Complex Hyperbolic Functions(I) And Jiang-Santilli Theorem

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Abstract: This means: $x^n + y^n = z^n (n > 2)$ has finite integer solutions. In this paper using the complex hyperbolic functions we prove Jiang-Santill theorem(JST) for exponents $3P$ and P, where P is an odd prime.
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Using Euler proof n=3 of Fermat last theorem and Fermat proof =4 of Fermat last theorem Jiang proved Fermat last theorem.Euler proof n=3 of FLT and Fermat n=4 of FLT are wrong,Hence Jiang is not prove FLT.Using Jiang mathematics si=0,where i=3,...,n， n-2 indeterminate equations.One find n-2 indeterminate eqations with n-1 variables to have the

finite rational solutions,one find the finite rational solutions of FLT.

Example n=3 have two variables t1 and t2,(1) s1=/=0,s2=/=0,s3=0 have Fermat equation;(2) s2=/=0,s3=/=0,s1=0 have Fermat equation,(3)s1=/=0,s3=/=0.s2=0 have Fermat equation.One study three indeterminate equations s1=0,s2=0,s3=0 to find finite rational solutions t1 and t2 to prove that Fermat last theorem have finite rational solutions .

（3987）^12+（4365）^12=(4472)^12

Is JST solution

In 1974 Jiang found out Euler formula of the cyclotomic real numbers in the cyclotomic fields

$$\exp\left(\sum_{i=1}^{n-1} t_i J^i\right) = \sum_{i=1}^{n} S_i J^{i-1} \quad (1)$$

where J denotes a n th root of unity, $J^n = 1$, n is an odd number, t_i are the real numbers.

S_i is called th complex hyperbolic functions of order n with $n-1$ variables

$$S_i = \frac{1}{n}\left[e^A + 2\sum_{j=1}^{\frac{n-1}{2}} (-1)^{(i-1)j} e^{B_j} \cos\left(\theta_j + (-1)^j \frac{(i-1)j\pi}{n} \right)\right] \quad (2)$$

where i=1,2,…,n;

$$A = \sum_{\alpha-1}^{n-1} t_\alpha, \qquad B_j = \sum_{\alpha=1}^{n-1} t_\alpha (-1)^{\alpha j} \cos\frac{\alpha j\pi}{n},$$

$$\theta_j = (-1)^{j+1} \sum_{\alpha=1}^{n-1} t_\alpha (-1)^{\alpha j} \sin\frac{\alpha j\pi}{n}, \qquad A + 2\sum_{j=1}^{\frac{n-1}{2}} B_j = 0 \quad (3)$$

(2) may be written in the matrix form

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ \cdots \\ S_n \end{bmatrix} = \frac{1}{n}\begin{bmatrix} 1 & 1 & 0 & \cdots & 0 \\ 1 & -\cos\frac{\pi}{n} & -\sin\frac{\pi}{n} & \cdots & -\sin\frac{(n-1)\pi}{2n} \\ 1 & \cos\frac{2\pi}{n} & \sin\frac{2\pi}{n} & \cdots & -\sin\frac{(n-1)\pi}{n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & \cos\frac{(n-1)\pi}{n} & \sin\frac{(n-1)\pi}{n} & \cdots & -\sin\frac{(n-1)^2\pi}{2n} \end{bmatrix} \begin{bmatrix} e^A \\ 2e^{B_1}\cos\theta_1 \\ 2e^{B_1}\sin\theta_1 \\ \cdots \\ 2\exp B_{\frac{n-1}{2}} \sin\theta_{\frac{n-1}{2}} \end{bmatrix} \quad (4)$$

where $(n-1)/2$ is an even number.

