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Stand Diameter Distribution Modelling of a Mixed-Species Forest Plantation in North-Central Nigeria

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Abstract: Understanding stand structure is essential for sustainable forest management, yield prediction, and silvicultural planning. This study modelled the diameter distribution of trees in a mixed-species forest plantation to identify the statistical models that best describe the stand structure. A systematic sampling design was adopted to establish twenty-one 30 m × 30 m plots across the 35.3 ha plantation. All trees with a diameter at breast height (Dbh) $\geq$ 10 cm were measured. Descriptive statistics were used to characterize stand structure, while seven probability density functions, namely Weibull, Gamma, Lognormal, Normal, Cauchy, Exponential, and Logistic, were fitted using maximum likelihood estimation. Model performance was evaluated using Kolmogorov–Smirnov, Cramér–von Mises, and Anderson–Darling goodness of fit statistics. Tree density averaged 312 trees ha^{-1}, with Dbh ranging from 10.2 to 54.7 cm (mean = 22.9 cm). The diameter distribution exhibited positive skewness, indicating a predominance of small to medium diameter trees. Among the tested models, the Lognormal distribution provided the best overall fit, followed by the Gamma and Logistic models, while the Exponential model performed poorly. The results reflect the multiplicative nature of tree growth and the structural heterogeneity typical of mixed plantations. The Lognormal model is recommended for predicting stand structure and supporting management planning in the forest plantation and similar Guinea savannah ecosystems.

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Keywords: Diameter distribution; Lognormal model; Stand structure; Tropical plantation; Guinea savannah; Forest modelling.

Highlights

- Diameter distribution of a mixed plantation was modelled using seven PDFs.
- The stand exhibited a unimodal, positively skewed Dbh structure.
- Lognormal distribution provided the best overall model fit.
- Normal and Exponential models poorly represented Dbh variability.
- Lognormal modelling supports yield prediction and stand planning.

1. Introduction

Diameter distribution modelling is a fundamental tool in forest biometrics, providing insights into stand structure, regeneration dynamics, productivity, and management potential. The distribution of tree diameters shows ecological processes (such as recruitment, growth, competition, and mortality) and is widely used for yield regulation, carbon estimation, and silvicultural planning (Souza Barros *et al.*, 2025). In both natural and planted forests, diameter structure serves as an indicator of stand development stage and management history, making it essential for sustainable forest management and long-term productivity assessment (Ogana and Abwage, 2018; Bassil *et al*., 2019; Condés *et al*., 2022; Sun *et al*., 2025; Bončina *et al*., 2025). Earlier studies in Nigeria have also emphasized the relevance of diameter distribution analysis for understanding stand dynamics and supporting management decisions (Aigbe and Omokhua, 2014; Ogana *et al*., 2015; Egonmwan and Ogana, 2020).

Probability density functions such as the Weibull, Gamma, and Lognormal distributions have been widely applied in forestry because of their flexibility in describing skewed diameter distributions typical of uneven-aged, or mixed-species stands (Clutter *et al*., 1983; Zenner and Hibbs, 2000; Rijal *et al*., 2024; Sahin, 2023; Ogana *et al*., 2015). Forest managers use these functions to represent entire stand structures using a few sets of parameters in a model structure

(Corona, P., and Scotti, 1998), thereby simplifying predictions of stocking levels, biomass accumulation, and harvest yields (Gonçalves, 2022). Selecting an appropriate distribution model is critical for reliable growth and yield modelling, as different models vary in their ability to capture the biological variability inherent in forest stands (Newton, 2007).

Diameter distribution models determine stand characteristics by mathematically describing the frequency of trees across diameter classes using diameter at breast height (Dbh). Because Dbh is strongly correlated with tree volume, biomass, and competitive status, it is widely regarded as the most practical variable for stand-level modelling and inventory analysis (Zhou *et al.*, 2019). Such models are used to estimate tree numbers within specified Dbh classes and are often integrated with height–diameter, volume, or biomass equations to enhance the precision of forest resource assessments (Piotr *et al*., 2019).

Globally, diameter distribution modelling has been successfully applied in both natural and plantation forests to characterize stand structure, guide thinning regimes, and support sustainable forest utilization (Zarnoch and Dell, 1985; Cao, 2004; *Ciceu et al*., 2021). In tropical regions, where mixed-species plantations are increasingly promoted for ecological resilience and economic diversification, understanding stand diameter structure is particularly important for predicting growth patterns and carbon sequestration potential (FAO, 2020). Despite these advances, the application of robust diameter distribution modelling remains limited for mixed plantation forests in the Guinea savannah zone of Nigeria.

