Controlling Chaos Using Recursive Backstepping Technique And Synchronization Via Reduced-Order Method In A Tunnel Diode Chaotic Oscillator Circuit.

Ogabi C. O[1]., Madufor M. A[2]., Olusa T. V[3].

1,2,3Department of Physics, Lagos State University, Ojo, PMB 1087, Apapa, Lagos, Nigeria.
1Email address: kunleogabi@yahoo.com or cornelius.ogabi@lasu.edu.ng

ABSTRACT: This research work investigates the control and synchronization in a fourth-order chaotic system derived from a tunnel diode oscillatory circuit. The chaotic behavior was controlled using a recursive backstepping design, based on the Lyapunov stability theory. Reduced-order synchronization between the fourth-order chaotic system and a second-order Duffing oscillator, deduced from a tunnel diode, and a non-linear resistor circuit was achieved using Lyapunov stability theory. Numerical simulations are implemented to show the effectiveness of the proposed method. The proposed approach ensured that global stability and exponential synchronization between the master and the slave systems can be achieved when the systems are of different order.
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1.0 INTRODUCTION

Chaotic systems are unpredictable systems, that are very sensitive to initial conditions, and a small change in initial conditions can bring about a great change in its output [33].

Recently, much attention has been attracted to the control of chaotic motions of electric circuits, particularly to synchronization of chaotic generators [27].

An electrical circuit element that is of interest is the tunnel diode. It has a non-linear negative resistance region in its current-voltage curve. The tunnel diode was introduced by Leo Esaki in 1958. It differs from an ordinary or "normal" diode in that the doping concentration in a p-n semiconductor junction is sufficiently large that suitable forward biasing causes the electrons to quantum mechanically tunnel through the junction barrier rather than jumping over it [7].

Although, tunnel diodes are capable of acting as very fast switching devices, it suffers from the problem of being susceptible to unwanted signals from stray capacitances and inductances contained in the wires and contact points.

This work involves the developing and analyzing various mathematical systems generated from a tunnel diode oscillator circuit that exhibits chaos, its control via recursive backstepping and reduced-order synchronization, between the generated fourth-order chaotic system and a second-order Duffing system.

Since 1990, the study of chaos control and synchronization have become the two most common and leading applications of chaos control theory [28, 29]. These two areas in the study of nonlinear systems is traceable to the pioneering classical chaos theory of Ott, Grebogi and Yorke [28], generally known as (OGY), and the seminal work of Pecora and Carrol [29]. They simultaneously reported in the same year, 1990. The OGY scheme [28] and the Pecora and Carroll scheme [29] introduced have been used to achieve chaos control. Subsequently, other methods have been proposed to control chaos.

These methods include: adaptive control [2, 8], backstepping design [3, 19, 39], and sliding mode control [16, 18], feedback controls [34, 35], to name a few. Chaos synchronization has been successfully carried out with identical systems, and non-identical systems using various algorithms.

The chaos theory approach has made it possible for extensive studies of chaos synchronization using various linear and nonlinear controls. The nonlinear controls include; the backstepping design [37] and active control [1] methods which have attracted the interest of researchers in this field due to the efficiency of the methods. These methods have been employed to synchronize both identical and non-identical systems.

The process of synchronization involves two identical or non-identical systems being coupled in such a way that the solution of one always converge to the solution of the other, independently of their initial conditions. This phenomenon can be termed as Master-Slave in order to differentiate it from the other phenomena, such as the Phase locking of population of coupled oscillators.

The Master-Slave Synchronization has been extensively applied in the control of chaos and chaotic signal masking.

2.0 THEORETICAL BACKGROUND

Chaos control in nonlinear dynamical systems may be achieved by various types of synchronization schemes, such as Adaptive control [9], Active control [13], Sliding mode control [17], Backstepping [38], Active-backstepping design [42], etc.

The backsteppiing method is a technique for stabilizing control of a special class of nonlinear dynamical systems that have been developed in 1990 [3]. Lyapunov technique has been proven to be the most efficient for investigating stability of equilibrium point [24].

Asymptotic stability can also be used to show boundedness of the solutions, even if the system has no equilibrium point. A combination of backstepping method and Lyapunov technique yield a flexible control [24]. The idea of synchronizing two identical chaotic systems that starts from different initial conditions consist of linking the trajectory of one system to the same values in the other system so that they remain in step with each other, through transmission of a signal [15]

The occurrence of chaos in non-autonomous systems can be in the two-dimensional models, such as in Lorenz [22] and Rössler [32] systems that have been widely studied. Electronic circuits that consist of two nonlinear element can used to verify theoretical predictions. As an example in nonlinear Duffing forced oscillation [12] and the nonlinear Chua's circuit, built and experimentally examined [23].

The chaotic dynamic system can be observed in many nonlinear circuits and mechanical systems, which has a significant research topic in physics, mathematics, and engineering communities, [24].

Chaos control and reduced-order synchronization in a third-order chaotic system derived from the rigid body dynamics has been carefully studied and successfully implemented [20]. A recursive backstepping control was designed based on Lyapunov stability theory to eliminate the chaotic behaviour. Reduced-order synchronization was achieved between the new system and a second-order Duffing oscillator, using a generalized active-backstepping approach based on Lyapunov stability theory.

Reduced-order synchronization of certain chaotic systems, based upon the parameters modulation and the adaptive control techniques was studied and successfully applied to two examples: generalized Lorenz system (fourth order) and Lu system (third order); Rossler hyperchaotic system (fourth order) and Rossler system (third order) [26]. The response system known as the drive system was controlled, even though their orders are different and their parameters are unknown.

The chaotic synchronization of third-order systems and second-order driven oscillator was studied [30]. Such a problem is related to synchronization of strictly different chaotic systems. This shows that dynamical evolution of second-order driven oscillators can be synchronized with the canonical projection of a third-order chaotic system. In this sense, it is said that synchronization is achieved in reduced order. Duffing equation is chosen as slave system whereas Chua oscillator is defined as master system. The synchronization scheme has nonlinear feedback structure. The reduced-order synchronization was attained in a practical sense.

2.1 Chaos control based on recursive Backstepping approach

A chaotic system in "strict-feedback" form as shown below:

$$\left.\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2), \\ \dot{x}_2 &= f_2(x_1, x_2, x_3), \\ &\vdots \\ \dot{x}_n &= f_n(x_1, x_2, ..., x_n) + f_{n+1}(t) \end{aligned}\right\} \quad (1)$$

where $x = [x_1, x_2, ..., x_n]^T \in \mathbb{R}^n$ are the state variables of the system f_1 is a linear function, $f_i (i = 2, 3, ..., n+1)$ are nonlinear functions and $f_{n+1}(t)$ is a periodic function of time.

