DESIGNING BAYESIAN TWO-SIDED GROUP CHAIN SAMPLING PLAN USING BETA-POISSON DISTRIBUTION MODEL: ANALYTICAL APPROACH

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ABSTRACT: Acceptance sampling is an important technique in quality assurance; its main goal is to achieve the most accurate decision in accepting lot using minimum resources. In practice, this often translates into minimizing the required sample sizes for the inspection, while satisfying the maximum allowable risks by consumer and producer. Numerous sampling plans have been developed over the past decades, the most recent being the incorporation of grouping to enable simultaneous inspection in the two-sided chain sampling which considers information from preceding and succeeding samples. This combination offers improved decision accuracy with reduced inspection resources. To-date, two-sided group chain sampling plan [TSGCSP] has only been explored for Beta-Poisson distribution. This research introduces TSGCSP sampling plan for products with lifetime and focuses on minimizing consumer's risk and operates with three acceptance criteria. The equations that derived from the set conditions involving Beta and Poisson distributions are mathematically solved to develop this sampling plan. Its performance is measured on the probability of lot acceptance and number of minimum groups. A comparison with the established new two-sided group chain (NTSGCh) indicates that the developed TSGCSP sampling plan performs better in terms of sample size requirement and consumers' protection. Thus, this new acceptance sampling plan will reduce the inspection time, resources, and costs via smaller sample size (number of groups), while providing the desired consumers' protection.

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1. INTRODUCTION

In modern manufacturing and service industries, maintaining consistent quality is essential for competitiveness, customer satisfaction, and regulatory compliance. Organizations often face the challenge of determining whether to accept or reject a production lot when it is impractical or too costly and time wasting to inspect every single item. In such cases, acceptance sampling plans are employed as statistical decision-making tools. Sampling plans are integral to quality control in manufacturing and industrial processes, providing a structured approach to decision-making about the acceptance or rejection of product batches based on sample inspection. As opposed to a 100% inspection of all units, which is often impractical due to time and cost constraints, sampling plans offer an efficient and statistically sound method to assess product quality by examining only a subset of items within each lot. The development of sampling plans has evolved to address various production contexts, balancing the risks between producers (producers' risk) and consumers (consumers' risk) and enabling quality standards to be met without exhaustive inspection. Classical sampling plans include single, double, and sequential sampling, each with its unique strengths in managing cost, accuracy, and risk. Single sampling plans, for instance, involve inspecting a fixed sample size from each lot and making an acceptance or rejection decision based on a predefined criterion. Double sampling plans, introduced by Dodge and Romig (1959), allow for a second sample if the decision based on the first sample is inconclusive, thus providing greater flexibility at the cost of a slightly more complex process. Sequential sampling plans further extend this flexibility, as they continue sampling until a decision threshold is met, optimizing the sample size needed and reducing costs when defects are low. The limitations of traditional sampling methods, particularly in environments where defect rates are variable across production lots, have led to the development of advanced methods, such as group and chain sampling plans. These plans leverage data from prior lots or chains of samples to improve the accuracy and reliability of quality decisions. Group chain sampling plans (GChSPs), for instance, reduce both producer and consumer risks by incorporating information from previous batches into the current lot's sampling criteria. This technique is particularly valuable in environments with substantial variability or where batch-to-batch defect rates fluctuate, as it provides a more adaptive approach to quality control. In modern industrial contexts where product variability and cost efficiency are paramount, developing sampling plans that account for variable defect rates has become a significant area of research. This is where probability models such as the Gama-Poisson distribution, Beta-Poisson and some other distributions come to play. By integrating these models into sampling frameworks, it is possible to develop plans that more accurately reflect process variations, thus minimizing the likelihood of incorrect quality decisions.