Tree diameter distribution models have been widely used to inform management decisions and quantify stand dynamics, yet research within guinea savannah forests in Nigeria is relatively scarce. This creates a gap in baseline quantitative information required for effective yield prediction, carbon stock estimation, and silvicultural planning in the region. Addressing this gap is essential given the increasing role of plantations in Nigeria's forest restoration and wood supply strategies. This study, therefore, aimed to: (i) Describe the diameter structure of the study Forest Plantation (ii) Fit selected statistical distribution models to Dbh data, and (iii) Identify the model that best represents the stand diameter distribution.

2. Material and Methods

2.1. Study Area

Tar-Ukpe Forest Plantation is in Yandev, Gboko Local Government Area of Benue State, North-Central Nigeria, between latitudes 7°21′25.2″ N and 7°21′50.4″ N and longitudes 9°02′30.8″ E and 9°03′07.2″ E. The plantation covers approximately 35.3 ha. The area lies within the Guinea savannah ecological zone, characterized by open woodland interspersed with shrubs and grasses. The climate has two distinct seasons: rainy season (April to October) and dry season (November to March). Annual rainfall ranges from 1,200–1,500 mm, with mean temperatures of 25–32 °C. Soils are typically ferruginous tropical soils derived from sedimentary parent materials. The plantation is mixed, dominated by *Gmelina arborea*, *Daniellia oliveri*, and Tectona grandis.

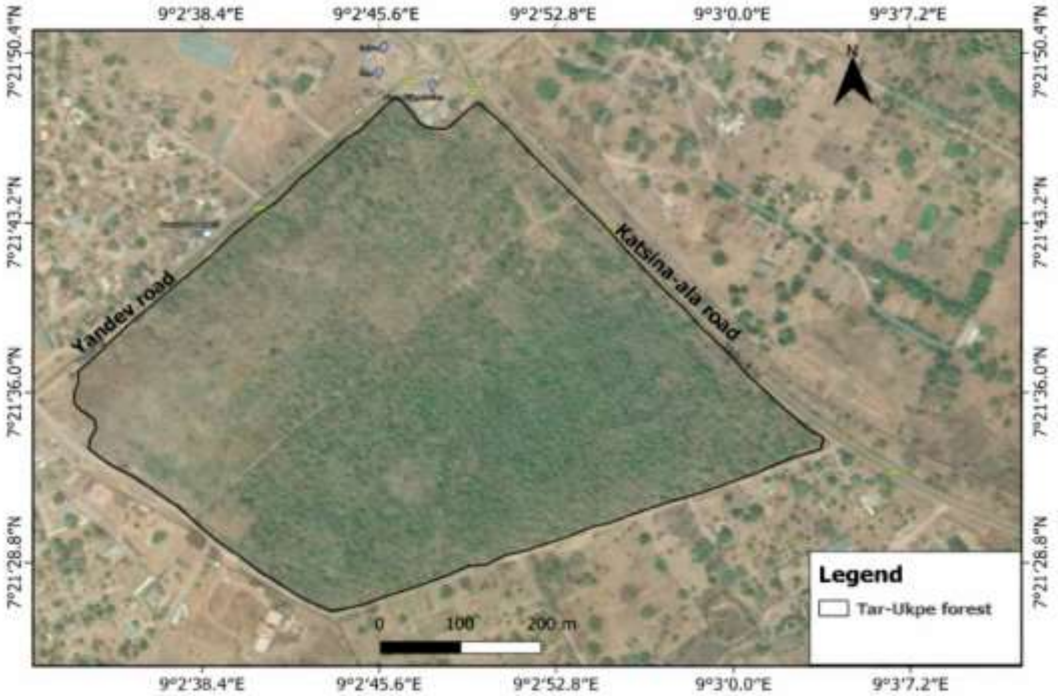

Figure 1: A Map of the *Tar-Ukpe* Forest Plantation, Gboko, Benue State, North-Central Nigeria

Data collection

The systematic sampling technique (Avery and Burkhart 2002) was adopted to ensure uniform coverage of the study area. A systematic grid was generated in a GIS environment and overlaid on the mapped boundary of Tar-Ukpe Forest Plantation. A total of twenty-one (21) sample plots, spaced at regular intervals of 133.8 meters, were distributed across the 35.3-ha plantation. Each sample plot measured 30 m × 30 m (0.09 ha) and was established at the grid point using a Garmin GPSMAP 78 for navigation and location verification. All live trees in the sample plots with stem diameter at breast height (Dbh)≥ 10cm were identified, and Dbh measurements were taken.