To control the chaotic system in the form (1) using recursive backstepping control, (1) is expressed as follows:

$$\left.\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2) + u_1, \\ \dot{x}_2 &= f_2(x_1, x_2, x_3) + u_2, \\ &\vdots \\ \dot{x}_n &= f_n(x_1, x_2, ..., x_n) + f_{n+1}(t) + u_n, \end{aligned}\right\} \quad (2)$$

Where $u_i(t)$ $(i = 2, 3, ..., n)$ are control functions. Now, consider a known, bounded and smooth reference model given as:

$$\dot{x}_{ri} = f_{ri}(x_r, t), \; 1 \le i \le m, n \le m$$

Where $x_r = [x_{r1}, x_{r2}, ..., x_{rm}]^T \in R^m$ are the state variables; $f_{ri}(:)$, (i = 1, 2, ..., m) are known smooth nonlinear function with their j^{th} derivatives uniformly bounded in t.

The objective is to design recursive backstepping controllers for system (1) that guarantees globally stability and forces the output x(t) of system (1) to asymptotically track the output $x_r(t)$ of the reference model, in order that; $|x - x_r| \to 0, as \quad t \to \infty$

2.2 **Reduced-order Synchronization**

In a case where the master and slave systems have known and same parameters as well as same order, it is convenient and easy to achieve synchronization using the active-backstepping design approach as proposed in [40]. Synchronization can still be possible when the systems to be synchronized are different in structure and order, as shown below.

Consider a driver chaotic system

Systems (15) and (16), three different cases of synchronization problem are possible.

$$\dot{x} = Ax + f(x) \qquad (3)$$

Where $x = (x_1, x_2, \ldots, x_n)^T \in R^n$, $A \in R^n \times R^n$, A is a constant system matrix and f(x) is nonlinear function which is continuous and differentiable. The slave system is given by

$$\dot{y} = By + f(y) + u(t) \qquad (4)$$

Where $y = (y_1, y_2, \ldots, y_m)^T \in R^m$, $B \in R^m \times R^m$, $u(t) = (u_1(t), u_2(t), \ldots, u_m(t))^T \in R^m$, $(m < n)$, and u(t) is the control inputs. f(y) satisfies the conditions of f(x) in equation (3). For the master-slave system (3) and (4) three different cases of synchronization problems are possible.

Case (1): If A = B, f(x) = f(y) and n = m, then system (3) and (4) are identical provided u(t) = 0; and the problem is identical synchronization as presented by [10]. Identical synchronization has been adequately investigated by various approaches.

Case (2): If $A \neq B$, and/or $f(x) \neq f(y)$ $(n = m)$, the problem is that of non-identical systems of the same order.

Case (3): If A more general and challenging case is when case (2) holds and $n \neq m$ $(n > m)$ studied in [20, 26, 30]. In these references the two systems are strictly different both in structure and order.

Definition: If there exist appropriate controller u(t) satisfying for all x, y, $e \in R^m$, $\lim_{t\to\infty} \| e \| = \lim_{t\to\infty} \| x - y \| = 0$, then the master-slave will synchronize.

Considering the case (3), the error dynamics between the two systems (3) and (4) is

$$\dot{e} = \dot{x} - \dot{y} = Ce + f(x) - f(y) + u(t), \qquad (5)$$

Where C = A – B is the matrix of the linear part of the error dynamics parameter and e = x – y. By choosing an appropriate Lyapunov function for the system (6), for instance

$$V = \frac{1}{2} \sum e^2; \qquad (6)$$

Its first derivative along the error dynamics being

$$\dot{V} = e[Ce + f(x) - f(y) + u(t)] \qquad (7)$$

Can be made negative definite.

Proposition: If u(t) is chosen such that u(t) = –e – Ce – f(x) + f(y), the $\dot{V} < 0$, is negative definite and the error states asymptotically converge to zero, i.e., the master system (3) and the slave system (4) asymptotically synchronized.

Since the controller u(t) does not change the equilibrium (0, 0, 0) of the error system (5), then the proposed controller achieves the required synchronization.

3.0 MODELS AND MATHEMATICAL FORMULATION

3.1 Model of the tunnel diode chaotic circuit (Master system)

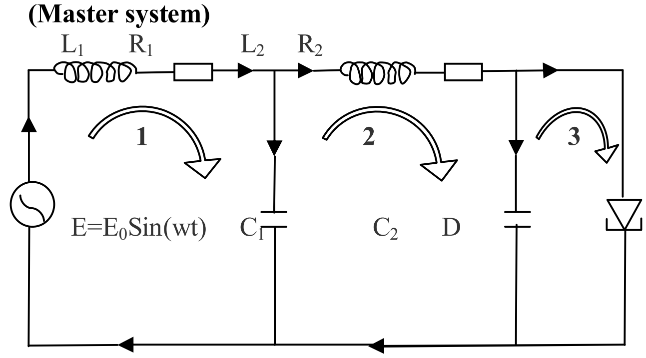


Figure 1

The nonlinear oscillator circuit is shown above (Figure 1). It consist of two linear resistors R_1 and R_2, two inductors L_1 and L_2, two capacitors C_1 and C_2, and a tunnel diode connected as shown in the figure with a sinusoidal voltage source.

3.2 **Electric circuit model of the Slave system**

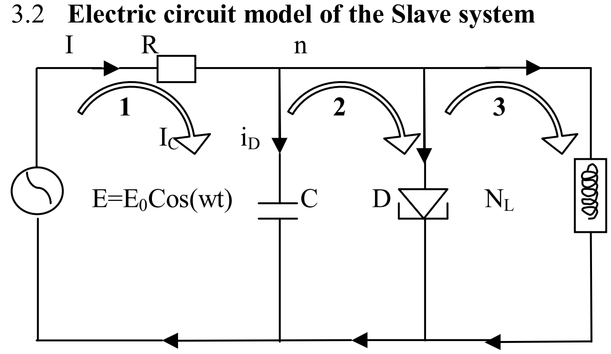


Figure 2

The circuit model of the slave system is shown above (Figure 2). It consist of a linear resistor R, capacitor C, a tunnel diode D, a nonlinear resistor N_L, and a sinusoidal voltage source E.

3.2 Mathematical formulation of the master system

Considering figure (1) and using Kirchoff's Voltage Law (KVL), From Loop 1:

$$E = L_1 \frac{d}{dt} I_{L1} + I_{L1} R_1 + V_{C1} \qquad (8)$$

$\frac{d}{dt} I_{L1} = \frac{1}{L1}(E_0 Sin(wt) - I_{L1}R_1 - V_{C1})$ (9)

From Loop 2:

$L_2\frac{d}{dt} I_{L2} = V_{C1} - V_{C2} - I_{L2}R_2$ (10)

Using Kirchoff's Current Law (KCL),

$C_2\frac{d}{dt} V_{C2} = I_{L2} - I_D$ (11)

Where $I_D = -aV_{C2} + bV_{C2}^3$

$C_2\frac{d}{dt} V_{C2} = I_{L2} + aV_{C2} + bV_{C2}^3$ (12)

and

$C_1\frac{d}{dt} V_{C1} = I_{L1} - I_{L2}$ (13)

re-writing (10), (11), (12), and (13), we have

$\frac{d}{dt} I_{L1} = \frac{1}{L1}(E_0 Sin(wt) - I_{L1}R_1 - V_{C1})$ (14)

$\frac{d}{dt} I_{L2} = \frac{1}{L2}(V_{C1} - V_{C2} - I_{L2}R_2)$ (15)

$\frac{d}{dt} V_{C1} = \frac{1}{C1}(I_{L1} - I_{L2})$ (16)

