

In order to investigate the contribution of the partial waves the value of phase shifts for each partial wave should be calculated. The calculated phase shifts that are plotted in figure 2 can be used to determine the significant partial waves. For fixed energy, the number of partial waves that can contribute to the cross section is calculated by SCAT2000 according to this relation [7].

$$l_{max} = [5(\rho_m^{0.8}) + 7]$$

$$\rho_m = kr_m \quad \& \quad r_m = \frac{3}{2} max(r_j + 7a_j) \quad j = 1, 2, 3 \quad (13)$$

where r_j and a_j are the radius parameter and diffuseness respectively. The index j refers to the types of the real volume, imaginary volume, and imaginary surface potentials. This relation determines the number of significant partial up to l_{max} i.e. l_{max} is the upper limit of contribution of partial waves to cross sections.

According to the phase shifts analysis in figure 2, the significant number of partial waves can be cut at $l_{Cut-Off}$ which is defined as the angular momentum l at which the value of its phase shift equals 1% of the maximum value of δ_l at a given energy. In such case all partial waves that have $l > l_{Cut-Off}$ can be ignored. The difference between l_{max} of SCAT2000 calculations and $l_{Cut-Off}$ of phase shift calculation is plotted in figure 3. The number of significant partial waves at 9.86 MeV equals 6 and 8 at 21.68 MeV as shown in figure 3. The values of total cross section according to l_{max} and $l_{Cut-Off}$ are shown in tables 2.

Table 2. Numerical values of cross section calculated for l_{max} and $l_{Cut-Off}$ partial waves

E (MeV)	$\sigma_T(mb)$ for l_{max}	$\sigma_T(mb)$ for $l_{Cut-Off}$
9.860	1671.100	1671.000
13.950	1740.400	1740.200
17.570	1698.000	1697.900
21.680	1720.000	1719.980

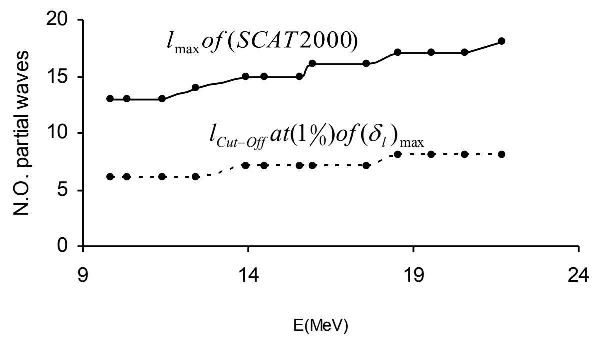

Figure 3. Number of partial waves $u_l(r)$ at l_{max} and $l_{Cut-Off}$

On the other hand, the behavior of vector magnitude of phase shifts for the three OMPs are studied to know the dependence of the phase shift on energy and the differences between the OMPs sets as shown in figure 4. The behavior of phase shifts, for $s_{1/2}$ at $E > 12$ MeV in addition to $p_{1/2}$ and $p_{3/2}$ at 9.86, 19.58, and 21.68 MeV, displays broad peaks. These peaks may be attributed to the effect of the interaction that takes place in the interior region of the nucleus, where low partial have high contribution. The similarity of phase shifts behavior for Xiaohua and our OMPs may be attributed to the similarity of our OMPs and those of Xiaohua. While as directed outword of the nuclear surface at higher $d_{3/2}$, $d_{5/2}$,…… $g_{9/2}$ partial waves, the behavior of this partial waves become smoother [17-18].

