Energy content of Kerr black hole emitting non thermal radiation or scalar radiation

Manjunath. R. (Reader in physics)
#16, 8th Main road
Shivanagar, Rajajinagar
Bangalore-560010
Karnataka, India
manjunathr1988@yahoo.in

Abstract: The rate of loss of mass of Kerr blackhole in the form of non thermal hawking radiation or scalar radiation is termed as Kerr black hole evaporation. It is shown that the loss in mass of Kerr blackhole has adhverse effects on its basic characteristics .The mathematical equation for calculation of total energy content of Kerr blackhole emitting non thermal radiation or scalar radiation accounts for change in its angular momentum,change in its time period of rotation and its spin parameter . The above expressions was developed based on basic concepts of Kerr blackhole, rotational mechanics and blackhole thermodynamics. [Report and Opinion. 2010;2(3):36-43]. (ISSN: 1553-9873).

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Kerr black hole is an uncharged black hole that rotates about a central axis. It is named after the New Zealand mathematician Roy Kerr who, in 1963, became the first person to solve the field equations of Einstein's general theory of relativity for a situation of this kind. Kerr black holes are probably the commonest in nature, since the massive stars from which they typically form possess rotation (but no overall charge) before they collapse at the end of their lives. By the principle of conservation of angular momentum, much of this spin is then retained by the black hole following the star's terminal collpase.

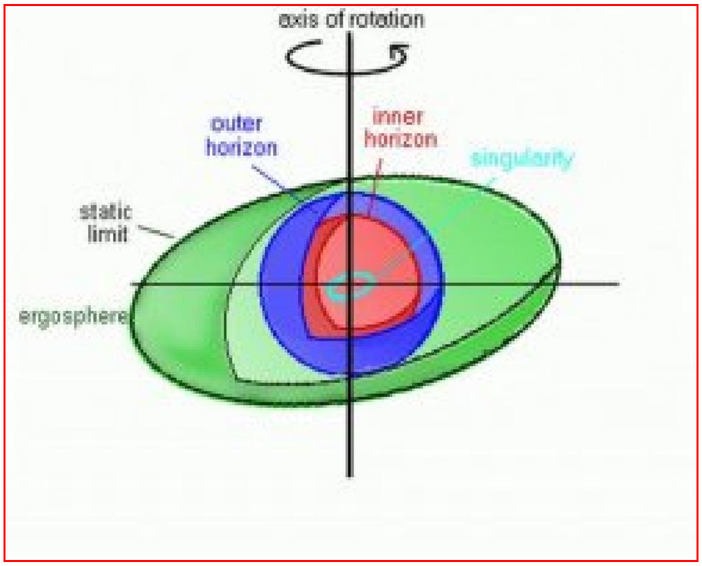

Figure-1: Kerr black hole

We study the evolution of an evaporating rotating black hole, described by the Kerr metric, which is emitting either solely massless scalar particles or a mixture of massless scalar and nonzero spin particles. Allowing the hole to radiate scalar particles increases the mass loss rate and decreases the angular momentum loss rate relative to a black hole which is radiating nonzero spin particles.

The presence of scalar radiation can cause the evaporating hole to asymptotically approach a state which is described by a nonzero value of a j a=M . This is contrary to the conventional view of black hole evaporation, wherein all black holes spin down more rapidly than they lose mass.Kerr black hole loses a small amount of its energy and therefore of its mass (mass and energy are related by Einstein's equation **$E = mc^2$**).Loss of mass of Kerr black hole is through the emission of non thermal hawking radiation or scalar radiation .

Figure-2: Rate of energy loss from Kerr black hole in the form of radiation

Power of the emitted radiation is the rate of evaporation energy loss of the Kerr black hole:

$$P = -C^2 dM/dt \quad \ldots\ldots\ldots..(1)$$

Here **dM** be small change in mass of Kerr black hole with respect to time **dt** , **-** ve sign indicates rate of energy loss of Kerr black hole with evaporation .

According to **First Law of Black hole thermodynamics**

We have

$$dM = k dA/8\pi + \Omega dJ + \Phi\, dQ \quad \ldots\ldots\ldots\ldots..(2)$$

where **M** is the mass, **A** is the horizon area, **Ω** is the angular velocity, **J** is the angular momentum, **Φ** is the electrostatic potential, **k** is the surface gravity and **Q** is the electric charge.

Kerr black hole is a black hole that possesses angular momentum but not charge

So let us take **dQ= 0**

Then the (2) becomes

dM= kdA/8π + ΩdJ …………………..(3)

From (1) we know that **P= - C^2 dM/dt**

P = -C^2 [kdA/dt8π + ΩdJ/dt]…………….......(4)

According to rotational mechanics

Torque is the tendency of a force to rotate an object about an axis, fulcrum, or pivot. Just as a force is a push or a pull, a torque can be thought of as a twist.

