**A SIMULATION STUDY OF MODIFIED RIDGE ESTIMATORS FOR HANDLING MULTICOLLINEARITY PROBLEM**

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**Abstract**

The study is on Proposed Modified ridge estimators for solving Multicollinearity problems in Linear Regression. A monte Carlo experiment was conducted using TSP 5.0 Statistical Package with sample sizes of 10,20,30,50,100 and 250 and at different multicollinearity levels of 0.2, 0.4, 0.6, 0.8, 0.95 and 0.99 which are categorize as low, moderate and high levels. The data generated were replicated 1000 time with normally and uniformly distributed regressors. The methods employed the use Ordinary Ridge and Generalized Ridge estimators along with CORC and ML estimators. The proposed estimators used the appropriate estimators, CORC and ML, to obtain MSE and the regression coefficients which is now used in developing the proposed estimators. The results of the analysis revealed that among the proposed modified ridge estimators MLMOREKBAY, CORCMOREKBAY, CORCMOREKLA and MLMGRE perform excellently well at different sample sizes and can therefore be used to solve problem of multicollinearity in any dataset. Also, the existing estimator OREKBAY compete favorably well with the proposed estimators in addressing the problem of multicollinearity.

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**Keywords:** Multicollinearity, CORC -Cochrane-Orcutt Estimator, ML – Maximum Likelihood Estimator, MSE – Mean Square Error and Monte Carlo Experiment.

1. **Introduction**

Multicollinearity is one of the important problems in multiple regression analysis. It is usually regarded as a problem arising as a result of the violation of the assumption that explanatory variables are linearly independent. Though no precise definition of multicollinearity has been firmly established in the literature. Multicollinearity is generally agreed to be present if there is an approximate linear relationship among some of the predictor variables in the data. Bowerman and O ʹConnell (2006) stated that the term multicollinearity refers to a situation in which there is an exact (or nearly exact) linear relation among two or more of the explanatory variables. Exact relations may arise by mistake or lack of understanding. Multicollinearity can also be defined in the concept of orthogonality. When the predictors are orthogonal or uncorrelated, all eigenvalues of the design matrix are equal to one and the design matrix is full rank. If at least one eigenvalue is different from one, especially when equal to zero or near zero, then non-orthogonality exists, meaning that multicollinearity is present. Multicollinearity can lead to increasing complexity in the research results, thereby posing difficulty for researchers to provide interpretation (Chartterjee, Hadi and Price, 2000).

Multicollinearity is a matter of degree. The real issue is to determine the point at which the degree of multicollinearity becomes “harmful”. The econometric literature typically takes the theoretical position that predictor variables are not collinear in the population. Hence, any observed multicollinearity in empirical data is considered as a sample based “problem” rather than as representative of the underlying population relationship (Kmenta, 1986). Regardless of whether multicollinearity in data is assumed to be a sampling problem or true reflection of population relationships, it must be looked into when data are analyzed using regression analysis because it has several potential undesirable consequences on the parameter estimates. When multicollinearity is a problem, parameter estimates have wrong signs when compared with theoretical knowledge and variables have insignificant coefficients. The regression coefficients, though determinate when multicollinearity is imperfect, possess large standard errors which imply that the coefficients cannot be estimated with great precision. (Hawking and Pendleton, 1983; Gujarati and Porter, 2009).

Various authors have worked on estimators for solving multicllinearity among which are: James (1956) which introduced the stein estimator as a method for handling multicollinearity. Massey (1965) also apply Principal Components Regression to handle the problem of multicollinearity by eliminating the model instability and reducing the variances of the regression coefficients. The sample correlation for any pair of components is observed to be zero. Wold (1966) introduced the Partial Least Squares Regression into handling multicollinearity problem. This method is similar to the method of the Principal Components Analysis. However, it utilizes the dependent variable together with the explanatory variables. Hoerl and Kennard (1970). Proposed the ridge estimator for dealing with multicollinearity in a regression model. It is the modification of the OLS that allows biased estimation of the regression coefficients. Also some years later, Hoerl, *et al* (1975) Proposed ridge estimator for dealing with multicollinearity and provided optimal value K of the ridge parameter given as: ; where is an unbiased estimator of error-variance from OLS estimation and is also regression coefficient from OLS estimation

This research work is aimed at developing a modified ridge estimators and compare it with the existing estimators to address the problem of multicollineariy.