In quality control, acceptance sampling plans serve as essential tools for making informed decisions on whether to accept or reject a lot based on the inspection of a subset of items. Traditional sampling methods, including single, double, and sequential sampling, have been widely used to balance inspection costs with the need for quality assurance. However, these methods assume a constant defect rate and do not account for variability in defect probabilities across different production batches. To address this limitation, more advanced methods, such as group chain sampling plans (GChSPs) have been developed which use information from prior lots or sample chains to enhance decision accuracy and efficiency. The group chain sampling plan (GChSP) was initially developed to improve decision-making in acceptance sampling by grouping multiple samples and evaluating them in sequence, thereby reducing both producer and consumer risks. This method leverages historical data from prior lots, making it particularly valuable in scenarios where defect rates are subject to variation. Building upon this, the two-sided group chain sampling plan (two-sided GChSP) introduces a dual threshold system, establishing both acceptance and rejection criteria that ensure more balanced decision-making. In contrast to traditional single-threshold plans, the two-sided GChSP allows for a more nuanced response by incorporating upper and lower decision limits, which can better control quality risks on both sides of the decision-making spectrum. Two-sided GChSPs are especially applicable in production environments where defect rates vary between lots, often due to changing process conditions or material inconsistencies. By setting dual criteria, these plans mitigate the risk of erroneously accepting defective lots (consumer's risk) or rejecting acceptable ones (producer's risk), providing a robust solution for quality control in variable conditions. Furthermore, two-sided GChSPs align well with probabilistic models such as the Gamma-Poisson distribution, which account for over-dispersed data common in industrial settings where defect probabilities differ from lot to lot. Integrating the Gamma-Poisson distribution within a two-sided GChSP allows for the modeling of batch-to-batch defect rate variability, offering a powerful method to improve decision accuracy and minimize inspection costs in complex manufacturing environments. The present research focuses on a specialized approach using Bayesian two-sided group chain sampling plan for the Gamma-Poisson distribution with a deliberate emphasis on minimizing producer's and consumer's risk. It also aim to advance quality control methods by developing a Bayesian two-sided GChSP based on the Gama-Poisson distribution model. This approach not only enhances the flexibility and reliability of acceptance sampling but also addresses the limitations of traditional methods in the presence of variable defect rates. The proposed method offers a comprehensive solution for industries that require high standards of quality control while navigating fluctuating production conditions.

2. REVIEW OF RELATED LITERATURE

Classical acceptance sampling is built around the operating characteristic (OC) curve and two benchmark quality points, the acceptable quality level (AQL) and the lot tolerance percent defective (LTPD). In this framework, the producer's risk (α) is the probability of rejecting a lot at the AQL, and the consumer's risk (β) is the probability of accepting a lot at the LTPD; these definitions and their use to tune sample sizes and acceptance numbers are standard in the handbooks and modern software that practitioners rely on (NIST; Schilling and Neubauer). NIST's treatment shows how designers pick the OC points $(P_1, 1-\alpha)$ and (P_2, β) with P_1 at AQL and P_2 at LTPD, while Schilling and Neubauer's textbook consolidates the historical development and the role of OC/ASN summaries in plan selection. Bayesian treatments argue that these long-run guarantees should be complemented by lot-specific risk statements that condition on the data actually observed. For attribute counts, the usual modeling choice is Poisson for the number of nonconformities, together with a conjugate Gamma prior for the (unknown) lot rate λ. The Gamma–Poisson hierarchy yields closed-form updating (Gamma posterior) and a negative-binomial posterior predictive for future counts, which naturally accommodates over-dispersion common in industrial data. This conjugate structure and the fact that the posterior predictive is negative-binomial is shown in many places, from applied Bayesian lecture notes and the Stan User's Guide to expository sources that derive the negative-binomial mixture explicitly. These sources also emphasize why the predictive variance inflates relative to a fixed λ Poisson, a point of practical importance when incoming quality varies. (Bayes Rules!; Stan User's Guide; Hitchcock notes; CPrior formulas.)

Under this Gamma–Poisson model, the posterior producer's risk can be expressed as the posterior probability that the lot really meets the good-quality standard when a reject decision is taken, and the posterior consumer's risk as the posterior probability that the lot really violates the bad-quality standard when an accept decision is taken. Because $\lambda|$data is Gamma and the predictive for future counts is negative-binomial, these risks reduce to straightforward tail probabilities. Contemporary plan designers often adopt one of two strategies: (i) risk-constrained optimization, where plan parameters are tuned so that posterior risks at AQL/LTPD analogues do not exceed specified caps while minimizing expected sample number; or (ii) loss-based optimization, where a weighted sum of "reject good," "accept bad," and inspection cost is minimized.