τ = dJ/dt

Here **dJ** is change in angular momentum of black hole with respect to time **dt**

Thus (4) becomes

P = -C^2 [kdA/dt8π + Ω τ]……………(5)

Rotational energy or angular kinetic energy is the kinetic energy due to the rotation of an object and is part of its total kinetic energy

Rotational kinetic energy of Kerr black hole is given by:

K=I/2I Ω^2

Here **Ω** is the angular velocity,**I** is the moment of inertia,**K** is the kinetic energyof kerr blackhole.

dK/dt =1/2IdΩ^2/dt =I Ω d Ω/dt …………..(7)

dK/dt = I Ω d Ω/dt

dK =I Ω d Ω…………………………….......(8)

Thus (5)becomes

P = -C^2 [kdA/dt8π + dK τ/I dΩ] ………..........(9)

Let us multiply by **dt**

we get

Pdt = -C^2 [kdA/8π + dK τdt/I dΩ] ……….............(10)

Angular acceleration can be given by

$\alpha = d\Omega/dt$

Thus (7)becomes

$Pdt = -C^2 [kdA/8\pi + dK\ \tau/I\ \alpha]$…………………………(11)

For rotational motion, Newton's second law can be adapted to describe the relation between torque and angular acceleration:

$\tau = I\ \alpha$

where **τ** is torque , **I** is the mass moment of inertia of the blackhole.

Thus (11) becomes

$Pdt = -C^2 [kdA/8\pi + dK]$……………………….............(12)

From (1)

$Pdt = -dE$

$-dE = -C^2 [kdA/8\pi + dK]$

$dM = [kdA/8\pi + dK]$………………………….................(13)

From (7) we know

$dK/dt = 1/2Id\ \Omega^2/dt = I\ \Omega\ d\ \Omega/dt$

But $\Omega = 2\ \pi/T$

T=Time period of rotation of blackhole

$dK/dt = 1/2Id\ \Omega^2/dt = I\ \Omega\ d\ \Omega/dt$

$dK/dt = -2\ \pi\ I\ \Omega\ dT/T^2 dt$

$dK = -2\ \pi\ I\ \Omega\ dT/T^2$……………...................................(14)

Thus(13) becomes

$dM = [kdA/8\pi - 2\ \pi\ I\ \Omega\ dT/T^2]$…………………………..(15)

$2\ \pi\ I\ \Omega\ dT/T^2 = [kdA/8\pi - dM]$……………………...........(16)

Angular momentum of Kerr black hole is given by

$J = I\ \Omega$

Thus (16) becomes

$\mathbf{2\,\pi\,J\,dT/T^2 = [kdA/8\pi - dM\;]}$……………………..........(17)

From(3) we know $\mathbf{dM = kdA/8\pi + \Omega dJ}$

Thus(17) becomes

$\mathbf{2\,\pi\,J\,dT/T^2 = -\;\Omega dJ}$ ....(18)

As we know

$\mathbf{\Omega = 2\,\pi/T}$

$\mathbf{\Omega\,J\,dT/T = -\;\Omega dJ}$...(19)

$\mathbf{J\; = -dJ\;T/dT}$..(20)

Here **dJ**=change in angular momentum, **dT**=change in Time period of rotation of blackhole.

Mass of Kerr black hole is the measure of its total energy content

$\mathbf{E=MC^2}$……………………………..........(21)

Spin parameter of this black hole is given by

$\mathbf{a=J/MC}$

Here **a=**Spin parameter,**J=**Angular momentum of kerr black hole

Thus (21) becomes

$\mathbf{E=JC/a}$

Thus(20)becomes

$\mathbf{E\; = -dJ\;TC/dTa}$..**(22)**

Here **dJ**=change in angular momentum, **dT**=change in Time period of rotation of blackhole,**T**= Time of rotation of blackhole,**a**= Spin parameter of this black hole,**C**=speed of light in vaccum ($3*10^8$m/s).

The Schwarzschild radius (sometimes historically referred to as the gravitational radius' **rg**') is a characteristic radius associated with every quantity of mass.The radius of a collapsing celestial object at which gravitational forces require an escape velocity that exceeds the velocity of light, resulting in a black holeis also termed as gravitational radius

By comparison of $\mathbf{rg = 2GM/C^\wedge 2}$ and $\mathbf{E=MC^2}$

We get

$$\mathbf{E= rg\, C^4 /2G}$$

Thus (22) becomes

$$\mathbf{rg = -[2GdJ\, T/dTaC^3]} \text{.....................................(23)}$$

Here **rg** = Gravitational radius of kerr blackhole,**G**=Universal gravitational constant**.**

