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ELZAKI TRANSFORM TO DIFFERENTIAL EQUATIONS WITH DELTA FUNCTION

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Abstract: The differential equations with delta function are generally solved by adopting Laplace transform method. The paper inquires the differential equations with delta function by Elzaki transform. The purpose of paper is to prove the applicability of Elzaki transform to analyze differential equations with delta function.
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I. INTRODUCTION

Elzaki Transform approach has been applied in solving boundary value problems in most of the science and engineering disciplines [1, 2, 3, 4, 5, 6]. It also comes out to be very effective tool to analyze differential equations with delta function [7, 8, 9, 10, 11, 12, 13]. The differential equations are generally solved by adopting Laplace transform method or convolution method of residue theorem method 14, 15, 16, 17, 18, 19, 20]. In this paper, we present a new technique called Elzaki transform to analyze differential equations with delta function.

II. BASIC DEFINITIONS

2.1 Elzaki Transform

If the function ɦ(y), y ≥ 0 is having an exponential order and is a piecewise continuous function on any interval, then the Elzaki transform of ɦ(y) is given by

$$E\{ɦ(y)\} = \bar{ɦ}(p) = p\int_0^\infty e^{-\frac{y}{p}}ɦ(y)dy.$$

The Elzaki Transform [1, 2, 3] of some of the functions are given by

- $E\,\{y^n\} = n!\,p^{n+2}\,, where\; n = 0,1,2,..$
- $E\,\{e^{ay}\} = \frac{p^2}{1-ap}\,,$
- $E\,\{sinay\} = \frac{ap^3}{1+a^2p^2}\,,$
- $E\,\{cosay\} = \frac{ap^2}{1+a^2p^2}\,,$
- $E\,\{sinhay\} = \frac{ap^3}{1-a^2p^2}\,,$
- $E\,\{coshay\} = \frac{ap^2}{1-a^2p^2}\,.$

2.2 Inverse Elzaki Transform

The Inverse Elzaki Transform of some of the functions are given by

- $E^{-1}\{p^n\} = \frac{y^{n-2}}{(n-2)!}\,, n = 2, 3, 4 \ldots$
- $E^{-1}\{\frac{p^2}{1-ap}\} = e^{ay}$
- $E^{-1}\{\frac{p^3}{1+a^2p^2}\} = \frac{1}{a}\sin ay$
- $E^{-1}\{\frac{p^2}{1+a^2p^2}\} = \frac{1}{a}\cos ay$
- $E^{-1}\{\frac{p^3}{1-a^2p^2}\} = \frac{1}{a}\sin hay$
- $E^{-1}\{\frac{p^2}{1-a^2p^2}\} = \frac{1}{a}\cos hay$

2.3 Elzaki Transform of Derivatives

The Elzaki Transform [1, 2, 3] of some of the Derivatives of h(y) are given by

- $E\{h'(y)\} = \frac{1}{p}E\{h(y)\} - p\,h(0)$

  $or\; E\{ɦ'(y)\} = \frac{1}{p}\bar{ɦ}(p) - p\,ɦ(0),$
- $E\{ɦ''(y)\} = \frac{1}{p^2}\bar{ɦ}(p) - ɦ(0) - pɦ'(0),$

  and so on.

III. METHODOLOGY

APPLICATION I:

(A)

$$\ddot{y} + 4y = 51\delta(t)$$
$$and\; y(0) = 0\,, y'(0) = 1$$

Applying Elzaki Transform, we have

$$E\,\{\ddot{y}\} + 4E\{y\} = 51p$$

Or

$$\frac{\bar{y}(p)}{p^2} - y(0) - py'(0) + 4\bar{y}(p) = 51p$$

Or

$$\frac{\bar{y}(p)}{p^2} + 4\bar{y}(p) = 52p$$

Or

$$\bar{y}(p) = \frac{52p^3}{1+4p^2}$$

Hence

$$y = E^{-1}\left\{\frac{52p^3}{1+4p^2}\right\}$$

or

$$y = 26sin2t$$

(B)

$$\ddot{y} + \dot{y} = 21\delta(t)$$

$$and\ y(0) = 0\ , y'(0) = 1$$

Applying Elzaki Transform, we have

$$\text{E}\ \{\ddot{y}\} + E\{\dot{y}\} = 21p$$

Or

$$\frac{\bar{y}(p)}{p^2} - y(0) - py'(0) + \frac{\bar{y}(p)}{p} - py(0) = 21p$$

Or

$$\frac{\bar{y}(p)}{p^2} + \frac{\bar{y}(p)}{p} = 22p$$

Or

$$\bar{y}(p) = \frac{22p^3}{1+p}$$

Hence

$$y = E^{-1}\left\{\frac{22p^3}{1+p}\right\}$$

or

$$y = 22E^{-1}\left\{\frac{p^3}{1+p}\right\}$$

or

$$y = 22E^{-1}\left\{p^2 - \frac{p^2}{1+p}\right\}$$

or

$$y = 22(1-e^{-t})$$

(C)

$$\ddot{y} + 12\dot{y} + 36y = 21\delta(t)$$

$$and\ y(0) = 0\ , y'(0) = 5$$

$$\text{E}\ \{\ddot{y}\} + 12E\{\dot{y}\} + 36E\{y\} = 21p$$

Or

$$\frac{\bar{y}(p)}{p^2} - y(0) - py'(0) + 12\frac{\bar{y}(p)}{p} - 12py(0) + 36\bar{y}(p) = 21p$$