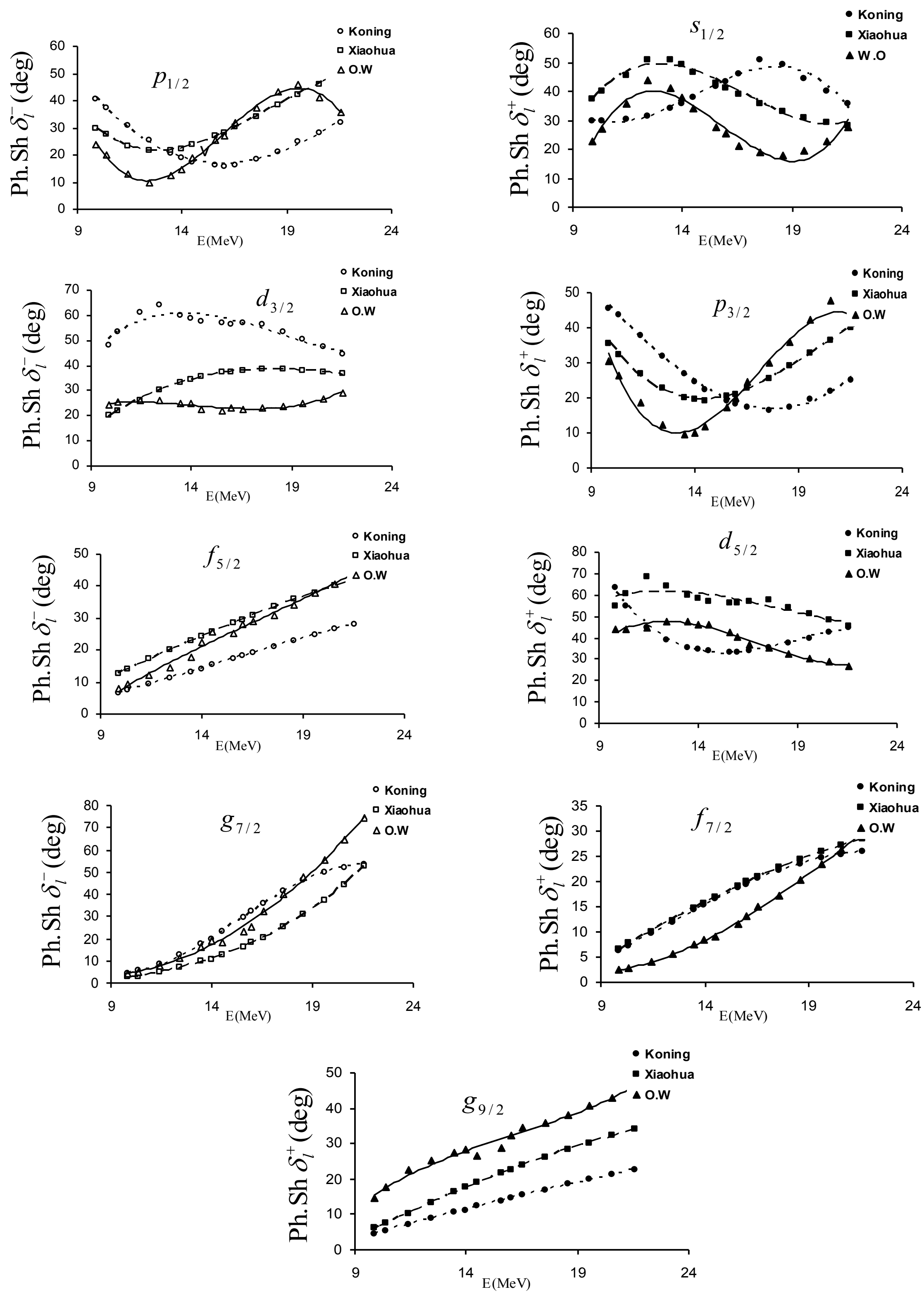

Figure 4. Phase shifts ($\delta_l^{\pm}$), that are calculated by Koning, Xiaohua, and Our OMPs, as function of incident proton energy for number of partial waves (l = 0, 1, 2, 3,4 and s = ± 1/2). Plus and minus signs mean that δ_l^+ at $j = l+1/2$ and δ_l^- at $j = l-1/2$

5. Conclusion

Three OMPs sets were used to calculate reflection coefficients and phase shifts of elastic scattering for p+^{40}Ca. The calculations were performed by means of SCAT2000. The calculated phase shifts were used to study of the contribution of partial waves to cross section. According to the results of phase shift analysis the significant number of partial waves to be taken in calculation has an upper limit $l_{Cut-Off}$, in contrary to SCAT 2000 in which calculations are performed up to l_{max} . Our used scheme to determine the $l_{Cut-Off}$ based on phase shift approach yealds cross sections that agree well with those of SCAT2000 based on l_{max}. Using $l_{Cut-Off}$ means less number of partial waves and hence reducing the calculation time.

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