1. **Methodology**

The general form of Linear Regression Model is: Y = X β + U (1)

Where,

Y is an (n x 1) vector of observations of the dependent variable. X matrix is an n x (k+1) full rank matrix of observable and fixed values of the explanatory variables. is a ((k+1) x1) vector of unknown parameters to be estimated. U is (n x 1) vector of random error. The parameter estimate is defined as:

 (2)

**The Ridge Estimators**

Ridge regression is a method of biased linear estimation which has been shown to be more efficient than the OLS estimator when data sets exhibit multicollinearity. Hoerl and Kennard (1970). The estimator of β is defined as:

-1 y (3)

The constant K is known as bias or ridge parameter and it yield minimum MSE compared to the OLS estimator.

This study employed the followings estimators:

1. **Generalized Ridge Estimator**

The optimum value of K had been obtained by Hoerl, Kennard and Baldwin (1975) as:

(4)

Since and are generally unknown, the needs to be estimated.

1. **The Ordinary Ridge Estimator**

The Ordinary ridge regression (ORR) estimator requires a fixed value of the ridge Parameter, K. Several Ks have also been proposed by authors including that of Lukman and Ayinde (2017) and Sclove (1973) given as:

= (5)

Sclove (1973) suggested an empirical K-Bayesian Ridge Parameter given as:

= (6)

Where SSR = Sum of Square of Regression

1. **Cochrane-Orcutt Estimator**
2. **Maximum Likelihood Estimator**

**Model Formulation**

Consider the linear regression model given as:

(7)

Where.

The regressors are fixed and exhibit different degree of multicollinearity.

**The Monte Carlo Experiments**

The experiment were replicated (R) one thousand time (1000) and at sample sizes of n = 10,20,30,50,100 and 250.

**Correlated Normally distributed Variables**

The equations provided and used by Ayinde (2007a, b) and Ayinde and Adegboye (2010) were used to generate normally distributed random variables with specified inter-correlation. With p = 3, the equations are:

(8)

(9)

(10)

Where

and ; and ,

,

Using the generated correlated normally distributed variables above,, ; the study further utilized the properties of random variables that cumulative distribution function of normal distribution produce U (0, 1) without affecting the correlation among variables to generate correlated uniform distributed variables , (Schumann, 2009)

The error terms were generated with .

The model parameter values were taken as= 0.8,and

**Modified Ridge Estimators:**

The ridge estimators discussed in (3) require estimation of

(12)

where,

 is the Mean Square Error based on OLS estimation

 is the regression coefficient i based on OLS estimation , i = 1,2.........p.

The proposed estimators used the appropriate estimators, CORC and ML, to obtain MSE and the regression coefficients. These result into the following proposed estimators. The algorithms required are as follows:

**Cochrane-Orcutt Modified Generalized Ridge Estimator (CORCMGRE):**

The procedures are as follows:

i. CORC estimator is used instead of the OLS estimator to obtain the regression co-efficients and the MSE,

ii. Thereafter, the different are obtained and used in the ridge estimator to obtain the regression co-efficient of the model.

All others proposed estimators follows the same procedure.

Cochrane Orcutt Modified Ordinary Ridge Estimator-KBAY

(CORCMORE-KBAY)

Cochrane Orcutt Modified Ordinary Ridge Estimator-KLA

(CORCMORE-KLA)

Maximum Likelihood Modified Generalized Ridge Estimator (MLMGRE)

Maximum Likelihood Modified Ordinary Ridge Estimator-KBAY

(MLMORE-KBAY)

Maximum Likelihood Modified Ordinary Ridge Estimator-KLA

(MLMORE-KLA)

The existing estimation techniques used are:

Ordinary Least Squares Estimator (OLSE)

Generalized Ridge Estimator (GRE)

Ordinary Ridge Estimator with K –Bayesian (OREKBAY)

Ordinary Ridge Estimator with K-Lukman and Ayinde (OREKLA)

**Criteria for Evaluation**

Mean Square Error (MSE) defined as:

MSE (i) = Bias(i)2 + Var(i) (13)

1. **Data Analysis and Results**

The table showing the Mean Square Error (MSE) at different sample sizes and levels of multicollinearity with Normally and Uniformly distributed Regressors are in tables 1 and 2





The table below shows the frequency of the Rank of the best Five (5) estimators at different levels of multicollinearity with Normally and Uniformly Regressors.