$\frac{d}{dt} V_{C2} = \frac{1}{C2}(I_{L2} + aV_{C2} + bV_{C2}^3)$ (17)

Let $I_{L1} = x_1$, $I_{L2} = x_2$, $V_{C1} = x_3$, $V_{C2} = x_4$, and setting $E_0\frac{1}{L2} = a_1$, $R_1\frac{1}{L1} = a_2$, $\frac{1}{L1} = a_3$, $\frac{1}{L2} = b_1$, $R_2\frac{1}{L2} = b_2$, $\frac{1}{C1} = k_1$, $\frac{1}{C2} = k_2$, $a\frac{1}{C2} = k_3$, and $b\frac{1}{C2} = k_4$

Hence, the chaotic system that figure 1 represents is shown below (18)

$$\begin{aligned}
\dot{x}_1 &= a_1 Sin(wt) - a_2x_1 - a_3x_3\\
\dot{x}_2 &= b_1x_3 - b_1x_4 - b_2x_2\\
\dot{x}_3 &= k_1(x_1 - x_2)\\
\dot{x}_4 &= k_2x_2 + k_3x_4 - k_4x_4^3
\end{aligned} \quad (18)$$

3.3 Mathematical formulation of the slave system

Considering figure 2, the current passing n, through the nonlinear resistor, N_L is given as

$n = a_1\phi + a_3\phi^3$ (19)

and for the tunnel diode D, we have

$i_D = -a_2V_D + b_2V_D^3$ (20)

Using KCL:

$I = C\frac{dVC}{dt} + n + i_D$ (21)

From Loops (2 and 3), figure 2:

$V_C = V_L = V_D = L\frac{dn}{dt} = \frac{d\phi}{dt} = \dot{\phi}$ (22)

$\frac{dVC}{dt} = \frac{dVL}{dt} = L\ddot{n} = \ddot{\phi}$ (23)

Substituting, (19), (20), and (23), into (21), results to

$I = C\ddot{\phi} + a_1\phi + a_3\phi^3 - a_2V_D + b_2V_D^3$ (24)

Also, substituting (23) into (24), gives

$I = C\dddot{\phi} + a_1\dot{\phi} + a_3\dot{\phi}^3 - a_2\dot{\phi} + b_2\dot{\phi}^3$ (25)

Using KVL and considering Loop (1) of figure 2

$E_0Cos(wt) = IR + V_1$ (26)

Substituting (25) in (26) results to

$\frac{1}{R}(E_0Cos(wt)) = C\ddot{\phi} + a_1\phi + a_3\phi^3 - a_2\dot{\phi} + b_2\dot{\phi}^3 + \frac{1}{R}\dot{\phi}$

and

$\frac{1}{RC}(E_0Cos(wt)) = \ddot{\phi} + \frac{1}{C}(\frac{1}{R} - a_2)\dot{\phi} + \frac{1}{C}b_2\dot{\phi}^3 + \frac{1}{C}a_1\phi + \frac{1}{C}a_3\phi^3$ (27)

If we define $\dot{\phi} = x$, $\epsilon = \frac{1}{C}(\frac{1}{R} - a_2)$, $k = \frac{1}{C}b_2$, $a = \frac{1}{C}a_1$, $b = \frac{1}{C}a_3$, and $B = \frac{1}{RC}E_0$, we have

$BCos(wt) = \ddot{x} + \epsilon\dot{x} + b\dot{x}^3 + ax + kx^3$,

or

$\ddot{x} + \epsilon\dot{x} + b\dot{x}^3 + ax + kx^3 = BCos(wt)$ (28)

Equation (28), is a form of Duffing equation, depending on the value of k, and can be expressed as system of equations as follows;

$$\begin{aligned}
\dot{x} &= y\\
\dot{y} &= BCos(wt) - \epsilon y - ax - bx^3 - ky^3
\end{aligned} \quad (29)$$

Hence, equation (29) is the slave system to be used in this work.

3.4 Chaos control of the master system

The time evolution of the system currents and voltages are represented by equation (18) re-written as shown below as equation (30).

$$\begin{aligned}
\dot{x}_1 &= a_1 Sin(wt) - a_2x_1 - a_3x_3\\
\dot{x}_2 &= b_1x_3 - b_1x_4 - b_2x_2\\
\dot{x}_3 &= k_1(x_1 - x_2)\\
\dot{x}_4 &= k_2x_2 + k_3x_4 - k_4x_4^3
\end{aligned} \quad (30)$$

The above system is chaotic and can be controlled by introducing control functions $U_1(t)$, $U_2(t)$, $U_3(t)$, and $U_4(t)$ at the inputs which are to be determined. So equation (30) becomes

$$\begin{aligned}
\dot{x}_1 &= a_1 Sin(wt) - a_2x_1 - a_3x_3 + U_1(t)\\
\dot{x}_2 &= b_1x_3 - b_1x_4 - b_2x_2 + U_2(t)\\
\dot{x}_3 &= k_1(x_1 - x_2) + U_3(t)\\
\dot{x}_4 &= k_2x_2 + k_3x_4 - k_4x_4^3 + U_4(t)
\end{aligned} \quad (31)$$

The objective is to design recursive backstepping controllers for the system (30) that guarantees global stability in such a way as to force the output of x(t) of the system (30) to asymptotically track the output x_{id} of the reference model, which means that

$|x - x_{id}| \to 0$, as $t \to \infty$

The control input $U_i(t)$, i = 1, 2, 3, 4 to be determined such that the state variables x_i (i = 1, 2, 3, 4) of the system (31), can be assigned desired values x_{id} , i = 1, 2, 3, 4 respectively.

The error states are defined as follows

$$\left.\begin{aligned}
e_1 &= x_1 - x_{1d}\\
e_2 &= x_2 - x_{2d}\\
e_3 &= x_3 - x_{3d}\\
e_4 &= x_4 - x_{4d}
\end{aligned}\right\} \quad (32)$$

In order to design the control function $U_i(t)$, i = 1, 2, 3, 4, we set

$$\left.\begin{aligned}
x_{1d} &= x_1 - f(t)\\
x_{2d} &= C_1e_1\\
x_{3d} &= C_2e_1 + C_3e_2\\
x_{4d} &= C_4e_1 + C_5e_2 + C_6e_3
\end{aligned}\right\} \quad (33)$$

where C_i, i = 1, 2, 3, 4, 5, 6 are control parameters to be chosen appropriately, by substituting (32) into (33), we get the error dynamics

$e_1 = x_1 - f(t)$
$e_2 = x_2 - C_1e_1$
$e_3 = x_3 - C_2e_1 + C_3e_2$
$e_4 = x_4 - C_4e_1 + C_5e_2 + C_6e_3$ (34)

Differentiating (34) w.r.t. time

$\dot{e}_1 = \dot{x}_1 - \dot{f}(t)$
$\dot{e}_2 = \dot{x}_2 - C_1\dot{e}_1$
$\dot{e}_3 = \dot{x}_3 - C_2\dot{e}_1 - C_3\dot{e}_2$
$\dot{e}_4 = \dot{x}_4 - C_4\dot{e}_1 - C_5\dot{e}_2 - C_6\dot{e}_3$ (35)