Data Analysis

Descriptive statistics were computed to summarize the characteristics of tree diameter at breast height (Dbh). These included measures of central tendency (mean), dispersion (standard deviation), shape (skewness and kurtosis), minimum and maximum values, and selected percentiles. Such statistics provide preliminary insight into stand structure and distributional form before probabilistic modelling (Chenge *et al.*, 2025).

Diameter Distribution Modelling

A variety of statistical approaches exist for estimating the parameters of probability density functions (PDFs), and the choice of method depends on the distribution form and desired estimation efficiency (Piqué-Nicolau *et al*., 2011). In this study, seven probability density functions commonly applied in forestry were evaluated for modelling tree diameter distributions: Weibull (two-parameter), Gamma (two-parameter), Lognormal (two-parameter), Normal, Cauchy, Exponential and Logistic. The form of the fitted (Table 1). These distributions were selected based on their documented flexibility in representing positively skewed and unimodal forest stand structures. Model fitting was conducted at the stand level using pooled Dbh data from all sample plots.

Parameter Estimation

Parameter estimation for each distribution was performed using the Maximum Likelihood Estimation (MLE) method. MLE identifies parameter values that maximize the likelihood function, thereby making the observed data most probable under the assumed distribution (Johnson *et al*., 1994; Xu, 2025). MLE is widely preferred in forestry applications because it provides asymptotically unbiased, efficient, and consistent estimators under regularity conditions (Hadi and Sahib, 2023).

Table 1: Diameter Distribution Fitted for the Study Area

Distribution	Equation	Parameter	Reference
Weibull	$f(x) = \left(\frac{c}{b}\right)\left(\frac{x}{b}\right)^{c-1} \; e^{-\left(\frac{x}{b}\right)^{c}}$	b = scale, c = shape	Bailey *and* Dell (1973)
Gamma	$f(x) = \frac{1}{\Gamma(a)b^{a}} e^{-\frac{x}{b}} x^{a-1}$	a = shape, b = scale	Krishnamoorthy (2006)
Lognormal	$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \; e^{-(lnx-\frac{\mu)^2}{2\sigma}}$	μ = location, σ = scale	Limpert *et al.* (2001)
Normal	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \; e^{-(x-\mu)^2/2\sigma^2}$	$\mu =$ mean, $\sigma =$ standard deviation	Krishnamoorthy (2006)
Cauchy	$f(x) = \frac{1}{\pi b[1+((x-a)/b)^2]}$	a = location, b = scale	Krishnamoorthy (2006)
Exponential	$f(x) = \frac{1}{b} e^{-\frac{x}{b}}$	b = scale	Krishnamoorthy (2006)
Logistics	$f(x) = \frac{1}{b} \cdot \frac{e^{-\frac{x-a}{b}}}{[1+e^{-\frac{x-a}{b}}]^2}$	a = location, b = scale	Johnson *et al.* (1994)

Goodness-of-Fit Evaluation

Model performance was evaluated using three empirical distribution function (EDF) statistics: (1) Kolmogorov–Smirnov (KS), (2) Cramér–von Mises (CVM), and (3) Anderson–Darling (AD) (Table 2). These statistics compare the empirical cumulative distribution function (ECDF) with the theoretical cumulative distribution function (CDF) of each fitted model (Stephens, 1974). Smaller values of these statistics indicate better agreement between observed and expected distributions, and therefore a more suitable model.

Distribution	Equation	Reference
Kolmogorov–Smirnov (KS)	$D_n = Supx\lvert F_n(x) - F(x) \rvert$	Kolmogorov (1933); Stephens (1974)
Cramér–von Mises (CVM)	$w^2 = n \int_{-\infty}^{\infty} [F_n(x) - F(x)]^2 \, dF^*(x)$	von Mises (1931)
Anderson–Darling (AD)	$A^2 = -n - \frac{1}{n}\sum_{i-1}^{n}(2i-1)[ln\, F(X_i) + \ln\,(1 - F(X_{n+1-i}))]$	Anderson *and* Darling (1954)

Where $F_n(x)$*is the empirical cumulative distribution function,* $F(x)$*is the theoretical cumulative distribution function, and* n *is the sample size.*

3. Results

3.1 Summary Statistics of Tree Diameter

Descriptive statistics of diameter at breast height (Dbh) are presented in Table 2. The mean Dbh was 22.9 cm, indicating that the stand is dominated by small- to medium-sized trees. The relatively high standard deviation (6.6 cm) reflects substantial variability in tree sizes, suggesting a structurally heterogeneous stand composed of individuals at different growth stages.