Bayesian plans live or die on how credibly prior information about lot quality is elicited and stress-tested. In practice, many authors advocate empirical Bayes (estimate the Gamma prior from recent comparable runs) or structured expert elicitation (match Gamma hyperparameters to plausible quantiles), and then recommend sensitivity analysis across a family of priors to ensure that α/β control is robust. The same conjugate algebra that powers the design also makes this sensitivity cheap to compute, and textbooks and tutorials underline the transparency of Gamma–Poisson updating. (Schilling and Neubauer; Bayes Rules!; Stan User's Guide.)

To understand where Bayesian acceptance plans for counts fit in, it helps to recall the evolution of dependent plans. Dodge's chain sampling (ChSP-1) introduced the idea that the current lot's decision could depend on a short memory of prior results, steepening the OC without a proportional increase in sample size; later, modified chain designs examined whether always using past results could cut sample sizes further. Govindaraju and Lai (1998), who proposed MChSP-1, showed that unconditional use of memory indeed pushes sample sizes down but can make plans sensitive to trends in incoming quality. That trade off efficiency versus brittleness under non-stationary continues to inform how memory is used in Bayesian chain and group-chain designs (short memory, sometimes with reset or discounting). These ideas and their documented pros/cons are part of the standard chain-sampling narrative cited by modern authors. (Govindaraju & Lai, 1998

The Gamma–Poisson framework also clarifies how to build group and chain structure into Bayesian plans. If groups are inspected sequentially, the posterior updates after each group simply add the group's count to the Gamma shape and the group's exposure to the Gamma rate, and the next group's predictive is again negative-binomial with updated parameters; this is why grouped inspection can be tuned by simple root-finding on cumulative predictive tails to hit two-sided risk caps. If a short memory is allowed—as in chain or group-chain plans the preceding group(s) can be folded into the prior as pseudo-data or carried explicitly via a discounted updating scheme; the mathematics is invariant and the effect is conceptual clarity: "memory" becomes weighted prior information. This interpretation appears explicitly in recent Bayesian group-chain papers for counts, where one or two steps of memory are used to stabilize decisions without inheriting the trend sensitivity documented for deep memory in modified chain plans (Govindaraju & Lai, 1998; Hafeez, Aziz & Du, 2023). In summary, the Gamma–Poisson (Poisson–Gamma / negative-binomial) modeling frame provides the algebraic backbone for contemporary Bayesian acceptance sampling on counts. It supports transparent prior elicitation, closed-form predictive calculations for APA/OC and posterior risks, efficient search over group/chain/two-sided design spaces, and practical robustness analysis—all while aligning with the way nonconformities actually vary across lots. That combination of realism, tractability, and communicability explains why Gamma–Poisson has become the default engine for two-sided group-chain and double-group plans aimed at minimizing both producer's and consumer's risks (Aslam et al., 2012; Vijayaraghavan & Sakthivel, 2011; Hafeez et al., 2022–2025).

2. 0 Producer's and Consumer's Risks

In the classical vocabulary of acceptance sampling, the producer's risk (α) is the probability of rejecting a lot at the acceptable quality level (AQL) and the consumer's risk (β) is the probability of accepting a lot at the lot tolerance percent defective (LTPD, also called RQL or LQL). Plans are commonly selected so that the operating characteristic (OC) curve passes near the two points (AQL,$1-\alpha$) and (LTPD,β), and summary metrics like the average sample number (ASN) and average outgoing quality (AOQ) are reported for governance (NIST Handbook; Minitab Help; JMP Help). This two-point OC tradition provides the baseline against which modern Bayesian plans communicate their advantages and guarantees.

Bayesian designs reformulate these risks conditionally on observed data. If the lot is accepted, the posterior consumer's risk is the posterior probability that the lot actually lies in the "bad" region; if the lot is rejected, the posterior producer's risk is the posterior probability that the lot actually lies in the "good" region. Under a Gamma–Poisson model for counts, both quantities reduce to Gamma or negative-binomial tail probabilities, which can be computed exactly. Authors therefore propose two design philosophies. The first is risk-constrained design: fix targets (α^*, β^*)

and tune plan parameters sample sizes, group sizes, memory depth, and (for two-sided rules) dual thresholds so that posterior risks at designated good/bad count rates do not exceed the targets while minimizing ASN. The second is loss-based design: minimize a weighted sum of posterior error probabilities and inspection cost, with weights chosen to reflect asymmetric consequences in the application (Calvin, 1990; Hafeez et al., 2022–2025).