From (34)

$x_1 = e_1 + f(t)$
$x_2 = e_2 + C_1e_1$
$x_3 = e_3 + C_2e_1 + C_3e_1$
$x_4 = e_4 + C_4e_1 + C_5e_2 + C_6e_3$ (36)

Substituting (36) into (31), gives

$\dot{x}_1 = a_1Sin(wt) - a_2(e_1 + f(t)) - a_3(e_3 + C_2e_1 + C_3e_1) + U_1(t)$
$\dot{x}_2 = b_1(e_3 + C_2e_1 + C_3e_1) - b_1(e_4 + C_4e_1 + C_5e_2 + C_6e_3) - b_2(e_2 + C_1e_1) + U_2(t)$
$\dot{x}_3 = k_1(e_1 + f(t) - e_2 - C_1e_1) + U_3(t)$
$\dot{x}_4 = k_2(e_2 + C_1e_1) + k_3(e_4 + C_4e_1 + C_5e_2 + C_6e_3) - k_4(e_4 + C_4e_1 + C_5e_2 + C_6e_3)^3 + U_4(t)$ (37)

Substituting (37) into (35), results to

$\dot{e}_1 = a_1Sin(wt) - a_2(e_1 + f(t)) - a_3(e_3 + C_2e_1 + C_3e_1) - \dot{f}(t) + U_1(t)$
$\dot{e}_2 = b_1(e_3 + C_2e_1 + C_3e_1) - b_1(e_4 + C_4e_1 + C_5e_2 + C_6e_3) - b_2(e_2 + C_1e_1) - C_1\dot{e}_1 + U_2(t)$
$\dot{e}_3 = k_1(e_1 + f(t) - e_2 - C_1e_1) - C_2\dot{e}_1 - C_3\dot{e}_2 + U_3(t)$
$\dot{e}_4 = k_2(e_2 + C_1e_1) + k_3(e_4 + C_4e_1 + C_5e_2 + C_6e_3) - k_4(e_4 + C_4e_1 + C_5e_2 + C_6e_3)^3 - C_4\dot{e}_1 - C_5\dot{e}_2 - C_6\dot{e}_3 + U_4(t)$ (38)

In order to stabilize the error system (38) at the equilibrium position, we consider the Lyapunov function of the form

$V = \frac{1}{2}\sum_{i=1}^{4} e_i^2$ (39)

$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2)$ (40)

$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4$ (41)

To satisfy the condition for asymptotic stability of the error dynamics (38) necessary for controlling chaos, we substitute (38) into (41) and choose $U_i(t)$, i = 1, 2, 3, 4 such that the derivative of the Lyapunov function (39) is negative definite as follows

$\dot{V} = e_1[a_1Sin(wt) - a_2(e_1 + f(t)) - a_3(e_3 + C_2e_1 + C_3e_1) - \dot{f}(t) + U_1(t)] + e_2[b_1(e_3 + C_2e_1 + C_3e_1) - b_1(e_4 + C_4e_1 + C_5e_2 + C_6e_3) - b_2(e_2 + C_1e_1) - C_1\dot{e}_1 + U_2(t)] + e_3[k_1(e_1 + f(t) - e_2 - C_1e_1) - C_2\dot{e}_1 - C_3\dot{e}_2 + U_3(t)] + e_4[k_2(e_2 + C_1e_1) + k_3(e_4 + C_4e_1 + C_5e_2 + C_6e_3) - k_4(e_4 + C_4e_1 + C_5e_2 + C_6e_3)^3 - C_4\dot{e}_1 - C_5\dot{e}_2 - C_6\dot{e}_3 + U_4(t)]$ (42)

From Lasalle-Yoshizawa Theorem,

$\dot{V} = -(e_1^2 + e_2^2 + e_3^2 + e_4^2) < 0$ (43)

is negative definite for the controller to effectively control the system and not change the equilibrium of the system.

Comparing (42) and (43),

$a_1Sin(wt) - a_2(e_1 + f(t)) - a_3(e_3 + C_2e_1 + C_3e_1) - \dot{f}(t) + U_1(t) = -e_1$
$b_1(e_3 + C_2e_1 + C_3e_1) - b_1(e_4 + C_4e_1 + C_5e_2 + C_6e_3) - b_2(e_2 + C_1e_1) - C_1\dot{e}_1 + U_2(t) = -e_2$
$k_1(e_1 + f(t) - e_2 - C_1e_1) - C_2\dot{e}_1 - C_3\dot{e}_2 + U_3(t) = -e_3$
$k_2(e_2 + C_1e_1) + k_3(e_4 + C_4e_1 + C_5e_2 + C_6e_3) - k_4(e_4 + C_4e_1 + C_5e_2 + C_6e_3)^3 - C_4\dot{e}_1 - C_5\dot{e}_2 - C_6\dot{e}_3 + U_4(t) = -e_4$ (44)

From (44), we have that

$U_1(t) = -a_1Sin(wt) + a_2(e_1 + f(t)) - a_3(e_3 + C_2e_1 + C_3e_1) + \dot{f}(t) - e_1$
$U_2(t) = -b_1(e_3 + C_2e_1 + C_3e_1) + b_1(e_4 + C_4e_1 + C_5e_2 + C_6e_3) + b_2(e_2 + C_1e_1) + C_1\dot{e}_1 - e_2$
$U_3(t) = -k_1(e_1 + f(t) - e_2 - C_1e_1) + C_2\dot{e}_1 + C_3\dot{e}_2 - e_3$
$U_4(t) = -k_2(e_2 + C_1e_1) - k_3(e_4 + C_4e_1 + C_5e_2 + C_6e_3) + k_4(e_4 + C_4e_1 + C_5e_2 + C_6e_3)^3 + C_4\dot{e}_1 + C_5\dot{e}_2 + C_6\dot{e}_3 - e_4$ (45)

Where $U_i(t)$, i = 1, 2, 3, 4 in (45) are the respective controllers for which the parameters C_i (i = 1, 2, 3, 4, 5, 6), and functions f(t) and $\dot{f}(t)$ are to be determined in computer simulation in order to effectively control the chaotic system (30).

Thus inserting the respective values of the controllers into (31), results to

$\dot{x}_1 = a_1Sin(wt) - a_2x_1 - a_3x_3 + [a_1Sin(wt) + a_2(e_1 + f(t)) - a_3(e_3 + C_2e_1 + C_3e_1) + \dot{f}(t) - e_1]$
$\dot{x}_2 = b_1x_3 - b_1x_4 - b_2x_2 + [-k_1(e_1 + f(t) - e_2 - C_1e_1) + C_2\dot{e}_1 + C_3\dot{e}_2 - e_3]$
$\dot{x}_3 = k_1(x_1 - x_2) + [-k_1(e_1 + f(t) - e_2 - C_1e_1) + C_2\dot{e}_1 + C_3\dot{e}_2 - e_3]$
$\dot{x}_4 = k_2x_2 + k_3x_4 - k_4x_4^3 + [-k_2(e_2 + C_1e_1) - k_3(e_4 + C_4e_1 + C_5e_2 + C_6e_3) + k_4(e_4 + C_4e_1 + C_5e_2 + C_6e_3)^3 + C_4\dot{e}_1 + C_5\dot{e}_2 + C_6\dot{e}_3 - e_4]$ (46)

Equation (46), is to be simulated with the control parameters and functions to determine the controllers in (45).