Observed Dbh values ranged from 10.2 cm to 54.7 cm, indicating the coexistence of younger trees and a limited number of mature trees. The distribution exhibited positive skewness (1.2), indicating a higher frequency of smaller-diameter trees relative to large stems. The kurtosis value (2.4) suggests a moderately peaked distribution with heavier tails, indicating that while most trees cluster around the mean diameter, a small number of larger individuals extend the upper tail of the distribution. Overall, these statistics indicate a stand structure characterized by smaller trees, with few larger size trees. The observed diameter distribution (Figure X) shows a unimodal but positively skewed pattern, with most stems concentrated within the 15–25 cm Dbh classes. The frequency declines progressively toward the larger diameter classes, reflecting limited representation of mature trees. This pattern is typical of managed or mid-rotation plantation stands where recruitment and growth dominate over mortality.

Table 2: Summary statistics of measured growth variables

Descriptive Statistics	**Dbh (cm)**
Mean	22.9
Standard Deviation	6.6
Kurtosis	2.4
Skewness	1.2
Minimum	10.2
Maximum	54.7
Sample size	590

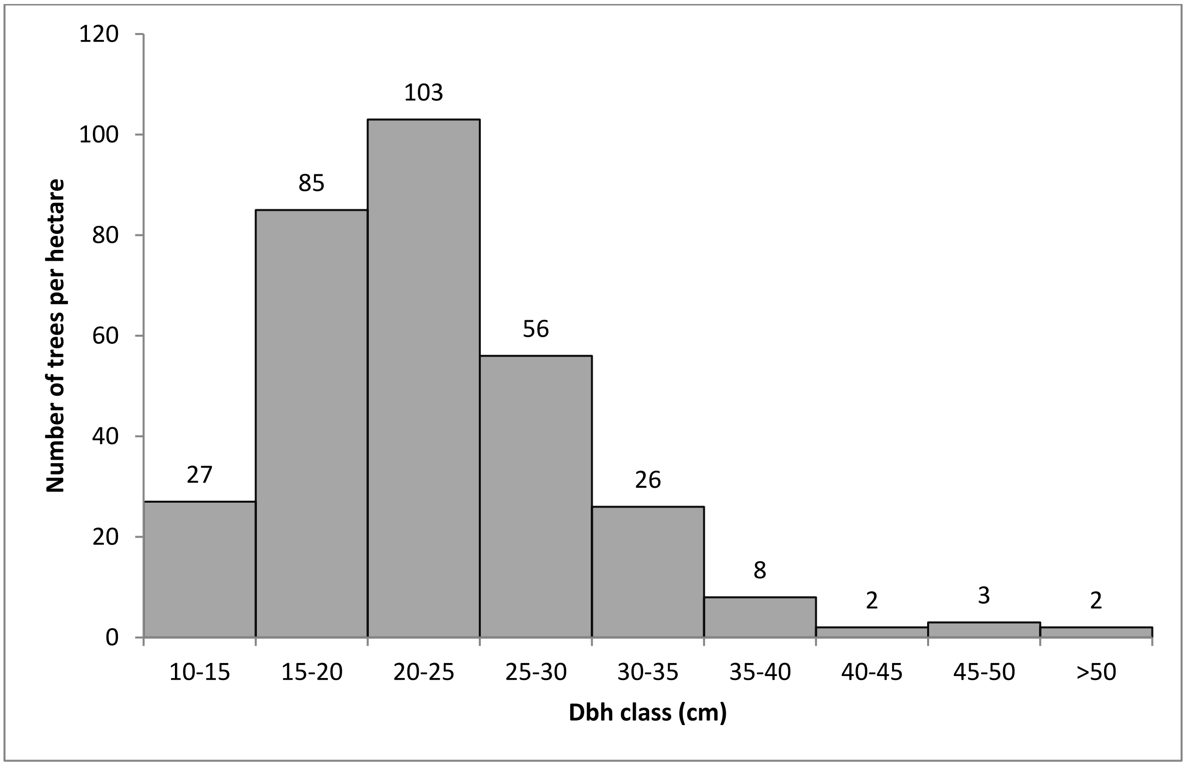

Figure 2: Stem diameter distribution of trees

Parameter Estimates of Fitted Distribution Models

Parameter estimates for the seven fitted probability distributions are presented in Table 3. The models produced varying shape, scale, location, and rate parameters, reflecting differences in their ability to represent the observed Dbh data.