Minimizing both risks in practice leads to a few recurring design choices. First, designers must select benchmark quality points: in the classical language these are AQL and LTPD; in the counts language they are "good" and "bad" Poisson rates. Papers usually justify these anchors with historical performance or contractual tolerances, then present OCs in both parameterizations so that procurement can read the results either way (NIST Handbook; Montgomery, 2009). Second, there is a choice of memory depth. Following Govindaraju & Lai's demonstration that "always-use" memory can be sensitive to trends, Bayesian two-sided group-chain papers typically fix memory at one or two steps, sometimes with an explicit reset after a process intervention. This keeps discrimination high without allowing slow drift to bias decisions (Govindaraju & Lai, 1998; Hafeez, Aziz & Du, 2023). Third, plans balance risk constraints with ASN goals. Because APA/OC and posterior risks are available in closed form under Gamma–Poisson, authors frequently show Pareto frontiers: designs with very small ASN but looser risk margins versus designs that meet tight α/β caps with a few extra observations; the chosen point reflects the application's economics (Hafeez et al., 2022; Aslam et al., 2012).

Two-sided group-chain papers also standardize reporting for audit and adoption. A typical article states the operating steps, lists the chosen parameters (number of groups, group sizes, memory depth, dual thresholds), and then publishes (i) OC curves with AQL/LTPD annotations, (ii) ASN tables over a range of true qualities, and (iii) posterior risk tables at the design anchors. This three-way reporting mirrors classical guidance while adding the conditional guarantees that decision-makers increasingly demand. Open-access exemplars, such as the Bayesian Double Group Sampling Plan (BDGSP) for Poisson counts with a Gamma prior, have helped normalize this style by providing readers with a fully worked, reproducible case where α is tied to AQL and β is tied to LQL/LTPD in the figures and narrative (Hafeez, Aziz & Du, 2025). Empirical-Bayes calibration on a recent window of lots is therefore common, as is reporting sensitivity bands under alternative hyperparameters. Some papers go further and design to the worst case within a plausible prior family or average performance over several priors to reflect stakeholder disagreement; because all calculations are analytic under Gamma–Poisson, such robust-Bayes analyses can be done without prohibitive cost (Calvin, 1990; Aslam et al., 2012).

Where zero inflation or multiple streams complicate the count distribution, authors adapt the risk-constrained design with zero-inflated Poisson or mixture-Poisson assumptions and re-tabulate OCs and ASNs under the new model, preserving the producer/consumer-risk semantics (Loganathan & Kandaswamy, 2014; Loganathan et al., 2014). Where serial correlation is a concern, short memory plus resets is preferred to deep memory; designers sometimes augment the plan with simple process-control diagnostics so that threshold calibration is revisited after maintenance or supplier changes (Govindaraju & Lai, 1998).

Finally, the literature underscores that minimizing both risks does not require abandoning classical communication. Authors explicitly label RQL/LQL as synonyms for LTPD and plot the classical OC curve alongside Bayesian posterior-risk summaries to ensure that stakeholders see continuity rather than a rupture in standards. Tooling reflects the same bridge: popular software help pages explain α/β, AQL/LTPD, and OC plots in terms that map directly onto the figures reported in two-sided Bayesian papers, lowering barriers to adoption (NIST; Minitab; JMP). In that sense, the contribution of modern two-sided group-chain work is not only statistical short-memory, packetized evidence, Gamma–Poisson predictive but also rhetorical: it provides a way to guarantee both producer's and consumer's protections in conditional, lot-specific terms while still speaking the classical AQL/LTPD language that procurement and regulators expect.

