Asymptotic Expansion method for solving variable viscous Flow of Maxwell Fluid

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ABSTRACT: The flow of visco-elastic fluid under steady pressure gradient in a region bounded by two parallel porous plates, it is assumed that at one plate fluid is injected with a certain constant velocity and sucked off with same velocity at the other; we also assumed a variable viscosity. The non linear dimensionless equation is then solved numerically by asymptotic expansion for fixed injection Reynolds number and suction parameter. The effects of visco-elastic K and the viscosity variation parameters on the velocity field are presented and discussed.
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INTODUCTION

The non Newtonian flow of blood through the artery is a source of major concern to all and sundry in the medical field because of the high mortality rate especially if it is within the coronary network. In an earlier work, (Singh, 1983), had applied the visco-elastic model to study blood flow in the artery by assuming a constant blood viscosity however for non-Newtonian blood flow we observe that changes in viscosity is highly significant. Flow of blood with variable viscosity had been studied by [2-5] under different flow conditions.

The objective of this paper is to study the steady non-Newtonian blood flow using the Maxwell viscoelastic model. But the modification we are proposing is that of variable viscosity by (Makinde, 2008)

The paper is organized in this form; in section 1 we give brief introduction and the statement of problem, in section 2 of the work, the problem is formulated and non-dimensionalized, in section 3, the problem is solved and numerical results are presented are discussed. While section 4 gives some concluding remarks.

MATHEMATICAL ANALYSIS

We consider the visco-elastic fluid model given by (Singh, 1983) whose constitutive equation is characterized by

$$\left(1+\lambda\frac{\delta}{\delta t}\right)\tau^{ij} = 2\mu e^{ik} \tag{2.1}$$

where

$$e_{ik} = \frac{1}{2}\left(v_{i,j} + v_{k,i}\right) \tag{2.2}$$

τ^{ij} -is the stress tensor, λ -is the relaxation, μ -is the dynamic viscosity, e^{ik} the rate of strain tensor.

For any contra-variant tensor b^{ik}

$$\frac{\delta\, b^{ik}}{\delta t} = \frac{\partial b^{ik}}{\partial t} + v^m\frac{\partial b^{ik}}{\partial x_m} - \frac{\partial v^k}{\partial x_m}b^{im} - \frac{\partial v^i}{\partial x_m}b^{mb} \tag{2.3}$$

The continuity equation for the incompressible unsteady flow of fluid of density ρ is

$$\left(\rho v^i\right)_{,i} = 0 \tag{2.4}$$

Let us assume that v is every where negative that is $v = -v_0$ (constant) and u=u(y,t) and then the momentum equation gives

$$\rho\left(\frac{\partial u}{\partial t}-v_0\frac{\partial u}{\partial y}\right)=-\frac{\partial P}{\partial x}+\frac{\partial \tau^{xy}}{\partial y} \tag{2.5}$$

Equation (1) gives,

$$\tau^{xy}=\frac{\mu\frac{\partial u}{\partial y}}{\left(1+\lambda\frac{\partial}{\partial t}-v_0\lambda\frac{\partial}{\partial y}\right)} \tag{2.6}$$

We assume a variable viscosity (Makinde, 2008)

$$\mu(y)=e^{\beta\left(1-y^2\right)} \tag{2.7}$$

Where

μ is the dynamic viscosity, u = velocity, ρ = density, p = pressure, x = co-ordinate in the direction of flow, y= coordinate across the flow, v_0 is the constant vertical velocity, τ^{xy} is the shear stress tensor

Substituting (6) in (5) we have

$$\rho\left(\frac{\partial u}{\partial t}-v_0\frac{\partial u}{\partial y}\right)=-\frac{\partial P}{\partial x}+\lambda\frac{\partial}{\partial t}\left(-\frac{\partial P}{\partial x}\right)+2\rho\lambda V_0\frac{\partial^2 u}{\partial t\partial y}-\rho\lambda\frac{\partial^2 u}{\partial t^2}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\rho\lambda V_0^2\frac{\partial^2 u}{\partial y^2}$$

Now introducing the following dimensionless parameters

$$u'=\frac{u}{v_0},\quad x'=\frac{x}{h},\quad y'=\frac{y}{h},\ t'=\frac{v_0 t}{h},\ p'=\frac{p}{\rho v_0^2},\mathrm{Re}=\frac{v_0 h}{\nu},\nu=\frac{\mu_0}{\rho},K=\frac{v_0^2\lambda}{\nu}$$

We obtain

$$\frac{\partial u}{\partial t}-\frac{\partial u}{\partial y}=-\frac{dP}{dx}+K\frac{\partial}{\partial t}\left(-\frac{dP}{dx}\right)+2K\frac{\partial^2 u}{\partial t\partial y}-K\frac{\partial^2 u}{\partial t^2}+\frac{\partial}{\partial y}\left(e^{\beta\left(1-y^2\right)}\frac{\partial u}{\partial y}\right)-K\frac{\partial^2 u}{\partial y^2} \tag{2.10}$$