From the computational result carried out, system (30) was effectively controlled with only $C_1 = C_2 = 1$ and $C_3 = C_4 = C_5 = C_6 = f(t) = \dot{f}(t) = 0$, which simplify the controllers in (45) to

$U_1(t) = -a_1Sin(wt) + a_2e_1 - a_3(e_3 + e_1) - e_1$
$U_2(t) = -b_1(e_3 + e_1) + b_1e_4 + b_2(e_2 + e_1) + \dot{e}_1 - e_2$
$U_3(t) = k_1e_2 + \dot{e}_1 - e_3$
$U_4(t) = -k_2(e_2 + e_1) - k_3e_4 + k_4e_4^3 - e_4$ (47)

and (46) results to

$\dot{x}_1 = a_1Sin(wt) - a_2x_1 - a_3x_3 + [-a_1Sin(wt) + a_2e_1 - a_3(e_3 + e_1) - e_1]$
$\dot{x}_2 = b_1x_3 - b_1x_4 - b_2x_2 + [-b_1(e_3 + e_1) + b_1e_4 + b_2(e_2 + e_1) + \dot{e}_1 - e_2]$
$\dot{x}_3 = k_1(x_1 - x_2) + [k_1e_2 + \dot{e}_1 - e_3]$ (48)
$\dot{x}_4 = k_2x_2 + k_3x_4 - k_4x_4^3 + [-k_2(e_2 + e_1) - k_3e_4 + k_4e_4^3 - e_4]$

3.5 REDUCED-ORDER SYNCHRONIZATION OF THE MASTER SYSTEM AND THE DUFFING SYSTEM.

In this case, system (18) is taken as the master system, while the Duffing's equation (29) is used as the slave system. The synchronization will be obtained in reduced-order as follows;

MASTER SYSTEM:

$\dot{x}_1 = a_1 Sin(wt) - a_2x_1 - a_3x_3$
$\dot{x}_2 = b_1x_3 - b_1x_4 - b_2x_2$
$\dot{x}_3 = k_1(x_1 - x_2)$
$\dot{x}_4 = k_2x_2 + k_3x_4 - k_4x_4^3$ (47)

SLAVE SYSTEM:

$\dot{y}_1 = y_2$
$\dot{y}_2 = BCos(wt) - \epsilon y_2 - ay_1 - by_1^3 - ky_1^3$ (48)

To drive the state variables (y_1, y_2) to (x_1, x_2), we introduce the controllers $U(t) = (U_1(t), U_2(t))$ to (48) and set k = 0 to give the required Duffing system.

$\dot{y}_1 = y_2 + U_1(t)$
$\dot{y}_2 = BCos(wt) - \epsilon y_2 - ay_1 - by_1^3 + U_2(t)$ (49)

The synchronization error is defined as

$e_1 = x_1 - y_1$
$e_2 = x_2 - y_2$ (50)

and

$\dot{e}_1 = \dot{x}_1 - \dot{y}_1$
$\dot{e}_1 = \dot{x}_2 - \dot{y}_2$ (51)

Subtracting (49) from (47), we have the following error dynamics

$\dot{e}_1 = a_1 Sin(wt) - a_2x_1 - a_3x_3 - y_2 - U_1(t)$
$\dot{e}_2 = b_1x_3 - b_1x_4 - b_2x_2 - BCos(wt) + \epsilon y_2 + ay_1 + by_1^3 - U_2(t)$ (52)

Let the Lyapunov function be defined as

$V(e_1, e_2) = \frac{1}{2}(e_1^2 + e_2^2)$ (53)

$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2$ (54)

Substituting (52) into (54), results to

$\dot{V} = e_1(a_1 Sin(wt) - a_2x_1 - a_3x_3 - y_2 - U_1(t)) + e_2(b_1x_3 - b_1x_4 - b_2x_2 - BCos(wt) + \epsilon y_2 + ay_1 + by_1^3 - U_2(t))$ (55)

To ensure that $\dot{V}$ is negative definite and thus satisfy the Lasalle-Yoshisawa theorem

$\dot{V} = -(e_1^2 + e_2^2) < 0$ (56)

Hence ,

$a_1 Sin(wt) - a_2x_1 - a_3x_3 - y_2 - U_1(t) = -e_1 b_1x_3 - b_1x_4 - b_2x_2 - BCos(wt) + \epsilon y_2 + ay_1 + by_1^3 - U_2(t) = -e_2$ (57)

and,

$U_1(t) = a_1 Sin(wt) - a_2x_1 - a_3x_3 - y_2 + e_1$
$U_2(t) = b_1x_3 - b_1x_4 - b_2x_2 - BCos(wt) + \epsilon y_2 + ay_1 + by_1^3 + e_2$ (58)

(58), represents the controllers necessary to synchronize the master and slave systems by reduce-order technique.

The error dynamics e_1 and e_2, between the fourth and the second order chaotic systems, showing reduced-order synchronization under the action of the control is shown in the next paragraph.

4.0 NUMERICAL RESULTS AND DISCUSSION

4.1 NUMERICAL RESULTS

The solution of the set of equations in chapter three can be shown graphically with each of the dependent variables x_i and y_i as a function of time t, the independent variable.

The numerical solution of the chaotic system (47), (the master system) and (48), (the slave system) are shown using MATLAB numerical simulation [4, 5, 14, 25, 31, 36], Figure 3 and Figure 4 respectively in appendix.

The parameters of the master system are; w=77, $a_1 = 37$, $a_2 = 0.01$, $a_3 = 0.2$, $b_1 = 0.001$, $b_2 = 0.02$, $k_1 = 10000$, $k_2 = 4000$, $k_3 = 1000$, $k_4 = 70000$, and the initial conditions (x_1, x_2, x_3, x_4) = (– 0.005, – 0.001, 0.1, – 0.5). The parameters for the slave system are; B = 0.3, w = 1.0, $\epsilon = 0.15$, a = 1, b = 1, k = 0, and the initial conditions are (y_1, y_2) = (0, 0). The initial conditions for the error states (e_1, e_2) = (0, 0). The parameters of the systems above are used in all the results plotted.

4.2 DISCUSSION OF RESULTS

The chaotic behaviour of the master system is shown in Figure 3(a) to (d). These are the time series that depicts the chaotic states of the system when controllers are not activated. When the time series was plotted at various time range, the systems still show chaotic behaviour. Also, one positive Lyapunov exponent is a clear evidence of a chaotic system as shown in Figure 12.

Figure 4, shows the phase portrait of the chaotic system when controllers are not activated. Figure 5 (a – f), represent the Poincaré sections of the master system, while Figure 5(g), shows that of the slave system. These Poincaré sections/maps are evidence of the chaotic nature of the systems.

The time series and phase portraits of the slave system is clearly shown in Figure 6. The time series here shows unrepeated states of the slave system, which is an attribute of a chaotic system.