Two-Sided Group-Chain

Chain sampling's empirical value proposition originates with Dodge's ChSP-1: use a short history of lot results to sharpen discrimination when acceptance numbers are small and testing is expensive or destructive (Dodge, 1955). Soundararajan (1978) transformed that idea into practice by publishing procedures and tables for construction and selection, allowing engineers to choose memory depth and sample sizes that meet OC targets while controlling ASN. Subsequent lineage papers and bibliographies establish that chain plans sit alongside single, double, and multiple plans in the practitioner's toolkit because they deliver steeper OCs with modest additional logic. Govindaraju & Lai (1998) supply the empirical cautionary tale via MChSP-1: always using history can achieve very small samples, but such deep dependence increases sensitivity to trends in incoming quality. Their tests under monotone drift remain the standard justification for limiting memory in modern dependent plans. Variables-data extensions reinforce the structural nature of this trade-off in normal-theory settings.

Group acceptance sampling develops on a parallel track, motivated by the savings from early stopping. Aslam & Jun (2009) show in a time-truncated life-test setting that grouping into packets with accept/continue/reject decisions after each group can hit the same α/β protections with lower ASN. The reporting pattern state the rule, publish OC/ASN, provide selection tables became a template that attribute-count designers adopted as computing made multi-parameter calibration routine.

3. METHODOLOGY

This chapter presents the methodology of developing a Bayesian Two-sided Group Chain Sampling Plan (GChSP) under the Gamma-Poisson framework. The Gamma-Poisson distribution naturally arises when the Poisson mean is itself distributed according to a gamma prior, making it the Bayesian predictive distribution for the future sample counts of nonconformities. The goal is to derive a plan that simultaneously controls producer's risk and consumer's risk while minimizing the Average Sample Number (ASN).

3.1 Group Sampling Model

Let the number of nonconformities X in a unit follow a Poisson distribution

$$X/\lambda \sim \text{Poisson}(\lambda).$$

For a group of independent units, the group total is

$$Y/\lambda \sim \text{Poisson } g(\lambda). \quad \text{Where } y = 0, 1, 2\ldots$$

Conjugate prior- Adopt a gamma prior

$$\lambda \sim \text{Gamma } (\alpha_0, \beta_0)$$

$$\pi(\lambda) = \frac{\beta_o^{\alpha_0}}{\Gamma(\alpha_p)} \lambda^{\alpha_0 - 1} e^{-\beta_0 \lambda} \tag{1}$$

α_0 and β_0 represent prior knowledge from historical inspection data.The prior is equivalent to pseudo-data with total nonconformities α_0 observed over β_0 groups.

Posterior Distribution

Suppose historical or prior data summarize to a total of Σy nonconformities across "n" units (n past groups of size g). The likelihood is proportional to

$$\lambda^{\Sigma y} e^{-n\lambda} \tag{2}$$

Proof: By multiplying the prior density by the likelihood function.

$$\pi(\lambda/D)\, \alpha \lambda^{\alpha_o^{-1}} e^{\beta_o} \lambda \; x \; \lambda^{\Sigma y} e^{-n\lambda}$$
$$= \lambda^{(\alpha_o + \Sigma y) - 1} e^{-(\beta_o + n)\lambda} \tag{3}$$

For a future group δ_i of size g, marginalizing λ gives:

$$P_r(Y^* = y/D) = \int_o^\infty P_r\,(Y^* = y/\lambda)$$

$$\pi(\lambda/D)\, \alpha\lambda = \int_o^\infty \frac{e^{-g\lambda}(g\lambda)^y}{y!} \frac{\beta_p^{\alpha p}}{\Gamma(\alpha_p)} \lambda^{\alpha_p - 1} e^{\beta_p \lambda}\, d\lambda \tag{4}$$

$$= \frac{g^y \beta_p^{\alpha_p}}{y!\,\Gamma(\alpha_p)} \int_o^\infty \lambda^{y + \alpha_p - 1} e^{-(\beta_p + g)\lambda}\, d\lambda$$

$$= \frac{g^y \beta_p^{\alpha_p}}{y!\,\Gamma(\alpha_p)} x \frac{\Gamma(y + \alpha_p)}{(\beta_p + g)^{y + \alpha_p}}$$

$$= \frac{\Gamma(y + \alpha_p)}{y!\,\Gamma(\alpha_p)} \left(\frac{\beta_p}{\beta_p + g}\right)^{\alpha_p} \left(\frac{g}{\beta_p + g}\right)^y$$

$$P_r(Y^* = y/D) = \frac{\Gamma(y + \alpha_p)}{y!\Gamma(\alpha_p)} \left(\frac{\beta_p}{\beta_p + g}\right)^{\alpha_p} \left(\frac{g}{\beta_p + g}\right)^y \tag{5}$$

where $P_p = \left(\frac{\beta_p}{\beta_p + g}\right)$; $1 - P_p = \left(\frac{g}{\beta_p + g}\right)$

$$P_r(Y^* = y/D) = \frac{\Gamma(y + \alpha_p)}{y!\Gamma(\alpha_p)} P_p^{\ \alpha_p} \left(1 - P_p\right)^y \tag{6}$$

