Generating the Peano axioms from two principles and one undefined term — Hilbert's second problem

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Abstract: The aim of this paper is to construct a model that only uses two principles and one undefined term from which the Peano axioms (five in total) can be derived. Plurality within the undefined term makes it possible to form new axioms from these two principles and by introducing certain substitutions the undefined term becomes multiplied into three entities. The Peano axioms arise from applying these substitutions to the two principles and the new axioms. The simplicity of the model makes it possible to take a look behind the scenery of mathematical thinking and to reveal a glimpse of its principles (foundation). [Report and Opinion. 2010;2(2):43-46]. (ISSN: 1553-9873).

Key words: self; self-evident; image; independence; singularity

1. Introduction
Although rational thinking and dreaming are both mental activities they differ in the fact that rational thinking obeys certain strict laws and therefore can be experienced as a discipline. Rational thinking can be analysed by itself and the apparent laws of rational thinking can be expressed in a rational way. For example, the law of identity states that an object is the same as it*self*.

2. Discussion
The rigidity of rational thinking is sustained by the concept 'self'. The peculiarity of this concept can be made clearer with the following thought experiment. A movable webcam is capable of imaging (reflecting) an object upon a screen but if one tries to image the screen it*self* upon the screen (self-reflection) then this method of imaging encounters some kind of oddity. When the webcam, while viewing the screen, is still far away from the screen then the image encountered on the screen is the surroundings of the screen and the screen which images a smaller screen with its surroundings which images a smaller screen with its surroundings and so forth. If one now moves the webcam towards the screen then the image of the screen on the screen becomes bigger and bigger. At the point where the image of the screen on the screen becomes exactly the same size as the screen (call this point P and while the webcam is in point P viewing the screen call this situation S) a peculiarity occurs. For consistency reasons *assume* that the clear screen (without an image, without a colour) is black, that an image covers the complete screen and that an image is neither flattened nor stretched. What is imaged on the screen will be an image of the screen which already carries an image. So, in situation S whatever image is imaged on the screen will depend on what image is encountered on the screen because this image will be re-imaged upon the screen. How, in this experiment, this re-imaged image will look depends on the image that is encountered on the screen just before the webcam arrives in situaion S, which depends on the image encountered just before that and so forth. Hence, it depends on the route taken towards situation S but also on the internal factors as the brightness of the screen, the sensitivity of the webcam, etc. *Assume* that the internal factors are set in such a way that in situation S the image on the screen equals the image of this image on the screen.[1]

The result of this thought experiment in situation S is a static self-imaging image (self-reflecting reflection) on the screen which is *independent* of the clear screen (without an image). This peculiar self-imaging image which is supposed to image the screen only re-images itself and therefore this method of imaging contains a bug with respect to situation S. The flaw arises from the ambiguity of the term 'screen' which in the process of imaging is experienced as an image and can no longer be considered an object. It is possible, though, to accept this peculiar self-imaging image on the screen as the correct image of the screen without need of proof or discussion because it is self-evident (in situation S the screen is imaged by definition). Self-evident is in this case independent of reality.

If, however, the screen in situation S would be without any image at all then this could be considered the true 'image' (without an image) because what is imaged on the screen is nothing and hence the black screen reveals its true self. This option would imply three things. First, that the possibility of 'imaging without an image' would in a mathematical context mean that the projection

function (imaging of the webcam on the screen) has a singularity in point P (the projection function is without an image for the object screen like a gap in a graph). Second, that this singularity would reveal a profound truth which lies beyond (actually before) the concept 'image', i.e., the screen without an image reveals the clear screen. Third, removing this singularity by allowing the webcam in situation S to image a self-imaging image on the screen instead (fulfillment of the singularity) would mean that the projection function would be complete (every object has an image) but in situation S independent of reality.

Accordingly, the following observation can be made: 'the image of an object is an image'. This can be formulated with only one term as 'the image of not an image (object) is an image'. The hoax, as pointed out, is created by the fact that the following principle also holds: 'the image of an image is an image'. This principle creates the existence of some kind of image which is independent of reality. In order to remain dependent of reality the method of imaging should instead obey the principle that 'the image of an image is not an image'.[2]

3. Generating the Peano Axioms
The following two principles will be considered:

P1.   The image of not an image is an image.
P2.   The image of an image is an image.

Plurality within the term 'image' can make the resulting 'an image' of principle P1 differ from the resulting 'an image' of principle P2. This can be formulated as an axiom by connecting the first part of principle P1 with the first part of principle P2 as follows:

A3.   The image of not an image *does not equal* the image of an image.

Plurality within the term 'image' can further be elaborated by suggesting that each image is unique. This can be formulated as an axiom by connecting the first part of principle P2 with itself as follows:

A4.   If the image of an image *equals* the image of an image then an image also *equals* an image (different images have different images).