Table 3: Parameter estimates of the fitted statistical distribution models

Model	**Parameter Estimates (Standard error in brackets)**			
	Shape	**Scale**	**Location**	**Rate**
Weibull	3.45 (0.10)	25.39 (0.32)		
Gamma	13.07 (0.75)			0.570 (0.03)
Lognormal		0.278(0.01)	3.09 (0.01)	
Normal		6.64 (0.19)	22.94 (0.27)	
Cauchy		3.68 (0.19)	21.66 (0.25)	
Exponential				0.04 (0.002)
Logistics		3.60 (0.12)	22.39 (0.27)	

3.4 Goodness-of-Fit Evaluation

Goodness-of-fit statistics for the fitted models are summarized in Table 4. Model performance was assessed using the Kolmogorov–Smirnov (KS), Cramér–von Mises (CVM), and Anderson–Darling (AD) tests, where smaller values indicate closer agreement between observed and theoretical distributions. The models were subsequently ranked based on their performance across the tests. The Lognormal distribution consistently provided the best fit across all three statistics (KS = 0.024; CVM = 0.044; AD = 0.39), ranking first overall. The Gamma and Logistic distributions also showed good agreement with the observed data, ranking second and third, respectively. The Weibull distribution demonstrated moderate performance, while the Normal and Cauchy models showed weaker fits. The Exponential model performed poorly, exhibiting substantially higher test statistics and failing to adequately represent the observed diameter structure. These results indicate that flexible, positively skewed distributions, particularly the Lognormal model, are most suitable for describing the Dbh distribution of the study stand.

Table 4: Goodness of fit statistics of fitted statistical distribution models

Model	Kolmogorov-Smirnov		Cramer-von Mises		Anderson-Darling	
	statistic	Rank	statistic	Rank	statistic	rank
Weibull	0.086	5	1.376	6	10.00	5
Gamma	0.040	2	0.179	2	1.32	2
Lognormal	0.024	1	0.044	1	0.39	1
Normal	0.113	6	1.222	5	10.98	6
Cauchy	0.076	4	0.950	4	6.29	4
Exponential	0.419	7	30.356	7	145.82	7
Logistics	0.051	3	0.325	3	3.51	3

Graphical comparisons further illustrate differences among model performances. Figure 1 compares fitted probability density curves, showing that the Lognormal, Gamma, and Logistic models closely follow the empirical distribution. Figure 2 overlays histograms with theoretical densities and shows that the Lognormal model most accurately captures both the central tendency and spread of the observed Dbh data. Figure 3 presents the empirical cumulative distribution function (CDF) against fitted CDFs, where the Lognormal curve aligns most closely across the full diameter range. Figure 4 shows probability–probability (P–P) plots, in which the Lognormal model exhibits points lying nearest the reference line, indicating minimal deviation between observed and predicted probabilities. These graphical diagnostics corroborate the statistical tests, confirming the Lognormal distribution as the most appropriate model for describing diameter structure in the Tar-Ukpe Forest Plantation.