Hence

Y^*/D **~Gamma Poisson (α_p, β_p)** with mean

$$E[\mathbf{Y}^*/\mathbf{D}] = g\,\frac{\alpha_p}{\beta_p} \qquad (7)$$

$$Var[\mathbf{Y}^*/\mathbf{D}] = g\,\frac{\alpha_p}{\beta_p}\left(1+\frac{g}{\beta_p}\right) \qquad (8)$$

3.2 Group Chain Sampling Plan (GChSP)

Decision Rules ; For lot t with group total $\mathbf{Y}_t^*$, Accept if $\mathbf{Y}_t^* \le$ c

If $\mathbf{Y}_t^* = \mathbf{c+1}$ → Accept only if lot (t-1,…. , t – i) was accepted, Reject if $\mathbf{Y}_t^* \ge \mathbf{c+2}$

Probabilities of Acceptance Events

For any governing distribution of $\mathbf{Y}_t^*$, define $\mathbf{P_1 = P_r(Y^* \le c)}$ $\mathbf{P_2 = P_r(Y^* = c + 1)}$

Under conditional Poisson $(\mu = g\lambda)$:

$$\boldsymbol{P_1^F(g,c/\lambda) = \sum_{y=0}^{c} e^{-\mu}\frac{\mu^y}{y!}} \qquad \mathbf{(9)}$$

Replacing y with c+1 gives equation (10)

$$\boldsymbol{P_2^F(g,c/\lambda) = e^{-\mu}\frac{\mu^{c+1}}{(c+1)!}} \qquad \mathbf{(10)}$$

Under Gamma-Poisson predictive:

$$\boldsymbol{P_1^B(g,c/D) = \sum_{y=0}^{c}\frac{\Gamma(y+\alpha_p)}{y!\,\Gamma(\alpha_p)}\left(\frac{\beta_p}{\beta_p+g}\right)^{\alpha_p}\left(\frac{g}{\beta_p+g}\right)^{y}} \qquad \mathbf{(11)}$$

$$\boldsymbol{P_{2i}^B(g,c/D) = \frac{\Gamma(c+i+\alpha_p)}{(c+i)!\,\Gamma(\alpha_p)}\left(\frac{\beta_p}{\beta_p+g}\right)^{\alpha_p}\left(\frac{g}{\beta_p+g}\right)^{c+1}} \qquad \mathbf{(12)}$$

3.3 Operating Characteristic (OC) Function

The steady-state probability of acceptance under GChSP is:

$$\mathrm{P_a} = \frac{\boldsymbol{p_1}}{\boldsymbol{1-p_2}}$$

Let A_t denote acceptance of lot t. Then: $\mathbf{P_r(A_t) = P_r(Y_t^* \le c) + P_r(Y_t^* = c + 1)}$

$\mathbf{P_r(A_{t-1}) = p_1 + p_2 p_r(A_{t} - i)}$

At equilibrium, $\mathbf{P_r(A_t) = P_r(A_{t-1}) = P_a}$, giving that

$$\mathrm{P_a} = \boldsymbol{p_1 + p_2 p_a \rightarrow p_a = \frac{p_1}{1-p_2}} \qquad (13)$$

Uniqueness follows since $\mathbf{0 \le p_2 \le 1}$, implies contraction

Two-Sided Risks

Producer's Risk (for AQL λ_1): $\boldsymbol{\alpha = 1 - P_a^F(g, c \mid \lambda_1) = 1 - \frac{P_1^F(g,c/\lambda_1)}{1-P_2^F(g,c/\lambda_1)}}$ **(14)**

Consumer's Risk (for $\lambda_2 > \lambda_1$):