**Table 1: Ranks of the Best 5 Estimators when there is Multicollinearity Problem**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | **Normally Distributed Regressors** | | **Uniformly Distributed Regressors** | |
| Multicollinearity. Levels | N | Estimators | Frequency | Estimators | Frequency |
| **Low**  **(0.2 – 0.4)** | 10 | GRE  OREKBAY  CORCMGRE  MLMGRE  MLMOREKBAY | 2  2  2  2  2 | OREKBAY  OREKLA  CORCMOREKLA  MLMOREKBAY  MLMOREKLA | 2  2  2  2  2 |
| 20 | GRE  OREKBAY  OREKLA  CORCMOREKBAY  MLMOREKBAY  MLMOREKLA | 1  2  2  2  2  1 | OREKBAY  OREKLA  CORCMOREKBAY  MLMOREKBAY  MLMOREKLA | 2  2  2  2  2 |
| 30 | OREKBAY  OREKLA  CORCMOREKBAY  CORCMOREKLA  MLMOREKLA | 2  2  2  2  2 | OREKBAY  CORCMOREKBAY  CORCMOREKLA  MLMOREKBAY  MLMOREKLA | 2  2  2  2  2 |
| 50 | GRE  OREKBAY  OREKLA  CORCMGRE  CORCMOREKLA  MLMGRE | 2  1  1  2  2  2 | OLSE  GRE  OREKBAY  CORCMGRE  CORCMOREKBAY  MLMGRE  MLMOREKBAY | 1  2  1  2  1  2  1 |
| 100 | OREKBAY  OREKLA  CORCMOREKBAY  CORCMOREKLA  MLMOREKLA | 2  2  2  2  2 | OREKBAY  OREKLA  CORCMOREKBAY  CORCMOREKLA  MLMOREKLA | 2  2  2  2  2 |
| 250 | OREKBAY  CORCMOREKBAY  CORCMOREKLA  MLMOREKBAY  MLMOREKLA | 2  2  2  2  2 | OREKBAY  CORCMOREKBAY  CORCMOREKLA  MLMOREKBAY  MLMOREKLA | 2  2  2  2  2 |
|  |  |  |  |  |  |
| **Moderate**  **(0.6 – 0.8)** | 10 | GRE  OREKBAY  CORCMGRE  MLMGRE  MLMOREKBAY | 2  2  2  2  2 | GRE  OREKBAY  OREKLA  CORCMGRE  CORCMOREKLA  MLMGRE  MLMOREKLA | 1  2  2  1  1  2  1 |
| 20 | GRE  OREKBAY  OREKLA  CORCMOREKBAY  MLMGRE  MLMOREKBAY | 2  2  1  2  1  2 | OREKBAY  OREKLA  CORCMOREKBAY  CORCMOREKLA  MLMOREKBAY  MLMOREKLA | 2  2  2  1  1  2 |
| 30 | GRE  OREKBAY  CORCMGRE  CORCMOREKBAY  MLMOREKBAY | 2  2  2  2  2 | GRE  OREKBAY  CORCMGRE  CORCMOREKBAY  CORCMOREKLA  MLMGRE  MLMOREKBAY  MLMOREKLA | 1  2  1  2  1  1  1  1 |
| 50 | GRE  OREKBAY  CORCMGRE  CORCMOREKBAY  MLMGRE  MLMOREKBAY | 2  2  2  2  1  1 | GRE  OREKBAY  CORCMGRE  CORCMOREKBAY  CORCMOREKLA  MLMGRE  MLMOREKLA | 2  1  2  1  1  2  1 |
| 100 | OREKBAY  CORCMGRE  CORCMOREKBAY  MLMGRE  MLMOREKBAY | 2  2  2  2  2 | OREKBAY  OREKLA  CORCMOREKBAY  CORCMOREKLA  MLMOREKBAY | 2  2  2  2  2 |
| 250 | GRE  OREKBAY  OREKLA  CORCMGRE  CORCMOREKBAY  CORCMOREKLA  MLMGRE  MLMOREKBAY | 1  2  1  1  2  1  1  1 | OREKBAY  CORCMOREKBAY  CORCMOREKLA  MLMOREKBAY  MLMOREKLA | 2  2  2  2  2 |
|  |  |  |  |  |  |
| **High**  **(0.95– 0.99)** | 10 | GRE  OREKBAY  CORCMOREKBAY  MLMGRE  MLMOREKBAY | 2  2  2  2  2 | GRE  OREKBAY  CORCMGRE  CORCMOREKBAY  MLMGRE  MLMOREKBAY | 2  2  1  1  2  2 |
| 20 | GRE  OREKBAY  CORCMOREKBAY  MLMGRE  MLMOREKBAY | 2  2  2  2  2 | GRE  OREKBAY  CORCMGRE  CORCMOREKBAY  MLMGRE  MLMOREKBAY | 2  2  1  1  2  2 |
| 30 | GRE  OREKBAY  CORCMGRE  CORCMOREKBAY  MLMGRE  MLMOREKBAY | 1  2  2  2  1  2 | GRE  OREKBAY  CORCMGRE  CORCMOREKBAY  MLMGRE  MLMOREKBAY | 2  2  2  2  1  1 |
| 50 | GRE  OREKBAY  CORCMGRE  CORCMOREKBAY  MLMOREKBAY | 2  2  2  2  2 | GRE  OREKBAY  CORCMGRE  CORCMOREKBAY  CORCMOREKLA  MLMGRE  MLMOREKBAY | 2  2  2  1  1  1  1 |
| 100 | GRE  OREKBAY  CORCMOREKBAY  MLMGRE  MLMOREKBAY | 2  2  2  2  2 | GRE  OREKBAY  OREKLA  CORCMGRE  CORCMOREKBAY  CORCMOREKLA  MLMOREKBAY | 1  2  1  1  2  1  2 |
| 250 | GRE  OREKBAY  CORCMGRE  CORCMOREKBAY  MLMOREKBAY | 2  2  2  2  2 | OREKBAY  OREKLA  CORCMOREKBAY  CORCMOREKLA  MLMOREKBAY  MLMOREKLA | 2  2  2  2  1  1 |