From (4) we have its inverse transformation

$$\begin{bmatrix} e^A \\ e^{B_1}\cos\theta_1 \\ e^{B_1}\sin\theta_1 \\ \cdots \\ \exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & -\cos\frac{\pi}{n} & \cos\frac{2\pi}{n} & \cdots & \cos\frac{(n-1)\pi}{n} \\ 0 & -\sin\frac{\pi}{n} & \sin\frac{2\pi}{n} & \cdots & \sin\frac{(n-1)\pi}{n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & -\sin\frac{(n-1)\pi}{2n} & -\sin\frac{(n-1)\pi}{n} & \cdots & -\sin\frac{(n-1)^2\pi}{2n} \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ \cdots \\ S_n \end{bmatrix} \quad (5)$$

From (5) we have

$$e^A = \sum_{i=1}^{n} S_i, \qquad e^{B_j}\cos\theta_j = S_1 + \sum_{i=1}^{n-1} S_{1+i}(-1)^{ij}\cos\frac{ij\pi}{n}$$

$$e^{B_j}\sin\theta_j=(-1)^{j+1}\sum_{i=1}^{n-1}S_{1+i}(-1)^{ij}\sin\frac{ij\pi}{n},\qquad (6)$$

In (3) and (6) t_i and S_i have the same formulas. (4) and (5) are the most critical formulas of proofs for JST. Substituting (4) into (5) we prove (5).

$$\begin{bmatrix} e^A \\ e^{B_1}\cos\theta_1 \\ e^{B_1}\sin\theta_1 \\ \cdots \\ \exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix}=\frac{1}{n}\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & -\cos\frac{\pi}{n} & \cos\frac{2\pi}{n} & \cdots & \cos\frac{(n-1)\pi}{n} \\ 0 & -\sin\frac{\pi}{n} & \sin\frac{2\pi}{n} & \cdots & \sin\frac{(n-1)\pi}{n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & -\sin\frac{(n-1)\pi}{2n} & -\sin\frac{(n-1)\pi}{n} & \cdots & -\sin\frac{(n-1)^2\pi}{2n} \end{bmatrix}\times$$

$$\begin{bmatrix} 1 & 1 & 0 & \cdots & 0 \\ 1 & -\cos\frac{\pi}{n} & -\sin\frac{\pi}{n} & \cdots & -\sin\frac{(n-1)\pi}{2n} \\ 1 & \cos\frac{2\pi}{n} & \sin\frac{2\pi}{n} & \cdots & -\sin\frac{(n-1)\pi}{n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & \cos\frac{(n-1)\pi}{n} & \sin\frac{(n-1)\pi}{n} & \cdots & -\sin\frac{(n-1)^2\pi}{2n} \end{bmatrix}\begin{bmatrix} e^A \\ 2e^{B_1}\cos\theta_1 \\ 2e^{B_1}\sin\theta_1 \\ \cdots \\ 2\exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix}$$

$$=\frac{1}{n}\begin{bmatrix} n & 0 & 0 & \cdots & 0 \\ 0 & \frac{n}{2} & 0 & \cdots & 0 \\ 0 & 0 & \frac{n}{2} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & \frac{n}{2} \end{bmatrix}\begin{bmatrix} e^A \\ 2e^{B_1}\cos\theta_1 \\ 2e^{B_1}\sin\theta_1 \\ \cdots \\ 2\exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix}$$

$$=\begin{bmatrix} e^A \\ e^{B_1}\cos\theta_1 \\ e^{B_1}\sin\theta_1 \\ \cdots \\ \exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix},\qquad (7)$$

where $1+\sum_{j=1}^{n-1}(\cos\frac{j\pi}{n})^2=\frac{n}{2}$, $\sum_{j=1}^{n-1}(\sin\frac{j\pi}{n})^2=\frac{n}{2}$.