Now multiply the term 'image' by introducing the following substitutions; the term 'a number' for the term 'an image', the term 'the immediate successor of' for the term 'the image of' and the term 'zero' for the term 'the image of not an image'. The two principles P1 and P2 and the two axioms A3 and A4 can be transformed by using these substitutions into the following four statements (the numbers correspond):

1.   Zero is a number.
2.   The immediate successor of a number is a number.
3.   Zero does not equal the immediate successor of a number.
4.   If the immediate successor of a number equals the immediate successor of a number then a number also equals a number (different numbers have different immediate successors).

Plurality has been imposed upon the term 'image' in situation S and it must be clear that at this point the analogy with the thought experiment collapses because it was assumed that "the internal factors are set in such a way that in situation S the image on the screen equals the image of this image on the screen". In order to maintain a model one should alter the thought experiment and assume, for example, that the image on the screen in situation S will with each new re-imaged image become a bit (unit) brighter. If the webcam now also becomes fixed in situation S (call this thought experiment II) then the only object that can be viewed by the webcam is the clear screen (if the screen is not clear then it must carry an image and is no longer experienced as an object). Principles P1 and P2 hold for thought experiment II and the term 'image' is in this case pluralised due to the fact that every re-imaged image is a bit brighter. Hence, axioms A3 and A4 also hold.

In the case of thought experiment II rational thinking would not impose plurality upon the concept 'not an image' because there is only one object; the clear screen. So, in this case 'the image of not an image' defines a unique image and connecting the first part of principle P1 with itself would give the following result:

B1.   The image of not an image *equals* the image of not an image.

Substituting twice the term 'zero' for the term 'the image of not an image' transforms axiom B1 into 'zero equals zero'. This axiom is a special case of a general statement about equality (for every natural number x, x=x).[3] [4]

In order to derive the principle (axiom) of mathematical induction it is necessary to *assume* that only principles P1 and P2 contribute to the concept 'all images'. This can be expressed by making use of the term 'the property of being an image' in the following way:

- The property of being an image only belongs to the image of not an image (that has not the property of being an image) and to the image of an image that has the property of being an image.

This statement can be rewritten by using an if-then construction and the term 'property' as follows:

A5. If a property belongs to the image of not an image (that has not the property) and also to the image of every image that has the property then this property belongs to all images.

The principle of mathematical induction can be derived from this axiom by using the same substitutions as before. This gives the following result:

5. If a property belongs to zero and also to the immediate successor of every number that has the property then this property belongs to all numbers.

4. Conclusion

The thought experiment of a fixed webcam in situation S with an increasing brightness of every re-imaged imaged (thought experiment II) is a model with only two principles (non-contradicting) and one undefined term from which the Peano axioms (1-5) can be derived. The simplicity of this model makes it possible to take a look behind the scenery of mathematical thinking and reveal a glimpse of its principles (P1 and P2).[5]

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[1] This paper does not discuss the possibility of the webcam viewing the screen under a certain angle. For example, the webcam viewing the screen under a 180 degree angle in situation S results into an image that continuously makes an upside down turnover (unless the image has top-bottom symmetry). This corresponds within rational thinking for example to the statement 'this sentence is false'.

[2] Compare this with one of Zeno's paradoxes: 'If everything that exists has a place, place too will have a place, and so on ad infinitum'. In order for rational thinking to remain rational it has to sustain the idea that 'the place of a place is a place'. Hence, rational thinking can only find the solution of this paradox by advocating the self-evident which leads to infinite regress, as pointed out by Zeno. The clear mind, however, would realise that although 'the place of not a place (a thing) is a place' it is also true that 'the place of a place is not a place' and this way cut through the paradox instead of fulfilling the term 'place' with an identity (self) which then can only be maintained by embracing the infinite.

[3] However, by using all three substitutions the statement 'the image of not an image equals the image of not an image' transforms into 'zero equals the immediate successor of not a number'. This statement (which differs from axiom 3) expresses the relationship between the three terms used in the substitutions and forms the key for transforming the Peano axioms back into the non-contradicting principles P1 and P2, that is, it forms the key for demonstrating the consistency of the Peano axioms. The statement can be considered a meta-mathematical statement which cannot be represented within the formalism of arithmetic and is unprovable from the Peano axioms. Although the statement appears as nonsense within mathematical thinking the clear mind is aware of the truth of this meta-mathematical statement because it realises that zero reflects (images) nothingness which is not a number.

[4] The four axioms that describe the equality relation (for every natural number x, x=x; for all natural numbers x and y, if x=y, then y=x; for all natural numbers x, y and z, if x=y and y=z, then x=z; for all A and B, if A is a natural number and A=B, then B is also a natural number) are in this paper considered axioms of pure logic and therefore not included within the Peano axioms.

[5] The set of principles can be extended by *assuming* that there is symmetry within rational thinking between A and not A, i.e., the term 'an image' and the term 'not an image' are interchangeable within the principles. Hence, two

new sub-principles arise; 'not the image of an image is not an image' derived from P1 and 'not the image of not an image is not an image' derived from P2. It can be checked that these two principles also hold for thought experiment II. Plurality can now be imposed upon the term 'not an image' resulting in that the screen of thought experiment II becomes embedded within another screen which is more sophisticated because it embodies the concept 'countably infinite'. This sophisticated screen is embedded within an even more sophisticated screen...