**3.1 Results of the analysis**

**Low Level of Multicollinearity**

With normally distributed regressors, the best estimator with highest frequency at different sample sizes are OREKBAY, MLMOREKBAY and CORCMOREKBAY.

With uniformly distributed regressors, the results of the analysis does not deviate much with the best estimators in normally distributed regressors except with addition of CORCMOREKLA which also compete favorably well. This is an indication that the above-named estimators can be used to address the problem of multicollinearity.

**Moderate Level of Multicollinearity**

With normally distributed regressors OREKBAY outperformed all other estimators. However the proposed modified ridge estimators like MLMOREKBAY, CORCMOREKBAY and MLMGRE also compete favorably well at different sample sizes.

With uniformly distributed regressors, while OREKBAY is still the best estimator CORCMOREKBAY and CORCMOREKLA compete favorably well among different sample sizes.

**High Level of Multicollinearity**

With normally distributed regressors the proposed modified ridge estimators of MLMOREKBAY and CORCMOREKBAY along with the existing estimator perform excellently well at all sample sizes.

With uniformly distributed regressors OREKBAY, MLMOREKBAY and CORCOREKBAY are the best estimators at different sample sizes. CORCMOREKLA also compete favorably well at sample size of n=50,100 and 250.

This study will best be illustrated with the aid of line graphas shown below. The levels of multicollinearity are categorize as Low (0.2 – 0.4), Moderate (0.6 – 0.8) and High (0.95 – 0.99) under Normal and Uniform Regressors.

Figure 1: MSE at Low Levels of Multicollinearity with Normal Regressors

From Fig.1 above, the best estimator is OREKBAY follows by the proposed modified estimators of MLMOREKBAY and CORCOREKLA.

Figure 2: MSE at Moderate Levels of Multicollinearity with Normal Regressors

From Fig.2 above, the best estimator is the proposed modified ridge estimators of MLMOREKBAY and CORCMOREKBAY. However, the existing estimator OREKBAY also compete favorably well.

Figure 3: MSE at High Levels of Multicollinearity with Normal Regressors

From Fig.3 above, the best estimators are MLMOREKBAY, CORCMOREKBAY and OREKBAY.

Figure 4: MSE at Low Levels of Multicollinearity with Uniform Regressors

The figure 4 above shows that the proposed estimators MLMOREKBAY perform excellently well follows by CORCMOREKBAY and CORCMOREKLA

Figure 5: MSE at Moderate Levels of Multicollinearity with Uniform Regressors

From Fig.5 above, the best estimator is the proposed modified ridge estimators of CORCMOREKBAY. However, MLMOREKLA and CORCMOREKLA are best at sample size of n= 10,20,30,50, and 250.

Figure 6: MSE at High Levels of Multicollinearity with Uniform Regressors

From Fig.6 above, the best estimator are GRE, OREKBAY, MLMOREKBAY and CORCMOREKBAY. However, at sample size of n=50,100 and 250 CORCMOREKLA do compete as the best.

**Conclusions**

A study on modified ridge estimators for solving multicollinearity in Linear Regression Model shows from the table that the best estimators among the ranks of the best five (5) estimators are categorize in to Low, Moderate and High Levels Multicollinearity. The study also revealed that among the proposed modified ridge estimator of MLMOREKBAY, CORCMOREKBAY, CORCMOREKLA and MLMGRE perform excellently well at different sample sizes and can therefore be used to solve problem of multicollinearity. Also, the existing estimator OREKBAY compete favorably well in addressing the problem of multicollinearity.

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