From (3) we have

$$\exp(A+2\sum_{j=1}^{\frac{n-1}{2}}B_j)=1. \quad (8)$$

From (6) we have

$$\exp(A+2\sum_{j=1}^{\frac{n-1}{2}}B_j)=\begin{vmatrix} S_1 & S_n & \cdots & S_2 \\ S_2 & S_1 & \cdots & S_3 \\ \cdots & \cdots & \cdots & \cdots \\ S_n & S_{n-1} & \cdots & S_1 \end{vmatrix}=\begin{vmatrix} S_1 & (S_1)_1 & \cdots & (S_1)_{n-1} \\ S_2 & (S_2)_1 & \cdots & (S_2)_{n-1} \\ \cdots & \cdots & \cdots & \cdots \\ S_n & (S_n)_1 & \cdots & (S_n)_{n-1} \end{vmatrix},$$

(9)

where $(S_i)_j=\frac{\partial S_i}{\partial t_j}$.

From (8) and (9) we have the circulant determinant

$$\exp(A+2\sum_{j=1}^{\frac{n-1}{2}}B_j)=\begin{vmatrix} S_1 & S_n & \cdots & S_2 \\ S_2 & S_1 & \cdots & S_3 \\ \cdots & \cdots & \cdots & \vdots \\ S_n & S_{n-1} & \cdots & S_1 \end{vmatrix}=1 \quad (10)$$

If $S_i\neq 0$, where $i=1,2,\cdots,n$, then (10) has infinitely many rational solutions.

Assume $S_1\neq 0$, $S_2\neq 0$, $S_i=0$ where $i=3,4,\cdots,n$. $S_i=0$ are $n-2$ indeterminate equations with $n-1$ variables. One can prove n-2 indeterminat equations to have finitely many rational solutions to prove finitely many rational solutions of JST

From (6) we have

$$e^A=S_1+S_2,\ e^{2B_j}=S_1^2+S_2^2+2S_1S_2(-1)^j\cos\frac{j\pi}{n}. \quad (11)$$

From (10) and (11) we have the JST equation

$$\exp(A+2\sum_{j=1}^{\frac{n-1}{2}}B_j)=(S_1+S_2)\prod_{j=1}^{\frac{n-1}{2}}(S_1^2+S_2^2+2S_1S_2(-1)^j\cos\frac{j\pi}{n})=S_1^n+S_2^n=1 \quad (12)$$

Example[1]. Let $n=15$. From (3) we have

$A=(t_1+t_{14})+(t_2+t_{13})+(t_3+t_{12})+(t_4+t_{11})+(t_5+t_{10})+(t_6+t_9)+(t_7+t_8)$