When the system controllers (47) are activated at t $\geq$ 0, the system is forced to a stable equilibrium as is shown in the error dynamics in Figure 7.

When controls are activated at t $\geq$ 30, the system is stabilized, and is totally controlled as shown in figure 8.

Figure 9 shows the synchronized states of master and the slave system, where Figure 9(a) represents the synchronization of x_1,y_1 against time while Figure 9(b) is the synchronization of x_2,y_2 against time. The error dynamics of the synchronized state is shown in Figure 10.The error dynamics in the two cases are not asymptotically zero, which show the effect of reduced-order method of synchronization. This means that reduced order synchronization may not lead to a total or complete synchronization of all the four state variables of the master system with the two variables of the slave system, but will only synchronize with the two considered variables of the master system and the two variables of the slave system.

Figure 11 shows a plot of the Lyapunov exponent versus time for the master system. It shows a clear evidence of the chaotic nature of the system. The Lyapunov exponents deduced are 0.011713, -0.017873, -0.021901, and -55.583, having only one positive value. The presence of a positive Lyapunov exponent satisfies one of the most important tests for the identification of a chaotic system.

5.0 CONCLUSION

The recursive backstepping technique employed in this work, based on Lyapunov stability theory, was able to control the chaotic oscillation of the system to a stable equilibrium. The performance of theoretically designed nonlinear controllers were verified by numerical simulations which confirmed the effectiveness of the proposed controllers.

Reduced-order synchronization was achieved between a new fourth-order system and a second-order Duffing oscillator. This study shows the effectiveness of the adopted technique for synchronization, and has been illustrated with numerical simulations. However, this ascertains that synchronization is not only possible with systems in the same order, but also possible with systems of different order.

6.0 REFERENCES

[1] Bal E. W., Lonngren K. E. (1997): Synchronization of two Lorenz systems using active control: Chaos, Solitons and Fractals. 8, 51-58.

[2] Cai G., Huang J., Tian L. (2006), Wang Q.: Adaptive control and slow manifold analysis of a new chaotic system. Int'l Journal of Nonlinear Science. 2(1), 50-60.

[3] Chaos A. M.,. Harb B. A. (2004): Chaos control of the third-order phase locked loops using backstepping nonlinear controller. Solitons and Fractals. 20, 719-723.

[4] David F. Griffiths (2005): An introduction to MATLAB. Version 2.3, Pages (1-37).

[5] Desmond J. Higham and Nicholas J.: Version of June 27, 2000. MATLAB Guide. Pages (1-302).

[6] Devaney, Robert L. (2003). An Introduction to chaotic Dynamical Systems. 2nd ed. Western Press. ISBN 0-8188-4085-3.

[7] Enns R. H. (2011): It's a nonlinear World: Pages (193-200).

[8] Feki M. (2003): An adaptive feedback control of linearizable chaotic systems, Chaos, Solitons and Fractals. 15, 883-890.

[9] Fotsin H. B. and Woafo P. (2005): Adaptive Synchronization of modified uncertain Van der Pol-Duffing Oscillator based on Parameter Identification., 14(5): 1363-1371.

[10] Haeri M. and Emadzadeh A. A. (2006): Comparism between different synchronization methods of identical chaotic systems. Chaos, Solitons and Fractals 29, 1002-1022.

[11] Hasselb Latt, Boris, Anatole katok (2003): A first course in Dynamics, with a Panorama of Recent Developments. Cambridge University Press: ISBN 0-521-58750-6.

[12] Hayashi C. (1964), "Nonlinear Oscillation in Physical systems". Mc Graw-Hill, new York.

[13] Ho M. C. and Hung Y. C. (1997): Synchronization of two Lorent Systems using active control [J].Chaos, Solitons and Fractals, , 8(11): 51-58

[14] Howard P. (2003): Ordinary Differential Equations in MATLAB. Fall. Pages (1-16).

[15] Idowu B. A., et al (2007): Control and Synchronization of Chaos in Nonlinear Gyros via backstepping design (1).

[16] Jang M. Chen C. K. (2002): Sliding mode control of chaos in the cubic Chua's circuit system. Int'l Journal of bifurcation and chaos. 12, 1437-1449.

[17] Jang M., et al (2002): Sliding mode control of chaos in the Cubic Chua's circuit system [J]. international Journal of Bifurcation and Chaos, , 12(6): 1437-1449.

[18] Konoshi K., et al (1998): Sliding mode control for a class of chaotic systems. Physics Letters. A 245, 511-571.

[19] Laoye J. A., et al (2008): Chaos control and reduced-order Synchronization of the Rigid body: Int'l Journal of Nonlinear Science, Vol. 6. No. 2, pp. 106-113.

[20] Laoye J. A., et al (2007): Chaos control of 4-D chaotic systems using recursive backstepping nonlinear controller. Chaos, Solitons and Fractals doi: 10, 1016/j. Chaos. 2007.04.020. In press.

[21] Lorenz, Edward N. (1963): "Deterministic non-periodic flow." Journal of the atmospheric Sciences 20(2): 130-141.

[22] Lorenz E. N. (1963), "Deterministic nonperiodic flow" [J], Atmospheric Science 20, 130-141.

[23] Madam R. N. (1993), "Chua's Circuit: A paradigm for Chaos". World Scientific,

Singapore.
[24] Mahsa K., et al (2001): Nonlinear Electric circuit oscillator control based on Backstepping method: A Generic algorithm Approach. (1-2).
[25] MATLAB (2009): The Language of Technical Computing: Computing, Visualization and Programming. www.mathworks.com. Pg.(1-190).
[26] Ming-Chung Ho, et al (2006): Reduced-order synchronization of chaotic systems with parameters unknown. Physics Letter A 348, 251-259.
[27] Orgazelek M. J. (2005): Taming Chaos-Part I: Synchronization.
[28] Ott E. C. Grebogi, Yorke J. A. (1990): Controlling Chaos. Physical Review Letter. 64, 1196-1199.
[29] Pecora L. M., Carroll T. L. (1990): Synchronization of chaotic systems. Physical review Letter, 64, 821-824.
[30] Ricardo Femat and Gualberto Solis-Perales (2002): Synchronization of chaotic systems with different order. Physical Review E, volume 65, 036226.
[31] Richard Johnson (2002). MATLAB Programming Style Guidelines.. Version 1.5, Pages (1-14).
[32] Rössler E. (1979), "An equation for hyperchaos". A 71,155-157.
[33] Rudrappa J. S. (2009): Chaotic Communication, their application and advantages over traditional method of communication..
[34] Sun M., et al (2007): feedback control and adaptive control of the Energy Resource chaotic system. Chaos, Solitons and Fractals. 32, 1725-1734.
[35] Sun M., Tian L., Xu J. (2006): Time-delayed feedback control of energy Resource Chaotic system. Int'l Journal of Nonlinear Science. 1(3), 172-177.
[36] Symbolic Maths Toolbox (2007), for use with MATLAB. www.mathworks.com. Pg.(1-296).
[37] Tan X., Zhang J., Yang Y. (2003): Synchronization of chaotic systems using backstepping design. Chaos, Solitons and Fractals. 16, 37-45.
[38] Tan X., et al (2003): Synchronization of chaotic Systems using backstepping design [J]. Chaos, Solitons and Fractals. 16(1): 37-45.
[39] Vincent U. E., et al (2007).: Controlling chaos and deterministic directed transport in inertia ratchets using backstepping control. Physica D, 231, 130-136.
[40] Vincent U. E., Laoye J. A. (2007): Synchronization and Control of directed transport in chaotic ratchets via active control. Physics Letters A 363, 91-95.
[41] Wikipedia: The free Encyclopedia. Chaos Theory. Note (28-32).
[42] Zhang H., et al (2004): Generalized synchronization of hyperchaos and chaos using active backstepping design. Chinese Physics, 14(1):86-94.