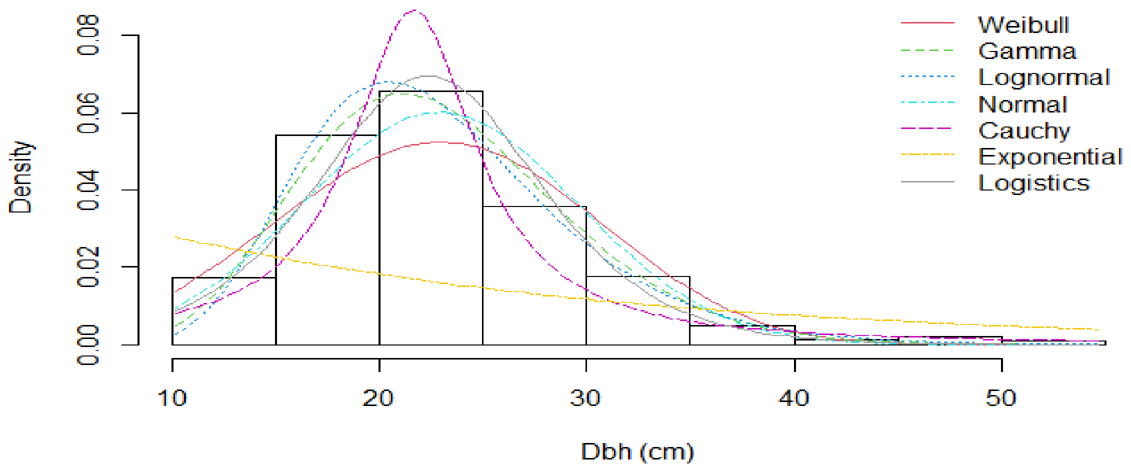

Figure 1: Graphical comparison of the fitted distribution model

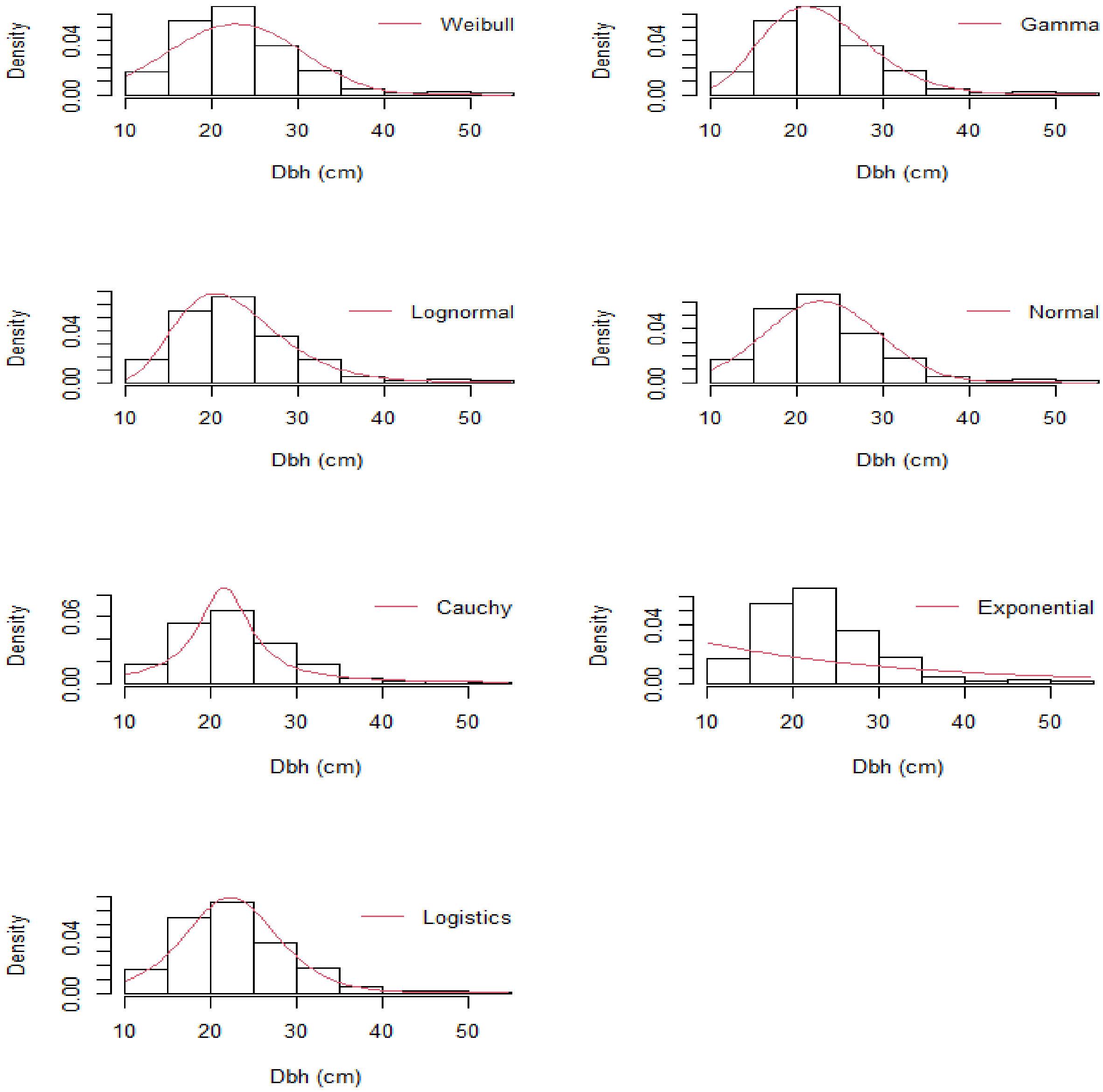

Figure 2: Histogram and theoretical densities of the fitted distribution models

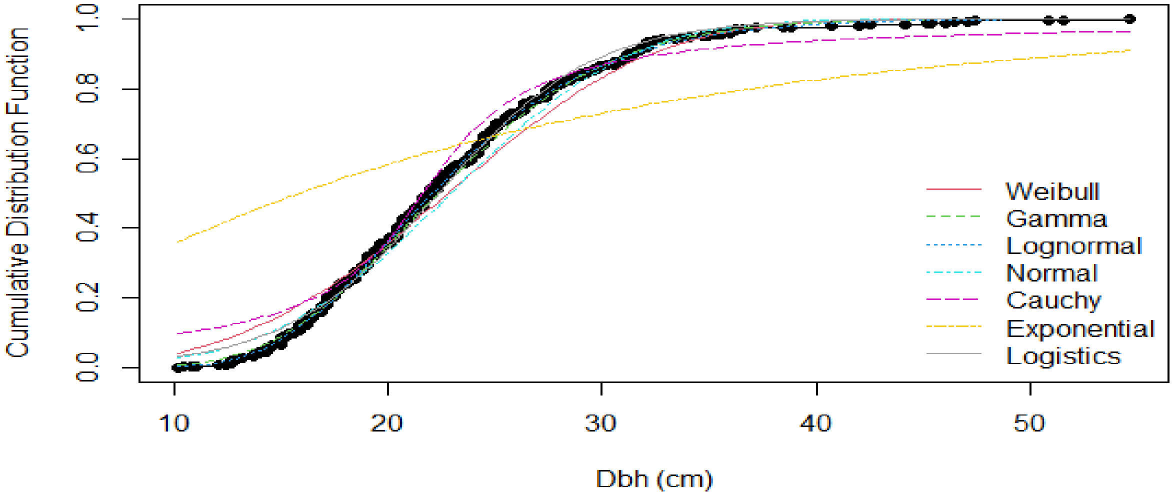