A black hole in general is surrounded by a surface, called the event horizon situated at the Schwarzschild radius (for a nonrotating black hole), where the escape velocity is equal to the velocity of light. Within this surface, no observer/particle can maintain itself at a constant radius. It is forced to fall inwards, and so this is sometimes called the static limit.A rotating black hole has the same static limit at the Schwarzschild radius but there is an additional surface outside the Schwarzschild radius named the "ergosurface" given by $\mathbf{(R - GM)^2 = G^2M^2 - J^2\cos^2\theta}$ in Boyer-Lindquist coordinates, which can be intuitively characterized as the sphere where "the rotational velocity of the surrounding space" is dragged along with the velocity of light. Within this sphere the dragging is greater than the speed of light, and any observer/particle is forced to co-rotate.

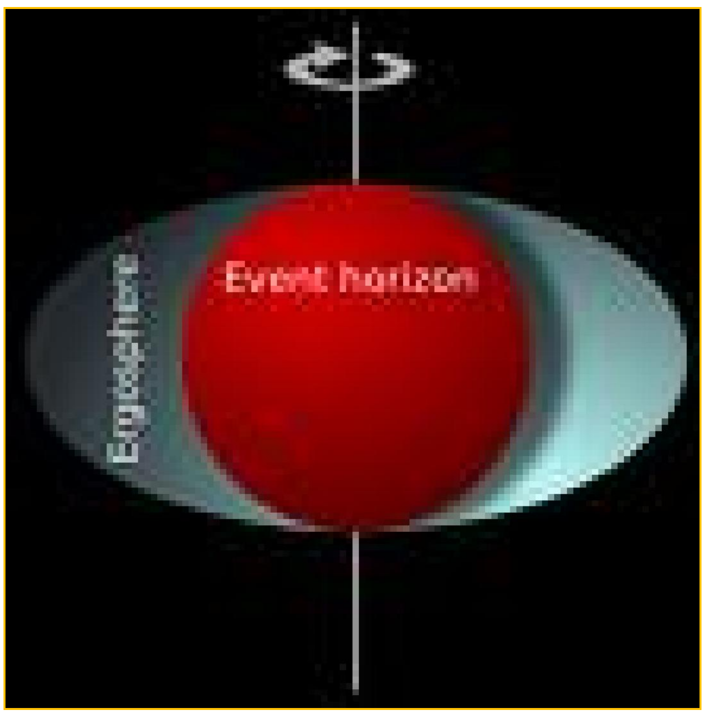

Figure-3: kerr black hole with ergosurface

$$\mathbf{(R- GM)^2 = G^2M^2 - J^2\cos^2\theta}$$

$$\mathbf{R^2+G^2 M^2-2GMR= G^2M^2 - J^2\cos^2\theta}$$

$$\mathbf{R^2-2GMR = - J^2\cos^2\theta}$$

Mutiplying the above equation by $\mathbf{C^2}$

we get

$$\mathbf{R^2\ C^2\text{-}2G(MC^2)R = -\ J^2\cos^2\theta\ C^2}$$ ....(24)

we know that $\mathbf{MC^2 = -dJ\ TC/dTa}$

By substitution

we get

$$\mathbf{(\ R^2 + J^2\cos^2\theta) = 2GdJ\ T/dTaC^2}$$ ....(25)

Here **R**,**θ** are the coordinates of standard spherical coordinate system.

Result:

1)The total energy content of Kerr blackhole emitting non thermal hawking radition or scalar radition is given by
E = -dJ TC/dTa
[dJ=change in angular momentum, **dT**=change in Time period of rotation of blackhole,**T**= Time periodof rotation of blackhole,**a**= Spin parameter of this black hole,**C** = speed of light in vaccum ($3*10^8$m/s)**]**.

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Correspondance to:

Manjunath. R.(Reader in physics)
#16,8th Main road,
Shivanagar, Rajajinagar,
Bangalore-560010
Karnataka, India
manjunathr1988@yahoo.in
manjunath5496@gmail.com

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Conclusion :The different concepts like d**J**,**J**,d**T**, **a** are brought togather in one frame of reference to explain the phenomenon of evaporation of Kerr blackhole .The new mathematical determination of total energy content of kerr black hole emitting radiation according to formula(**22**)which take into account change in angular momentum of black hole, change in time period of rotation of blackhole, time period of rotation of blackhole andSpin parameter of black hole.This theory predicts a new method for determination of total energy content of this black hole and its gravitational radius.The Boyer-Lindquist coordinates like **R**,**θ** are included in the paper to explain this phenomenon.The mathematical model also explains the negative effects of Kerr black hole evaporation on the properties of Kerr black hole like **J** and **M**.Loss in mass of Kerr black hole causes decrease in its angular momentum .The decrease in angular velocity is followed by the increase in its period of rotation .

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