7.0 APPENDIX

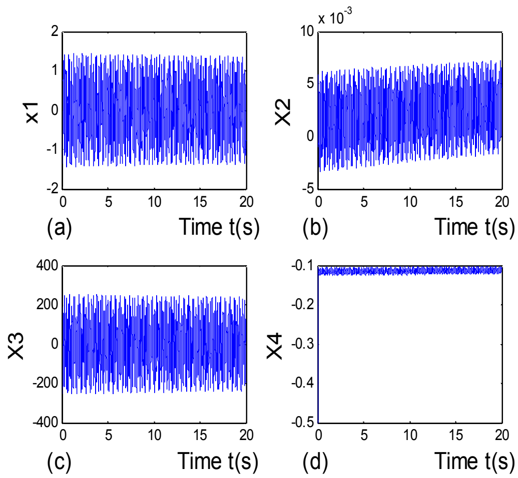


Figure 3: Time series for x_1, x_2, x_{3vb} , and x_4 respectively of the master system when controllers are not activated.

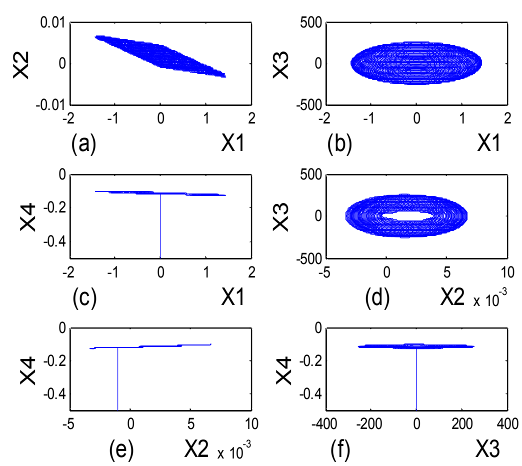
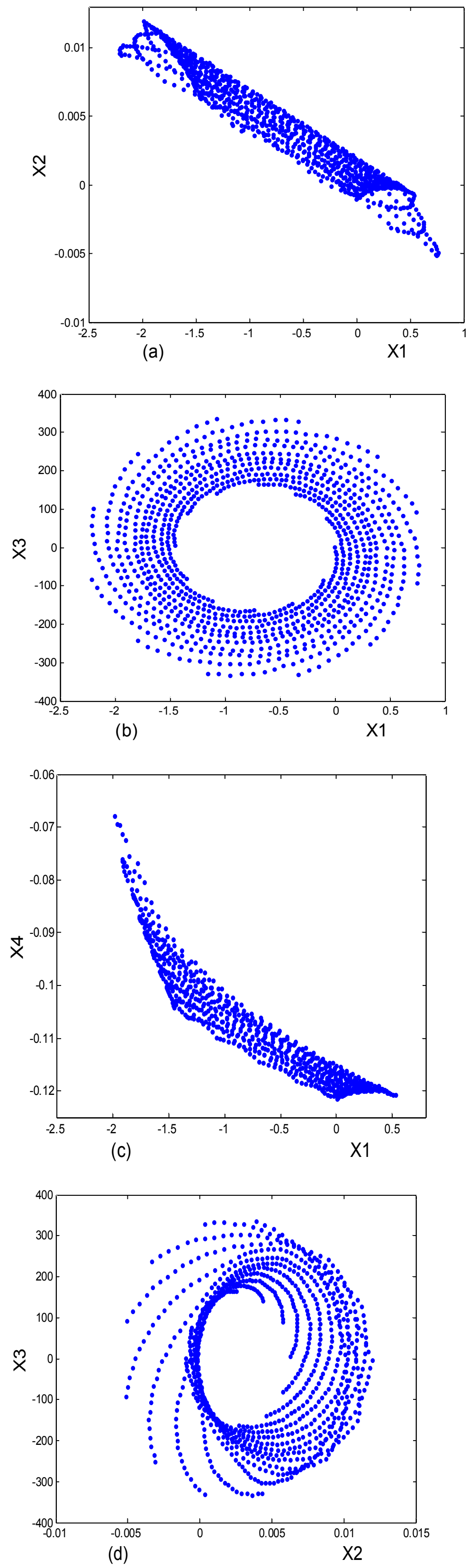


Figure 4: Phase portraits of the master system.



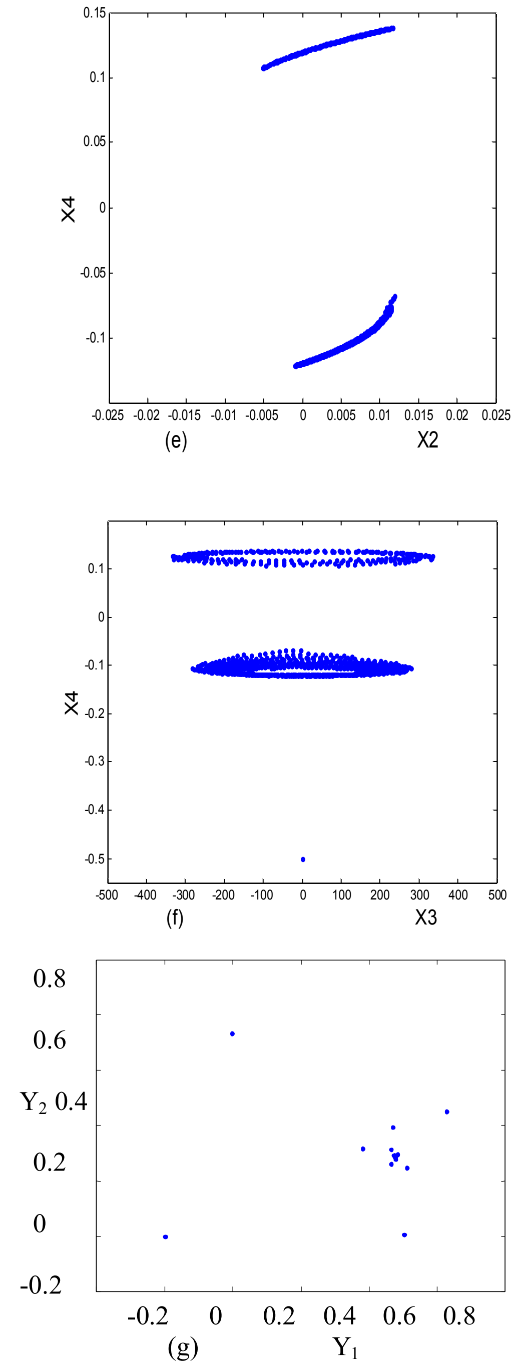


Figure 5: (a)-(f) are the Poincaré sections of the master system while (g) is that of the slave (Duffing) system.