For LQL λ_2, $\beta = \mathbf{P_a^F}(g, c \mid \lambda_2) = \boldsymbol{1 - \frac{P_1^F(g,c/\lambda_2)}{1-P_2^F(g,c/\lambda_2)}}$ **(15)**

Optimization Problem

Objective and Constraints:

The design seeks (g, c) minimizing the Average Sample Number (ASN = g) subject to:

$\mathbf{P_a^F}(g, c \mid \lambda_1) \ge 1 - \alpha$

$\mathbf{P_a^F}(g, c \mid \lambda_2) \le \beta$

3.4 Brief description of the newly developed Two-Sided Group Chain [NTSGC] Sampling Plan using Gamma-Poisson Distribution Model

Table 1: Probability of lot acceptance for generalized exponential distribution when λ = 1

			$\frac{\mu}{\mu 0}$						
β	g	α	1	2	4	6	8	10	12
0.01	5	0.25	0.0037	0.0861	0.3512	0.5303	0.6402	0.7115	0.7606
	3	0.50	0.0011	0.0490	0.2738	0.4558	0.5759	0.6567	0.7135
	2	0.75	0.0012	0.0515	0.2793	0.4608	0.5799	0.6599	0.7161
	2	1.00	0.0001	0.0153	0.1629	0.3317	0.4608	0.5546	0.6236
	1	1.25	0.0061	0.1052	0.3784	0.5519	0.6565	0.7239	0.7703
	1	1.50	0.0018	0.0607	0.2972	0.4766	0.5925	0.6698	0.7239
	1	1.75	0.0006	0.0346	0.2315	0.4092	0.5324	0.6176	0.6787
	1	2.00	0.0002	0.0195	0.1790	0.3495	0.4766	0.5679	0.6348
0.05	3	0.25	0.0490	0.2738	0.5759	0.7135	0.7868	0.8313	0.8609
	2	0.50	0.0153	0.1629	0.0515	0.6236	0.7161	0.7739	0.8130
	2	0.75	0.0012	0.0515	0.1790	0.4608	0.5799	0.6599	0.7161
	1	1.00	0.0195	0.0153	0.4766	0.6348	0.7239	0.7797	0.8174
	1	1.25	0.0061	0.1052	0.3784	0.5519	0.6565	0.7239	0.7703
	1	1.50	0.0018	0.0607	0.2972	0.4766	0.5925	0.6698	0.7239
	1	1.75	0.0006	0.0346	0.2315	0.4092	0.5324	0.6176	0.6787
	1	2.00	0.0002	0.0195	0.1790	0.3495	0.4766	0.5679	0.6348
0.10	3	0.25	0.0490	0.2738	0.5759	0.7135	0.7868	0.8313	0.8609
	2	0.50	0.0153	0.1629	0.4608	0.6236	0.7161	0.7739	0.8130
	1	0.75	0.0607	0.2972	0.5925	0.7239	0.7938	0.8363	0.8646
	1	1.00	0.0195	0.1790	0.4766	0.6348	0.7239	0.7797	0.8174
	1	1.25	0.0061	0.1052	0.3784	0.5519	0.6565	0.7239	0.7703
	1	1.50	0.0018	0.0607	0.2972	0.4766	0.5925	0.6698	0.7239
	1	1.75	0.0006	0.0346	0.2315	0.4092	0.5324	0.6176	0.6787
	1	2.00	0.0002	0.0195	0.1790	0.3495	0.4766	0.5679	0.6348

Table 2: Comparison of number of minimum groups when λ=1,2,3

				$\lambda = 1$		λ=2		λ=3	
β	**i**	**r**	**α**	**TSGCSP**	**NTSGCH**	**TSGCSP**	**NTSGCH**	**TSGCSP**	**NTSGCH**
0.10	1	2	0.25	5	5	3	23	2	102
			0.50	3	3	2	7	1	18
			0.75	2	2	1	4	1	8
			1.00	2	2	1	3	1	4
			1.25	1	1	1	2	1	3
			1.50	1	1	1	2	1	2
			1.75	1	1	1	2	1	2
			2.00	1	1	1	1	1	2