Or

$$\frac{\bar{y}(p)}{p^2} + 12\frac{\bar{y}(p)}{p} + 36\bar{y}(p) = 26p$$

Or

$$\left[\frac{1}{p^2} + \frac{12}{p} + 36\right]\bar{y}(p) = \frac{26p^3}{1+p}$$

Or

$$\bar{y}(p) = \frac{26p^3}{(1+6p)^2}$$

Hence

$$y = E^{-1}\left\{\frac{26p^3}{(1+6p)^2}\right\}$$

or

$$y = 26te^{-6t}$$

(D)

$$\ddot{y} + 4b\dot{y} + 4b^2y = 14\delta(t)$$

$$and\ , y(0) = 0\ , y'(0) = 10$$

Applying Elzaki Transform, we have

$$\text{E}\ \{\ddot{y}\} + 4bE\{\dot{y}\} + 4b^2E\{y\} = 14p$$

Or

$$\frac{\bar{y}(p)}{p^2} - y(0) - py'(0) + 4b\frac{\bar{y}(p)}{p} - 4bpy(0) + 4b^2\bar{y}(p) = 14p$$

Or

$$\frac{\bar{y}(p)}{p^2} + 4b\frac{\bar{y}(p)}{p} + 4b^2\bar{y}(p) = 14p + 10p$$

Or

$$\left[\frac{1}{p^2} + \frac{4b}{p} + 4b^2\right]\bar{y}(p) = 24p$$

Or

$$\bar{y}(p) = \frac{24p^3}{(1+2bp)^2}$$

Hence

$$y = E^{-1}\left\{\frac{24p^3}{(1+2bp)^2}\right\}$$

Or

$$y = 24te^{-2bt}$$

(*E*) $\ddot{y} + 8b^2\dot{y} + 16b^4y = 5\delta(t)$

$$and\ , y(0) = 0\ , y'(0) = 4$$

Applying Elzaki Transform, we have

$$\text{E}\ \{\ddot{y}\} + 8b^2E\{\dot{y}\} + 16b^4E\{y\} = 5\delta(t)$$

Or

$$\frac{\bar{y}(p)}{p^2} - y(0) - py'(0) + 8b^2\frac{\bar{y}(p)}{p} - 8b^2py(0) + 16b^4\bar{y}(p) = 5p$$

Or

$$\frac{\bar{y}(p)}{p^2} + 8b^2\frac{\bar{y}(p)}{p} + 16b^4\bar{y}(p) = 5p + 4p$$

Or

$$\left[\frac{1}{p^2} + \frac{8b^2}{p} + 16b^4\right]\bar{y}(p) = 9p$$

Or

$$\bar{y}(p) = \frac{9p}{(1+4b^2p)^2}$$

Hence

$$y = E^{-1}\left\{\frac{9p}{(1+4b^2p)^2}\right\}$$

Or

$$y = 9te^{-4b^2t}$$

(F)

$$7\ddot{y} + 3\dot{y} = 17\,\delta(t)$$

$and\ y(0) = 0\,, y'(0) = 13$

Applying Elzaki Transform, we have

$$7\mathrm{E}\,\{\ddot{y}\} + 3E\{\dot{y}\} = 17p$$

Or

$$7\frac{\bar{y}(p)}{p^2} - 7y(0) - 7py'(0) + 3\frac{\bar{y}(p)}{p} - 3py(0) = 17p$$

Or

$$7\frac{\bar{y}(p)}{p^2} + 3\frac{\bar{y}(p)}{p} = 24p$$

Or

$$\bar{y}(p) = \frac{24p^3}{7+3p}$$

Hence

$$y = E^{-1}\left\{\frac{24p^3}{7+3p}\right\}$$

or

$$y = 24E^{-1}\left\{\frac{p^3}{7+3p}\right\}$$

or

$$y = 24E^{-1}\left\{\frac{1}{3}p^2 - \frac{7}{3}\frac{p^2}{(7+3p)}\right\}$$

or

$$y = 8(1 - e^{-3/7t})$$

(G)

$$9\ddot{y} + 4y = 49\delta(t)$$

$and\ y(0) = 0\,, y'(0) = 8$

Applying Elzaki Transform, we have

$$9\mathrm{E}\,\{\ddot{y}\} + 4E\{y\} = 49p$$

Or

$$9\frac{\bar{y}(p)}{p^2} - 9y(0) - 9py'(0) + 4\bar{y}(p) = 49p$$

Or

$$9\frac{\bar{y}(p)}{p^2} + 4\bar{y}(p) = 121p$$

Or

$$\bar{y}(p) = \frac{121p^3}{9+4p^2}$$

Hence

$$y = \frac{121}{6}\,E^{-1}\left\{\frac{\frac{2}{3}p^3}{1+\frac{4}{9}p^2}\right\}$$

or

$$y = \frac{121}{6} sin\frac{2}{3}t$$

IV. CONCLUSION

In this paper, we have differential equations with delta function by Elzaki Transform technique**.** It may be finished that the technique is accomplished in analyzing the differential equations with delta function.

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