Figure 3: Plot of the empirical cumulative distribution against the fitted distribution functions

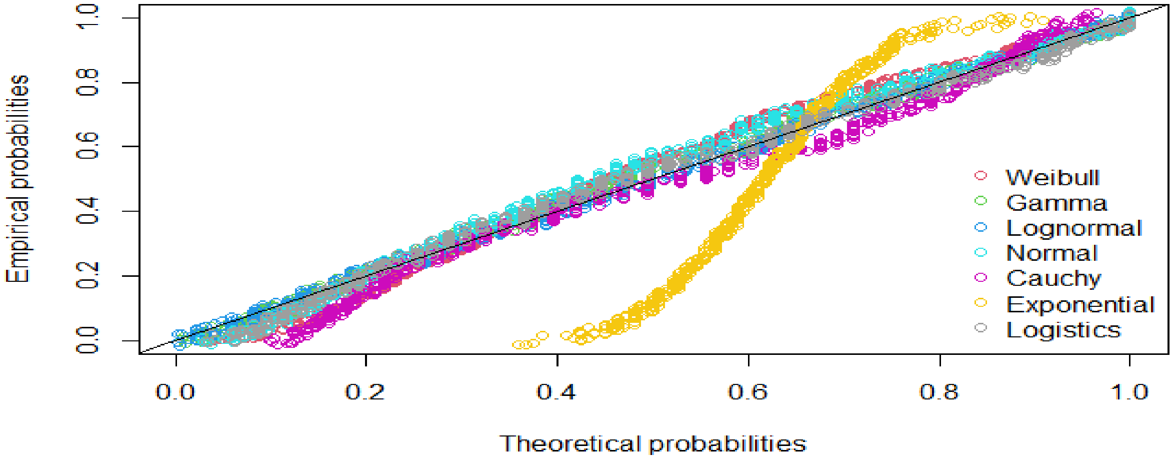

Figure 4: Plot of the probabilities of each fitted distribution against the empirical probabilities.