$$B_1 = -(t_1 + t_{14})\cos\frac{\pi}{15} + (t_2 + t_{13})\cos\frac{2\pi}{15} - (t_3 + t_{12})\cos\frac{3\pi}{15} + (t_4 + t_{11})\cos\frac{4\pi}{15}$$
$$-(t_5 + t_{10})\cos\frac{5\pi}{15} + (t_6 + t_9)\cos\frac{6\pi}{15} - (t_7 + t_8)\cos\frac{7\pi}{15},$$
$$B_2 = (t_1 + t_{14})\cos\frac{2\pi}{15} + (t_2 + t_{13})\cos\frac{4\pi}{15} + (t_3 + t_{12})\cos\frac{6\pi}{15} + (t_4 + t_{11})\cos\frac{8\pi}{15}$$
$$+(t_5 + t_{10})\cos\frac{10\pi}{15} + (t_6 + t_9)\cos\frac{12\pi}{15} + (t_7 + t_8)\cos\frac{14\pi}{15},$$
$$B_3 = -(t_1 + t_{14})\cos\frac{3\pi}{15} + (t_2 + t_{13})\cos\frac{6\pi}{15} - (t_3 + t_{12})\cos\frac{9\pi}{15} + (t_4 + t_{11})\cos\frac{12\pi}{15}$$
$$-(t_5 + t_{10})\cos\frac{15\pi}{15} + (t_6 + t_9)\cos\frac{18\pi}{15} - (t_7 + t_8)\cos\frac{21\pi}{15},$$
$$B_4 = (t_1 + t_{14})\cos\frac{4\pi}{15} + (t_2 + t_{13})\cos\frac{8\pi}{15} + (t_3 + t_{12})\cos\frac{12\pi}{15} + (t_4 + t_{11})\cos\frac{16\pi}{15}$$
$$+(t_5 + t_{10})\cos\frac{20\pi}{15} + (t_6 + t_9)\cos\frac{24\pi}{15} + (t_7 + t_8)\cos\frac{28\pi}{15},$$
$$B_5 = -(t_1 + t_{14})\cos\frac{5\pi}{15} + (t_2 + t_{13})\cos\frac{10\pi}{15} - (t_3 + t_{12})\cos\frac{15\pi}{15} + (t_4 + t_{11})\cos\frac{20\pi}{15}$$
$$-(t_5 + t_{10})\cos\frac{25\pi}{15} + (t_6 + t_9)\cos\frac{30\pi}{15} - (t_7 + t_8)\cos\frac{35\pi}{15},$$
$$B_6 = (t_1 + t_{14})\cos\frac{6\pi}{15} + (t_2 + t_{13})\cos\frac{12\pi}{15} + (t_3 + t_{12})\cos\frac{18\pi}{15} + (t_4 + t_{11})\cos\frac{24\pi}{15}$$
$$+(t_5 + t_{10})\cos\frac{30\pi}{15} + (t_6 + t_9)\cos\frac{36\pi}{15} + (t_7 + t_8)\cos\frac{42\pi}{15},$$
$$B_7 = -(t_1 + t_{14})\cos\frac{7\pi}{15} + (t_2 + t_{13})\cos\frac{14\pi}{15} - (t_3 + t_{12})\cos\frac{21\pi}{15} + (t_4 + t_{11})\cos\frac{28\pi}{15}$$
$$-(t_5 + t_{10})\cos\frac{35\pi}{15} + (t_6 + t_9)\cos\frac{42\pi}{15} - (t_7 + t_8)\cos\frac{49\pi}{15},$$

$$A + 2\sum_{j=1}^{7} B_j = 0, \qquad A + 2B_3 + 2B_6 = 5(t_5 + t_{10}). \tag{13}$$

Form (12) we have the JST equation

$$\exp(A + 2\sum_{j=1}^{7} B_j) = S_1^{15} + S_2^{15} = (S_1^5)^3 + (S_2^5)^3 = 1. \tag{14}$$

From (13) we have

$$\exp(A + 2B_3 + 2B_6) = [\exp(t_5 + t_{10})]^5. \tag{15}$$

From (11) we have

$$\exp(A + 2B_3 + 2B_6) = S_1^5 + S_2^5. \tag{16}$$

From (15) and (16) we have the JST equation

$$\exp(A+2B_3+2B_6)=S_1^5+S_2^5=[\exp(t_5+t_{10})]^5. \quad (17)$$

If (14) has a rational solutions for exponent 3. Therefore we prove that (17) has a rational solutions for exponent 5.

Theorem 1. Let $n=3P$, where $P>3$ is odd prime. From (12) we have the JST equation

$$\exp(A+2\sum_{j=1}^{\frac{3P-1}{2}}B_j)=S_1^{3P}+S_2^{3P}=(S_1^P)^3+(S_2^P)^3=1. \quad (18)$$

From (3) we have

$$\exp(A+2\sum_{j=1}^{\frac{P-1}{2}}B_{3j})=[\exp(t_P+t_{2P})]^P. \quad (19)$$

From (11) we have

$$\exp(A+2\sum_{j=1}^{\frac{P-1}{2}}B_{3j})=S_1^P+S_2^P. \quad (20)$$

From (19) and (20) we have the JST equation

$$\exp(A+2\sum_{j=1}^{\frac{P-1}{2}}B_{3j})=S_1^P+S_2^P=[\exp(t_P+t_{2P})]^P. \quad (21)$$

If (18) has a rational solutions for exponent 3. Therefore we prove that (21) has a rational solutions for $P>3$.

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