Subject to initial and boundary conditions

$$u(0,y)=Sin(\pi y),\qquad \frac{\partial u}{\partial t}(0,y)=0$$
$$u(t,-1)=0=u(t,\ 1), \tag{2.11}$$

3.1 STEADY STATE SOLUTION

We assume that blood flow steadily therefore (2.10) reduces to

$$0=\lambda+\frac{du}{dy}+\frac{d}{dy}\left(e^{\beta\left(1-y^2\right)}\frac{du}{dy}\right)-K\frac{d^2u}{dy^2} \tag{3.1.1}$$

Subject to the boundary conditions

$$u(-1)=0=u(1) \tag{3.1.2}$$

Let $0<\beta<<1$ therefore by asymptotic expansion,

We take $u=u_0+\beta u_1+\beta^2 u_2$ neglecting other terms (3.1.3)

Taking the Taylor's expansion of $e^{\beta\left(1-y^2\right)}$ about β

$$e^{\beta\left(1-y^2\right)}=1+\beta\left(1-y^2\right)+0.5\beta^2\left(1-y^2\right)^2 \tag{3.1.4}$$

Equating coefficients

$$0=1+\frac{du_0}{dy}+(1-K)\frac{d^2u_0}{dy^2} \tag{3.1.5}$$

$$u_0(-1)=0=u_0(1) \tag{3.1.6}$$

$$0=\frac{du_1}{dy}+\frac{d}{dy}\left(\left(1-y^2\right)\frac{du_0}{dy}\right)+\left(1-K\right)\frac{d^2u_1}{dy^2} \tag{3.1.7}$$

$$u_1(-1)=0=u_1(1) \tag{3.1.8}$$

$$0=\frac{du_2}{dy}+\frac{d}{dy}\left(\left(1-y^2\right)\frac{du_1}{dy}\right)+\frac{d}{dy}\left(\left(1-y^2\right)\frac{du_1}{dy}\right)+\left(1-K\right)\frac{d^2u_2}{dy^2} \tag{3.1.9}$$

$$u_2(-1)=0=u_2(1) \tag{3.1.10}$$

Using mathematical version 6 we the solution of (3.3) is given as Appendix A, while the graphical results are figures 1 and 2

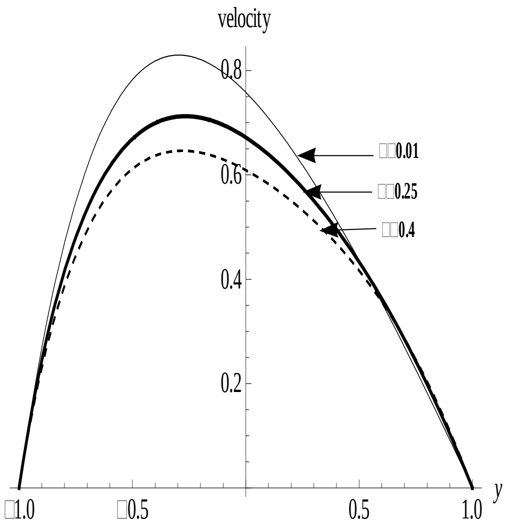

Figure 1

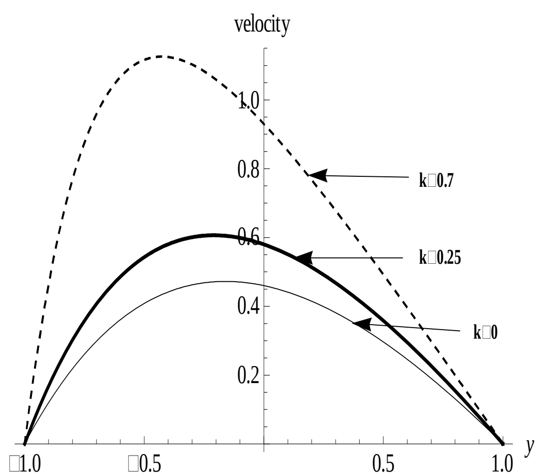

Figure 2

In figure 1, we observed that as the viscosity variation parameter increase there is reduction in the flow velocity.

While in Fig. 2 the effect of increase in the relaxation time parameter is to increase the flow velocity this result is in agreement with (Singh, 1983; Akhtar et al, 2008)

CONCLUDING REMARKS

We have studied the steady flow of Maxwell fluid at steady state from our result we observed that as relaxation time reduce the fluid shows Newtonian behaviour while velocity reduces with increase in viscosity. Possible application of this work is in the treatment and diagnosis of cardiovascular diseases and stenosis. The non-Newtonian flow using Maxwell model is still open for future research.

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