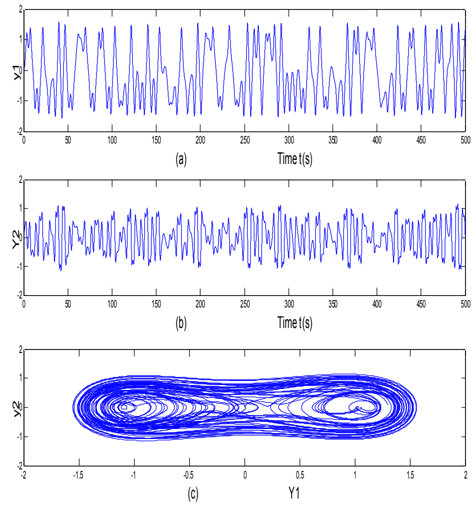


Figure 6: (a) and (b) are the respective time series of the slave system (Duffing system) and (c) is its phase portrait.

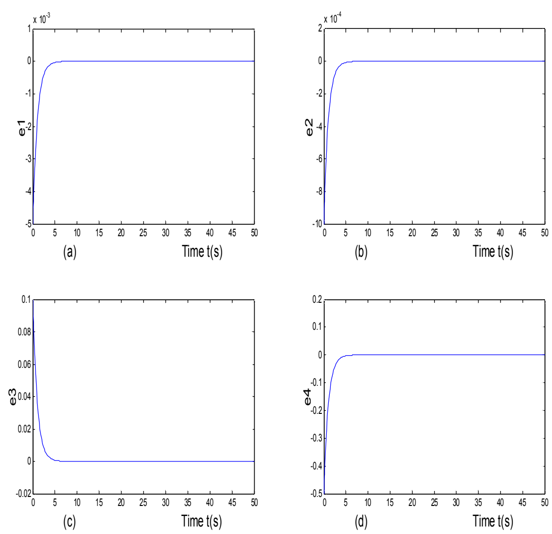


Figure 7: Error dynamics of the system when controllers are applied at t ≥ 0.

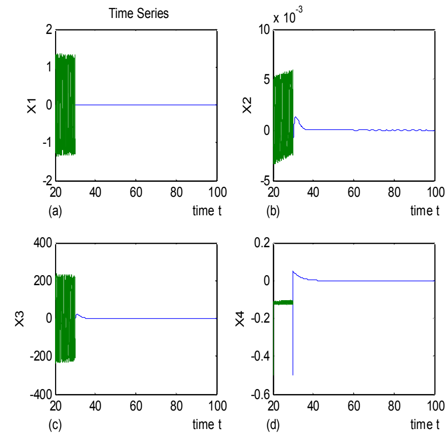


Figure 8: Time response of the state variables when control is activated at t ≥ 30 (a) x_1 variable (b) x_2 variable (c) x_3 variable (d) x_4 variable.

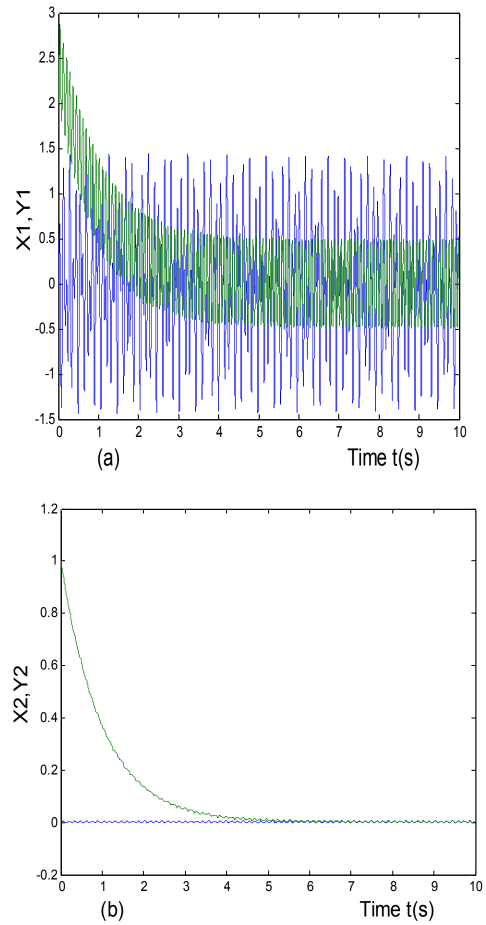


Figure 9 (a & b): Synchronization of the master and slave systems.

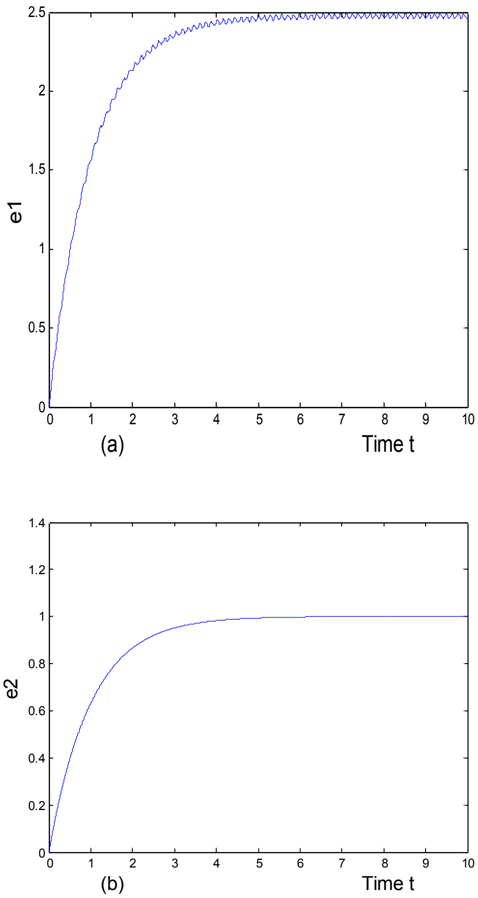


Figure 10 (a & b): The error dynamics of the synchronized state.

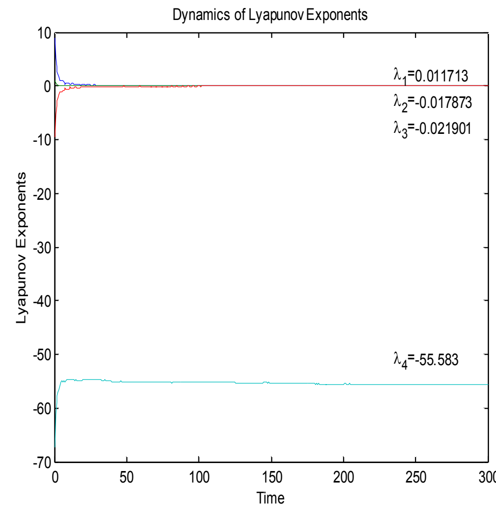


Figure 11: Lyapunov exponents versus time of the master system.

7/26/2013