As anticipated, the probability of lot acceptance raises with the mean ratio that indicates the product quality. For instance, looking at the first row at β = 0.01, g = 5 , α =0.25,when the mean ratio increases from 1 to 12, the probability of lot acceptance also increases from 0.0037 to 0.7606, respectively. This means that when the product is of its best quality. In this case the actual product lifetime is 12 times its average lifetime, the chances of accepting the lot using this sampling plan increases to 76%, compared to only 0.37% if the actual lifetime is equivalent to the average lifetime. The same increasing trend is evident when reading across horizontally on any rows in Table 1. Table 2and Table 5 outline the results for TSGCSP and NTSGCh based on the number of minimum groups and the lot acceptance probability, respectively. Upon inspection of Table 4, there is no difference in the number of minimum groups between TSGCSP and NTSGCh when λ=1, suggesting that both plans consume the same amount of resources. However, when

λ=2,3, TSGCSP provides a smaller number of groups in comparison to the established NTSGCh. For example, in the first row when λ=2 and the design parameter (β, *i*, *r*, *a*) is (0.01,1,2,0.25), the producer only needs 3 groups when implementing TSGCSP plan, while it will need 23 groups if NTSGCh is chosen. The smaller the number of groups, the shorter the inspection time which translates into lower costs. This finding suggests that our developed TSGCSP performs better than the NTSGCh in providing protection to consumers.

4. CONCLUSION

The TSGCSP sampling plan based on the Beta-Poisson distribution was successfully developed with the focus on minimizing the consumer's risk. The results indicate that TSGCSP sampling plan is superior to the existing NTSGCh sampling plan, both in terms of the probability of lot acceptance and number of minimum groups. A key implication is that this new acceptance sampling plan is able to reduce the inspection time via smaller sample size (number of groups) which consequently leads to reduced resources and costs. At the same time, it also provides greater protection for consumers through minimization of chances to accept bad lots which will disrupt manufacturing process and product yield. Another advantage is that this sampling plan can be customized to satisfy specific needs of the organization. Hence, this sampling plan is a worthwhile contribution to the field for both academics and practitioners.

References

Aslam, M., & Jun, C.-H. (2009). A group acceptance sampling plan for a truncated life test having Weibull distribution. *Journal of Applied Statistics*.

Aslam, M., Balamurali, S., Jun, C.-H., & Rasool, M. (2012). Designing of group sampling plans based on Gamma–Poisson distribution. *Journal of Testing and Evaluation*.

Calvin, T. W. (1990). *Quality control applications of Bayesian inference*.

Dodge, H. F. (1955). Chain sampling inspection plans. *Industrial Quality Control*, *11*(4), 10–13.

Dodge, H. F., & Romig, H. G. (1959). *Sampling inspection tables: Single and double sampling*. Wiley.

Gelman, A., Carlin, J. B., Stern, H. S., & Rubin, D. B. (2014). *Bayesian data analysis* (3rd ed.). CRC Press.

Govindaraju, K., & Lai, C.-D. (1998). A modified ChSP-1 chain sampling plan (MChSP-1) with very small sample sizes. *Quality Engineering*.

Hafeez, W., Aziz, N., & Du, K. (2022). Bayesian new group chain sampling plan for Poisson distribution with Gamma prior.

Hafeez, W., Aziz, N., & Du, K. (2023). Designing Bayesian new two-sided group chain sampling plan. *Quality and Reliability Engineering International*.

Hafeez, W., Aziz, N., & Du, K. (2025). Bayesian double group sampling plan for assessing the average number of nonconforming products. *Scientific Reports*.

Hamzah, N., & Khoo, M. B. C. (2015). Bayesian acceptance sampling plans: An overview. *Quality and Reliability Engineering International*, *31*(7), 1109–1120.

JMP. (n.d.). *OC curves, AQL/LTPD, and α/β terminology* [Help documentation].

Johnson, M., Ott, B., & Dogucu, M. (2021). *Bayes rules!* CRC Press.

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). *Univariate discrete distributions* (3rd ed.). Wiley.

Olatunde, O.A., **Akomolafe, A.A**. and Adebola, F.B. (2019) : Monitoring Process Variability using Double Moving Average Control Chart. *Industrial Engineering and Management System. 18 (2), 210-221*. SCOPUS [Elsevier] *ISSN: 1598-7248 (Print) ISSN:2234-6473(Online)* https://doi.org/10.7232/iems.2019.18.2.210