4. Discussions

4.1 Stand Structural Characteristics

The diameter distribution observed in the Tar-Ukpe Forest Plantation is characterized by a unimodal, positively skewed structure dominated by small to medium-sized stems. Such distributions are typical of developing or mid-rotation stands where recruitment and active growth exceed mortality, resulting in a concentration of individuals in intermediate diameter classes (Clutter *et al*., 1983; Chenge *et al.*, 2025). The relatively wide range of Dbh values, extending to over 50 cm, indicates that while the stand is largely composed of younger trees, a limited number of older individuals contribute to structural heterogeneity. This pattern is consistent with mixed-species plantations where interspecific differences in growth rates and competitive interactions create variability in size-class representation (Chenge *et al.*, 2025). Understanding such structural patterns can guide management interventions aimed at shaping future stand structure through targeted regeneration, restoration, or replanting to achieve desired management objectives (Wang *et al*., 2021; Li *et al*., 2021).

The positive skewness of the distribution indicates ongoing stand development and implies that mortality or harvesting pressures have not yet substantially reduced stem density. Similar structural patterns have been reported in tropical plantation systems undergoing canopy differentiation and resource-driven competition (Avery *and* Burkhart, 2002; Newton, 2007). From a management perspective, this indicates a transition stage in which silvicultural interventions, such as selective thinning, may influence future stand uniformity and productivity (Pretzsch, 2009)

4.2 Performance of Probability Density Functions

Among the evaluated models, the Lognormal distribution provided the best overall fit according to all goodness-of-fit statistics and graphical diagnostics. This finding is consistent with other studies in tropical for goodness-of-fit tests in Nigeria (Ezenwenyi *et al*., 2018; Oladoye *et al*., 2024). The strong performance of the Lognormal model is theoretically consistent with the multiplicative nature of biological growth processes, whereby relative growth rates depend on tree size, resource capture, and competitive status (Limpert *et al*., 2001; Chenge *et al.*, 2025). Because such processes generate proportional rather than additive changes, tree-size distributions in many natural and planted forests tend to approximate lognormality.

The Gamma and Logistic distributions also performed well, demonstrating flexibility in accommodating moderate skewness and dispersion. These distributions have been shown to effectively model heterogeneous forest stands where site conditions and species composition influence growth variability (Johnson *et al*., 1994; Cao, 2004). Although the Weibull function has historically been regarded as the standard model for diameter distribution analysis due to its flexible shape parameter (Bailey *and* Dell, 1973), its moderate ranking in this study suggests it may be less suitable for mixed plantations with complex growth patterns. Previous studies have observed that the Weibull model performs well in even-aged or intensively managed stands; alternative distributions may better represent structurally diverse forests (Newton, 2007).

The poor performance of the Exponential model was expected, as it assumes a constant rate of decline in tree frequency with increasing diameter, a condition rarely observed in biologically regulated forest stands. Forest growth is influenced by competition, microsite variability, and species-specific traits, producing more complex size distributions than those described by simple exponential decay (Clutter *et al*., 1983; Mayoral *et al*., 2018; Salekin *et al*., 2019).

Accurate representation of diameter distributions is central to forest growth modelling, inventory interpretation, and biomass estimation, as Dbh is strongly correlated with volume, basal area, and carbon storage (Zhou *et al.*, 2019). The identification of the lognormal model as the most appropriate descriptor for Tar-Ukpe Forest Plantation provides a practical analytical framework for predicting stand development and informing management decisions. The dominance of intermediate diameter classes implies that the stand is approaching a phase of intensified competition, during which silvicultural regulation can enhance growth allocation to selected crop trees (Li *et al.,* 2025). Such interventions are widely recommended to optimize stand productivity and structural stability in plantation forestry (Li *et al.,* 2025).

5. Conclusion

This study modelled the diameter distribution of a mixed-species plantation in North-Central Nigeria to identify the statistical function that best represents its stand structure. The observed Dbh distribution was positively skewed, indicating a developing stand dominated by small to medium-sized trees. Among the evaluated models, the Lognormal distribution provided the best overall fit, followed by the Gamma and Logistic functions, while the Normal and Exponential models were unsuitable for describing the observed variability.

The results highlight the importance of using flexible, skew-sensitive distributions to accurately characterize structurally heterogeneous plantations. The Lognormal model offers a reliable basis for estimating stand structure, supporting yield prediction, biomass assessment, and silvicultural planning. This study provides baseline biometric information for mixed plantations in the Guinea savannah zone and underscores the need for further growth-based modelling to enhance sustainable forest management in the region.

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