Gaps Among Products of *m* Primes

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Abstract: Using Jiang function we prove Gaps Among Products of *m* Primes.
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Theorem 1.

$$d(x)=d(x+1)=d(x+2)=2 \text{ infinitely-often.} \quad (1)$$

where $d(x)$ represents the number of distinct prime factors of x, $d(x)=\sum_{P|x}1, d(3)=1$, $d(15)=2$, $d(105)=3$.

Proof (see[1] p.146 theorem 3.1.154). Prime equations are

$$p_2=10p_1+1,\quad p_3=15p_1+2,\quad p_4=6p_1+1 \quad (2)$$

We have Jiang function

$$J_2(\omega)=3\prod_{7<P}(P-4)\neq 0, \quad (3)$$

where $\omega=\prod_{2\le P}P$

We prove that $J_2(\omega)\neq 0$ there exist infinitely many primes P_1 such that P_2, P_3, P_4 are primes.

We have asymptotic formula

$$\pi_4(N,2)=\left|\{P_1\le N:10P_1+1,15P_1+2,6P_1+1\}\right|\sim\frac{J_2(\omega)\omega}{\phi^4(\omega)}\frac{N}{\log^4 N}, \quad (4)$$

where $\phi(\omega)=\prod_{2\le P}(P-1)$.

From (2) we have $3p_2+1=30p_1+4=2p_3$, $3p_2+2=30p_1+5=5p_4$. We prove that there exist infinitely many triples of consecutive integers, each being the products of two distinct primes.

Theorem 2.

$$d(x)=d(x+1)=d(x+2)=m>1 \text{ infinitely-often} \quad (5)$$

Proof (see [1] p.148, theorem 3.1.158). Suppose that $u, u+1$ and $u+2$ are three consecutive integers, each being the products of $m-1$ distinct primes. Let $M=u(u+1)(u+2)$. We define the three prime equations

$$P_2=\frac{2M}{u}P_1+1,\quad P_3=\frac{2M}{u+1}P_1+1,\quad P_4=\frac{2M}{u+2}P_1+1 \quad (6)$$

Using Jiang function $J_2(\omega)$ we prove that there exist infinitely many primes P_1 such that P_2, P_3 and P_4 are primes.

From (6) we have

$uP_2 = 2MP_1 + u$, $uP_2 + 1 = 2MP_1 + u + 1 = (u+1)\left(\frac{2M}{u+1}P_1 + 1\right) = (u+1)P_3$

$$uP_2 + 2 = 2MP_1 + u + 2 = (u+2)\left(\frac{2M}{u+2}P_1 + 1\right) = (u+2)P_4$$

We prove

$d(x) = d(x+1) = d(x+2) = m > 1$ infinitely-often. (7)

Theorem 3.

$d(x) = d(x+2) = d(x+4) = 2$ infinitely-often (8)

Proof [1,2,3]. Prime equations are

$P_2 = 70P_1 + 1, \quad P_3 = 42P_1 + 1, \quad P_4 = 30P_1 + 1$ (9)

Using Jiang function $J_2(\omega)$ we prove that there exist infinitely many primes P_1 such that P_2, P_3 and P_4 are primes.

Frome (9) we have

$$3P_2 = 210P_1 + 3, \quad 3P_2 + 2 = 210P_1 + 5 = 5(42P_1 + 1) = 5P_3$$

$3P_2 + 4 = 210P_1 + 7 = 7(30P_1 + 1) = 7P_4$ (10)

We prove

$d(3P_2) = d(3P_2 + 2) = d(3P_2 + 4) = 2$ infinitely-often. (11)

Theorem 4.

$d(x) = d(x+2) = d(x+4) = m > 1$ infinitely-often. (12)

Proof [1, 2, 3]. Suppose that $u, u+2$ and $u+4$ are three odd integers, each being the products of $m-1$ distinct primes. Let $M = u(u+2)(u+4)$

We define three prime equations

$P_2 = \frac{2M}{u}P_1 + 1$, $P_3 = \frac{2M}{u+2}P_1 + 1$, $P_4 = \frac{2M}{u+4}P_1 + 1$ (13)

Using Jiang function $J_2(\omega)$ we prove that there exist infinitely many primes P_1 such that P_2, P_3 and P_4 are primes.

From (13) we have $uP_2 = 2MP_1 + u$,

$uP_2 + 2 = 2MP_1 + u + 2 = (u+2)\left(\frac{2M}{u+2}P_1 + 1\right) = (u+2)P_3$,

$uP_2 + 4 = MP_1 + u + 4 = (u+4)\left(\frac{2M}{u+4}P_1 + 1)\right) = (u+4)P_4$. (14)

We prove

$d(x) = d(x+2) = d(x+4) = m > 1$ infinitely-often. (15)

Theorem 5.

$d(x) = d(x+1) = d(x+2) = d(x+4) = m > 1$ infinitely-often. (16)

Proof. From (9) we have prime equations

$P_2 = 70P_1 + 1$, $P_3 = 105P_1 + 2$, $P_4 = 42P_1 + 1$, $P_5 = 30P_1 + 1$ (17)

Using Jiang function we prove there exist infinitely many primes P_1 such that P_2, P_3, P_4 and P_5 are primes.

From (17) we have

$$3P_2 = 210P_1 + 3$$
$$3P_2 + 1 = 210P_1 + 4 = 2(105P_1 + 2) = 2P_3$$
$$3P_2 + 2 = 210P_1 + 5 = 5(42P_1 + 1) = 5P_4$$
$$3P_2 + 4 = 210P_1 + 7 = 7(30P_1 + 1) = 7P_5 . \quad (18)$$

Using Jiang function we prove

$d(x) = d(x+1) = d(x+2) = d(x+4) = m > 1$ infinitely-often. (19)

$d(x) = d(x+1) = d(x+2) = d(x+4) = d(x+8) = d(x+10) = m > 1$ infinitely-often. (20)

Using Jiang function $J_2(\omega)$ we are able to prove

$d(x) = d(x+n) = m > 1$ infinitely-often. (21)

$d(x) = d(x+5-3) = d(x+7-3) = \cdots = d(x+P-3) = m > 1$ infinitely-often. (22)

Goldston *et. al* prove only $d(x) = d(x+n \le 6) = 2$ infinitely-often [4].

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Some descriptions by Chinese:

我们发现素数分布新的规律，这个问题比哥德巴赫猜想要难一万倍。这是人类最伟大数学发现。欧拉高斯没接触这个问题,20 世纪最伟大数学家埃尔德什开始关注这个问题，但也没得出有用结果。最近国际顶尖数学家 Goldston 等正在研究这个问题。得到国际数学界广泛的支持和关注，但文章都发表在著名杂志上，但没有得出任何实质性进展，蒋春暄在 2002 年[1]就彻底证明了它，但国内外数学家都读了它，都不说话，看到文献[4]后，我们决定写本文，如不用 Jiang 函数，再过两百年也不一定能证明它，国内更无人研究它，这才是研究方向！2009 年 1 月 10 日蒋春暄为休息去参加宋正海讲座在公共汽车上发现公式(22),1 月 11 日发现定理五。这样算一篇完整论文。

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